Nov. 16, 2007

Phys. 541, Homework Assignment 5

Please pass in your solutions at the beginning of class on Wed., Nov. 28.

1. (20) Plischke and Bergesen, problem 3.9 (a calculation of the latent heat in a first order transition, using the Landau-Ginzburg free energy functional).

2. (20) Consider the 3-state Potts model in thermal equilibrium at temperature T with zero external magnetic field, on a one-dimensional lattice. Recall that the Hamiltonian for this model may be written as $\mathcal{H} = -J\sum_{\langle i,j\rangle}\delta_{\eta_i\eta_j}$, where the classical spin variable η_i on each site i takes on the values $\eta_i = 1, 2$ or 3, $\langle ij\rangle$ denotes nearest-neighbor pairs of sites, and δ_{rs} is the Kronecker delta, equal to 1 if $r = s$ and 0 if $r \neq s$. In the thermodynamic limit, calculate (i) the Gibbs free energy $G(T, H)$ per site, (ii) the specific heat per site, C_H.

3. (40) Consider the Ising model in thermal equilibrium at temperature T with external magnetic field H, defined on a one-dimensional lattice. Recall from our earlier class discussion for a general lattice that the Hamiltonian for this model may be written as $\mathcal{H} = -J\sum_{\langle i,j\rangle}\eta_i\eta_j - \mu H\sum_i \eta_i$ where the classical Ising spin variable $\eta_i = \pm 1$ on each site i, and, as before, $\langle ij\rangle$ denotes nearest-neighbor pairs of sites. Calculate the following per-site quantities (i) the Gibbs free energy $G(T, H)$, (ii) the internal energy U, (iii) the magnetization M for $J > 0$, and (iv) the magnetization M for $J < 0$. Discuss the role of competing interactions where relevant.

4. (20) Consider a spin model with Hamiltonian $\mathcal{H} = \mathcal{H}_0 - \mu H\sum_i S_i^z$, where $\mathcal{H}_0$ contains the usual nearest-neighbor spin-spin interaction, $\vec{\mu} = \mu\vec{S}$ is the magnetic moment, and without loss of generality we have chosen the axis along which the external magnetic field $\vec{H}$ points as the z axis. Prove the following relation, which enables one to calculate the total magnetic susceptibility $\chi_T = (\partial M/\partial H)_T$ in terms of connected spin-spin correlation functions: $\chi_T = \beta\mu^2\sum_{i,j}\langle S_i^z S_j^z\rangle_c$, where $\beta = (k_B T)^{-1}$ and $\langle S_i^z S_j^z\rangle_c \equiv \langle S_i^z S_j^z\rangle - M^2$ is the connected spin-spin correlation function.

5. (40) Calculate the chromatic polynomial for the wheel graph Wh_{n+1} defined as the graph obtained by starting with the circuit graph C_n and adding a single additional vertex adjacent to all of the vertices of C_n. For what range of q is there nonzero ground state entropy in the q-state Potts antiferromagnet on this graph?