7/12/98

reading assignment: Chap. 7 of your book
many things there are of much interest, and I do not have time to discuss them in lectures

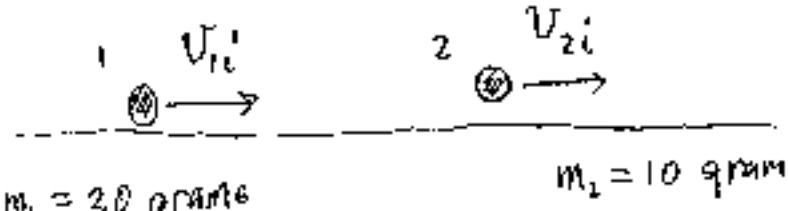
important:

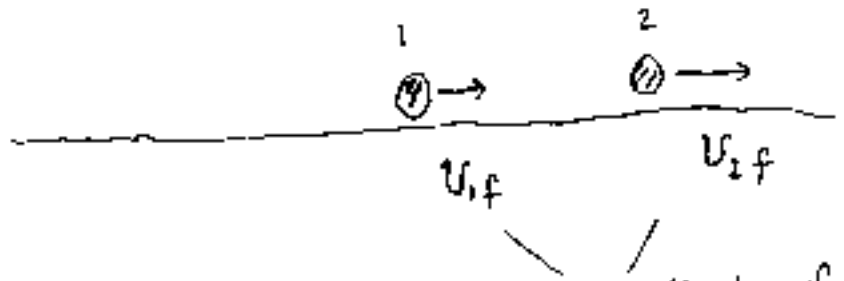
momentum conservation

elastic collision

inelastic collision

Ex. Head-on, elastic collision of 2 marbles:





Now we have 2 equations:

momentum conservation (0 = 0):
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad (1)$$

energy conserv. (0 = 0):
$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (2)$$

rewrite (1) as
$$m_1^2 (v_{1i} - v_{1f})^2 = m_2^2 v_{2f}^2 \quad (3)$$

rewrite (2) as
$$m_1 \underbrace{(v_{1i}^2 - v_{1f}^2)}_{(v_{1i} - v_{1f})(v_{1i} + v_{2f})} = m_2 v_{2f}^2 \quad (4)$$

(3)/(4) eliminates v_{2f}:
$$m_1 \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} = m_2$$

$$\Rightarrow \quad v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \quad \text{--- (5)}$$

put in numbers: $v_{1f} = \frac{20-10}{20+10} \cdot 5 = \frac{5}{3}$ m/s

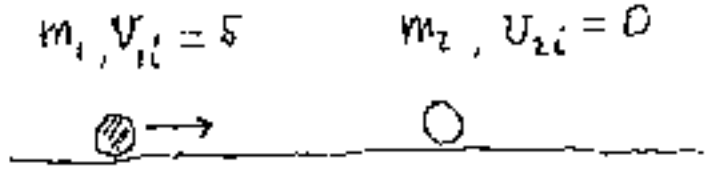
put ⑤ back to ①: ....

$$\rightarrow \quad v_{2f} = \frac{2m_1}{m_1+m_2} v_{1i} = \frac{40}{30} \cdot 5$$

$$= \frac{20}{3} \text{ m/s}$$

an interesting special case ($m_1 = m_2$)

before:



elastic collision, $m_1 = m_2$

after: from formulas we just had

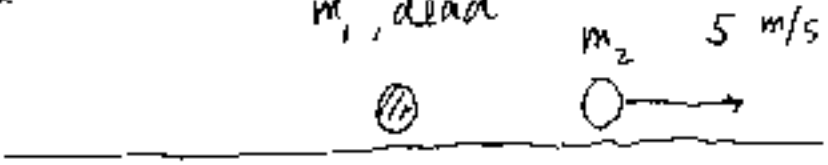
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = 0 \quad , \text{ as } m_1 = m_2$$

ie. ball-1 drops dead

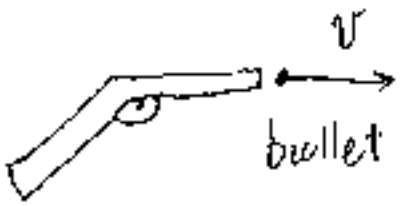
$$v_{2f} = \underbrace{\frac{2m_1}{m_1+m_2}}_{=1,\ \text{as } m_1=m_2} v_{1i} = v_{1i} = 5 \text{ m/s}$$

, ie. ball-2 goes on with 5 m/sec

Hence



Try the above with billiard balls.

