# Physics 308 Introduction to Quantum Mechanics Spring 2005

Homework 12 (LAST ONE!), Due AT BEGINNING OF CLASS, Friday 29 April

Hydrogen Atom

1. Prove $\nabla^2 f(r) = f''(r) + 2f'/r$. Use chain rule and product rule for differentiation, and start by showing $\partial_x r = x/r$. You need the definitions $\nabla^2 \equiv \partial_x^2 + \partial_y^2 + \partial_z^2$ and $r = \sqrt{(x^2 + y^2 + z^2)}$.

2. Find the normalization of the ground state hydrogen atom wave function. A simple way is to relate the normalization integral to the simpler integral $\int_0^\infty dx e^{-\lambda x}$.

3. Find the wave function corresponding to the first radial excitation of the ground state, meaning a wave function depending only on $r$, but with a factor $(1 - br)$ multiplying a declining exponential. Because the binding energy must be smaller than for the ground state, the exponential factor must decline more slowly, so you have to find both the new $\kappa$ and the value of $b$. Check that this wave function is orthogonal to the ground state wave function $e^{-r/a}$.

4. Find the lowest hydrogen wave function corresponding to $\ell = 1$. Now the Schrödinger equation has an extra term (the "centrifugal potential" term) $(\hbar^2 \ell(\ell+1)/2mr^2)\psi$. This time the radial dependence of the wave function is $f(r) = re^{-\kappa r}$, where again the value of $\kappa$ must be smaller than for the ground state.

Spin and Statistics

5. Two electrons ($a$ and $b$) both have orbital angular momentum $\ell = 1$ and the same radial wave functions. What simultaneous values of $L$ (= total orbital angular momentum) and $S$ (= total spin) are allowed by the Pauli exclusion principle, also called Fermi-Dirac statistics? Hint: Possible values are $L = 0, 1, 2$ and $S = 0, 1$.