**Physics 308 Introduction to Quantum Mechanics Spring 2005**

Homework 9, Due AT BEGINNING OF CLASS, Wednesday 6 April

Algebraic or Heisenberg approach to harmonic oscillator

1. The lowering operator is defined by $a = (\xi + \partial_\xi)/\sqrt{2}$, and the raising operator $a^\dagger$ is its adjoint. Given on the space of eigenstates of the harmonic oscillator that $\xi$ and $-i\partial_\xi$ are self-adjoint, what is $a^\dagger$?

2. What are the commutators $[a^\dagger, a]$, $[a, a]$ and $[a^\dagger, a^\dagger]$?

3. Define $N \equiv a^\dagger a$. What are the commutators $[N, a]$ and $[N, a^\dagger]$?

4. Given that $a^\dagger$ is the adjoint of $a$, show that $N$ cannot be negative. This implies, as explained in class, that the state $|0\rangle$ with the lowest value of $N$ must obey $a|0\rangle = 0$. What is $N$ for this state?

5. Show that the Hamiltonian $(\xi^2 - \partial_\xi^2)/2 = N + 1/2$. Then what is the energy of the lowest state? Does this agree with what you found earlier using the Schrödinger approach?