Phys. 541, Homework Assignment 3

Please pass in your solutions in class on Wed. Nov. 7.

1. (50) Consider a hypothetical classical model of a paramagnet, described by the Hamiltonian, for a given spin, \( H = -\mu \cdot H \). Take \( \mu = \mu_0 \mathbf{S} \) with \( \mathbf{S} \) a classical spin of unit magnitude. Calculate, for a system of \( N \) spins,

   (a) (10) the partition function \( Z \)
   (b) (10) the internal energy \( U \)
   (c) (10) the magnetization \( M \)
   (d) (10) the specific heat at constant field, \( C_H \)
   (e) (10) the entropy, \( S \)

   For each of these give the limits as \( T \to 0 \) and \( T \to \infty \). Which results are similar to, and which are different from, the results that we derived in class for the quantum spin 1/2 paramagnet.

2. (20) For a liquid-gas phase transition, let us denote the critical temperature \( T_c \), pressure \( p_c \), and critical volume per particle \( v_c = V_c/N \) (equivalently, critical molar volume, \( V_c \)), and let \( z_c = p_c v_c/(k_B T_c) = p_c V_c/(RT_c) \). Using the table handed out in class (Table 3.5 from p. 48 of H. E. Stanley, Intro. to Phase Transitions and Critical Phenomena) or another source of data, calculate \( z_c \) for water, CO\(_2\), Xe, and Ar, and compare with the van der Waals prediction for \( z_c \) which we derived in class.

3. (30) Calculate the prediction for the critical exponents \( \beta, \gamma, \) and \( \delta \) from the van der Waals equation of state.