Phys. 541, Homework Problems 1

Please pass in your solutions at the beginning of class on Wed., Oct. 3.

1. (20 pts)
   Consider an ideal monatomic gas confined to two spatial dimensions. Calculate (i) $C_V$, the molar specific heat at constant volume, for this gas at a temperature $T \simeq 300$ K. (ii) What is the relation between the pressure $p$ and the volume $V$ for adiabatic expansion or compression of this gas?

2. (20 pts)
   Same two questions as above, for an ideal diatomic gas confined to two spatial dimensions.

3. (20 pts)
   Recall the internal energy $U = U(S, V)$, which has the differential $dU = T dS - pdV$. By means of a Legendre transform, construct the Gibbs free energy, $G = G(T, p)$ in terms of $U$ and derive the form of the differential $dG$ (which involves calculating $\partial G/\partial T$ at constant $p$ and $\partial G/\partial p$ at constant $T$).

4. (30 pts)
   Consider a hydrogen-bonded crystal like water ice, but with the difference that the coordination number is 6 rather than 4. Use Pauling’s method, calculate an estimate of the ground state entropy per site for this crystal. How does this compare with the Pauling estimate for water ice?

5. (40 pts)
   Consider an antiferromagnetic 3-state Potts model in zero external magnetic field on a cubic lattice in the thermodynamic limit. Does this system obey or violate the third law of thermodynamics (in the form that states that the entropy per site goes to zero as the temperature $T \to 0$). Prove your answer either way.

(total pts. = 130.)