Rifle 

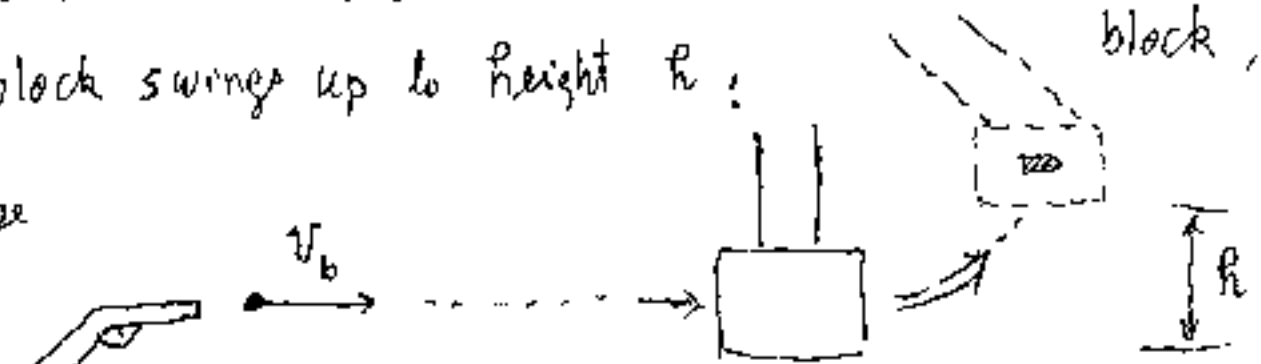
how to measure v ?

now, many high-tech methods (like using a radar gun)

olden days, how did one do ?

Ans ballistic pendulum ← { conservation of momentum; conservation of energy

bullet fired into and remained inside a wooden block, block swings up to height h :



V = velo. of (bullet + wood) right after collision

$$m_b v_b + 0 = (M_w + m_b) V$$

$$V = \frac{0.02 \times 300}{(2 + 0.02)} = 2.97 \text{ m/s}$$

Warning: do not use

$$\frac{1}{2} m_b v_b^2 = \frac{1}{2}(m_b + M_w) V^2$$

as firing bullet into wood is by no means "elastic"

now use energy conservation for the swinging-up of $\begin{cases} \text{wood} \\ + \text{bullet} \end{cases}$:

$$(KE + PE)_{\text{bottom}} = (KE + PE)_{\text{top}}$$

$$\frac{1}{2}(m_b + M_w) V^2 \qquad 0 \qquad 0 \qquad (m_b + M_w) g h$$

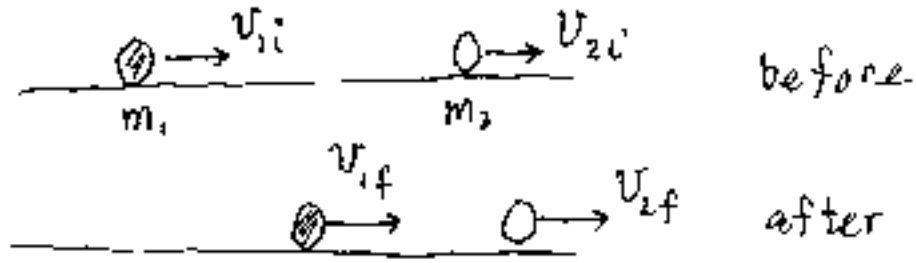
$$\therefore h = \frac{V^2}{2g} = \frac{2.97^2}{2 \times 9.8} \simeq 0.45 \text{ m}$$

See, if one measures h, one can deduce v_{bullet}

M

Supplement to Chap. 7 For CAPA-6, you need:

elastic head-on collision:



mtum conserv.
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad (1)$$

energy conserv.
$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (2)$$

m_1, m_2, v_{1i}, v_{2i} given

can solve (1) & (2), after messy algebra, get

$$v_{1f} = \frac{2 m_2 v_{2i} + v_{1i}(m_1 - m_2)}{m_1 + m_2}$$

$$v_{2f} = \frac{2 m_1 v_{1i} - v_{2i}(m_1 - m_2)}{m_1 + m_2}$$

outline of solution:

$$(1) \rightarrow \quad v_{2f} = \frac{1}{m_2}\left(m_1 v_{1i} + m_2 v_{2i} - m_1 v_{1f}\right) \quad \cdots (3)$$

