Oct. 26, 2007

Phys. 541, Homework Assignment 3

Please pass in your solutions in class on Wed. Nov. 7.

1. (50) Consider a hypothetical classical model of a paramagnet, described by the Hamiltonian, for a given spin, $\mathcal{H} = -\mu \cdot \mathbf{H}$. Take $\mu = \mu_0 \mathbf{S}$ with $\mathbf{S}$ a classical spin of unit magnitude. Calculate, for a system of N spins,

   (a) (10) the partition function Z

   (b) (10) the internal energy U,

   (c) (10) the magnetization M,

   (d) (10) the specific heat at constant field, C_H,

   (e) (10) the entropy, S

   For each of these give the limits as $T \to 0$ and $T \to \infty$. Which results are similar to, and which are different from, the results that we derived in class for the quantum spin 1/2 paramagnet.

2. (20) For a liquid-gas phase transition, let us denote the critical temperature T_c, pressure p_c, and critical volume per particle $v_c = V_c/N$ (equivalently, critical molar volume, $\mathcal{V}_c$), and let $z_c = p_c v_c/(k_B T_c) = p_c \mathcal{V}_c/(RT_c)$. Using the table handed out in class (Table 3.5 from p. 48 of H. E. Stanley, *Intro. to Phase Transitions and Critical Phenomena*) or another source of data, calculate z_c for water, CO_2, Xe, and Ar, and compare with the van der Waals prediction for z_c which we derived in class.

3. (30) Calculate the prediction for the critical exponents β, γ, and δ from the van der Waals equation of state.