Physics 308 Introduction to Quantum Mechanics Spring 2005

Homework 10, Due AT BEGINNING OF CLASS, Wednesday 13 April

Exploring the uncertainty principle

1. Assume $\psi(x) = Ne^{-(x^2/2a^2)}$. Here N is chosen to give $\langle\psi|\psi\rangle = 1$. Find $\langle\psi|x|\psi\rangle$ and $\langle\psi|x^2|\psi\rangle$. Determine $\int dxe^{-(ipx/\hbar)}\psi(x)$ for $p = 0$.

2. Determine $\tilde{\psi}(p) =\tilde{N} \int dxe^{-(ipx/\hbar)}\psi(x)$ for general p. What is the value of $\tilde{N}$ if $\tilde{\psi}$ is normalized to unity?

3. Now obtain the reverse Fourier transform $f(x') \equiv \int dpe^{i(px'/\hbar)}\tilde{\psi}(p)/\tilde{N}$. Use the ratio of $f(x)$ to $\psi(x)$ to deduce the Fourier integral theorem, $\int dkdxe^{ikx'}e^{-ikx}g(x) = 2\pi g(x')$ for any reasonably smooth function $g(x)$.

Orbital angular momentum

4. From the analogy with classical mechanics, we have the orbital angular momentum components $L_z = xp_y - yp_x, L_x = yp_z - zp_y, L_y = zp_x - xp_z$. Show $[L_x, L_y] = i\hbar L_z$. This suggests (correctly!) $[L_y, L_z] = i\hbar L_x$ and $[L_z, L_x] = i\hbar L_y$.

5. Define $L_\pm \equiv L_x \pm L_y$. Show $[L_z, L_\pm] = \pm\hbar L_\pm$.