.. substitute (3) into (1), get an equation of the form

$$A v_{1f}^2 + B v_{1f} + C = 0$$

$$v_{1f} = \frac{1}{2A}\left[-B \pm \sqrt{B^2 - 4AC}\ \right] = \cdots$$

Chap. 8 rotational motion

some basics:

circle of radius r

circumference $= 2\pi r$

$\theta_1 = 90^\circ = \frac{\pi}{2}$ radians [rad]

1 radian $\approx 57.3^\circ$

$\theta_2 = 180^\circ = \pi$ rad.

others: $60^\circ = \frac{\pi}{3}$ rad, $45^\circ = \frac{\pi}{4}$ rad, etc

angular velocity ω [rad/sec]

↑ omega

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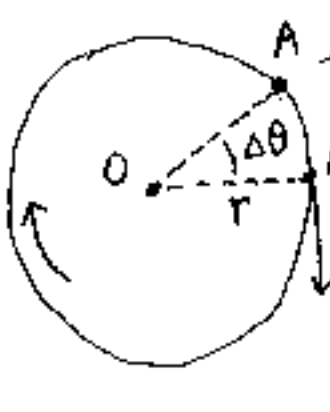
paint a point A on a rotating disk:



average angular velo

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \quad \frac{\text{rad}}{\text{sec}}$$

instantaneous angul velo

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

arc length $AA' \approx \Delta s = r\,\Delta\theta$

tangential velo. $v = \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = r\omega \quad \frac{m}{sec}$

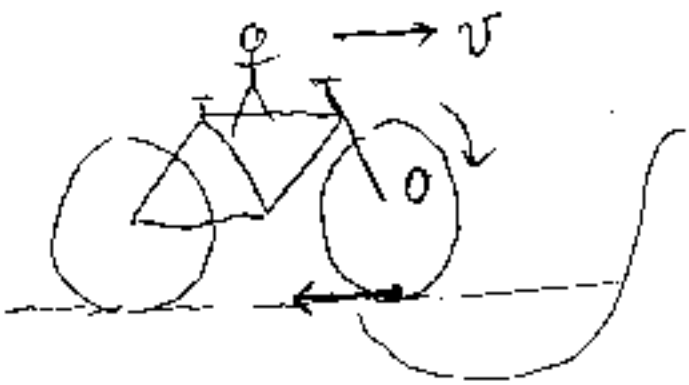
Ex. flywheel at 1000 rpm ← revolution per minute

$\omega = ?$ rad.

$$\omega = \frac{1000 \times 2\pi}{60}$$

$$= 104.72 \text{ rad/sec}$$

Ex. bicycle has $v = 40$ km/hr, $\omega_{wheel} = ?$



$r_{wheel} = 0.5$ m

$v = 40$ km/hr relative to O (be sure to see this point) ⟸

$$v = 40 \times \frac{1000}{3600} = r\omega = 0.5 \times \omega$$

$40 \times \frac{1000}{3600}$ = 11.11 m/s

$\omega = 22.22$ rad/sec

$= 3.54$ rev/sec

$= 212.1$ rpm

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translational	rotational
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
$l = v_i t + \frac{1}{2} a t^2$	$\theta = \omega_i t + \frac{1}{2} \alpha t^2$
$v_f^2 = v_i^2 + 2al$	$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

α = angul. acc. $\frac{\text{rad}}{s^2}$

do some examples to explain

Ex. flywheel

initially at rest

electric motor gives it

acc. $\alpha = 1.2$ rad/sec^2

at $t = 10$ sec, $\omega = ? = \omega_i + \alpha t = 15$ rad/sec

$\omega_i = 0$, $\alpha = 1.5$, $t = 10$

$\theta = ?$ after motor is on for 0.5 min

$= \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 1.2 \times (0.5 \times 60)^2$

$\omega_i = 0$

$= 540$ rad (1)

or $= \frac{540}{2\pi} = 85.9$ revolutions

Another way of doing the same problem:

