PHY 308 INTRODUCTION TO QUANTUM MECHANICS SPRING 2005

Homework 8 (Due Wednesday 30 March 2005)

Quantum Harmonic Oscillator and Schrödinger and Heisenberg pictures. [Griffiths Ch. 2.3 and Ch..3, p.130]

1. Find the Heisenberg operator equations of motion $dx/dt =?$ and $dp/dt = ?$ for the harmonic oscillator Hamiltonian $H = p^2/2m + kx^2/2$. The Heisenberg equation of motion for an operator $\mathcal{O}$ (with no explicit dependence on time) is $d\mathcal{O}/dt = i[H, \mathcal{O}]/\hbar$, where $[A, B] \equiv AB - BA$. [Use the definition of the momentum operator, $p \equiv -i\hbar\partial_x$.]

2. Use the result of Prob. 1 to find a second-order differential equation in time for the position coordinate operator x: $d^2x/dt^2 =?$, and compare with the corresponding classical equation for the position coordinate.

3. The ground-state wave function of the harmonic oscillator at $t = 0$ is $\langle x|\psi_0\rangle \equiv \psi_0(x) = Ne^{-x^2\sqrt{km}/2\hbar}$, where N is the required normalization constant. Obtain $\langle\psi_0|x|\psi_0\rangle(t)$ and $\langle\psi_0|p|\psi_0\rangle(t)$ for this wave function, using the Schrödinger picture. [Hint: For any operator $\mathcal{O}$ which is a function of x and p, by definition we have $\langle\psi|\mathcal{O}|\psi\rangle(t) \equiv \int dx\psi^*(x, t)\mathcal{O}\psi(x.t)$.]

4. The ground-state wave function of the harmonic oscillator at $t = 0$ is $\langle x|\psi_0\rangle \equiv \psi_0(x) = Ne^{-x^2\sqrt{km}/2\hbar}$, where N is the required normalization constant. Obtain $\langle\psi_0|x|\psi_0\rangle(t)$ and $\langle\psi_0|p|\psi_0\rangle(t)$ for this wave function, using the Heisenberg picture. Does this agree with the result of Prob. 3? [Hint: You can use the results for the expectation values in Prob. 3 at $t = 0$ to give initial conditions for this problem.]

5. Let the wave function at $t = 0$ for the harmonic oscillator be $\psi(x, t = 0)) = Ne^{-(x-a)^2\sqrt{km}/2\hbar}$. Find $\langle\psi|x|\psi\rangle(t = 0)$ and $\langle\psi|p|\psi\rangle(t = 0)$. Now use the Heisenberg equations of motion to obtain these expectation values at later times t. Would this be harder to do in the Schrödinger picture?