ω_f = angul velo after 0.5 min

$= 0 + 1.2 \times 0.5 \times 60 = 36$ rad/sec

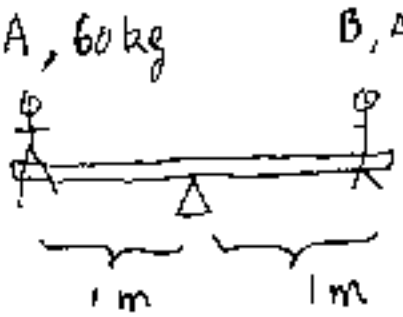
$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$36^2 \quad 0 \quad 1.2$

$\therefore \theta = \frac{36^2}{2 \times 1.2} = 540$ rad

same as (1)

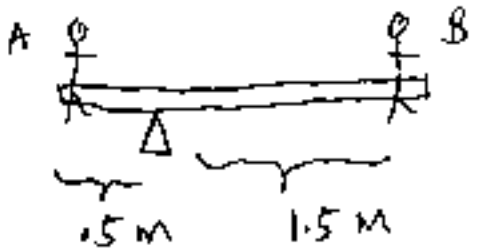
seesaw



A would rotate the seesaw ↶

B " " " " ↷

A is heavier, ∴ seesaw turns ↶



which direction does it turn?

τ (tau) = force $\perp$ × arm [Nm]

component of force $\perp$ arm

$\tau_A = 60 \times 9.8 \times .5 = 294$ Nm ↶

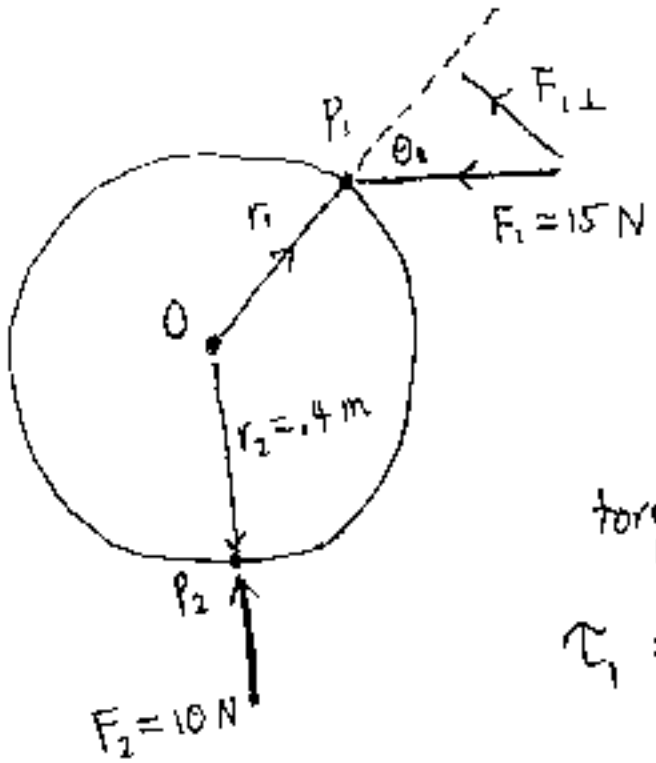
$\tau_B = 40 \times 9.8 \times 1.5 = 588$ Nm $> \tau_A$ ↷

↑ ↑
m g

∴ seesaw rotates ↷

Key: Torque (both force and arm) makes things rotate

let us make this point more precisely next page.



$\theta_1 = 40^\circ$

O = axis of rotation

P_1 = point of action

$r_1 = .3\ m$ = "arm"

torque produced by F_1 is

$$\tau_1 = \underbrace{F_{1\perp}}_{15\sin 40^\circ}\ r_1 \qquad \cdot 3$$

$= 0.289\ N\text{-}m$ ↺ direction

$\vec{F}_2$ is anti $\parallel$ to $\vec{r}_2$

$\therefore F_{2\perp} = 0$

i.e. no component $\perp$ the arm

$\therefore \tau_2 = \underbrace{F_{2\perp}}_{0}\ r_2 = 0$

$$\boxed{\tau = F_\perp\, r}\ Nm$$

W

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recall translation motion

$\vec{F} = m\vec{a}$

↑ force, causes m to accelerate

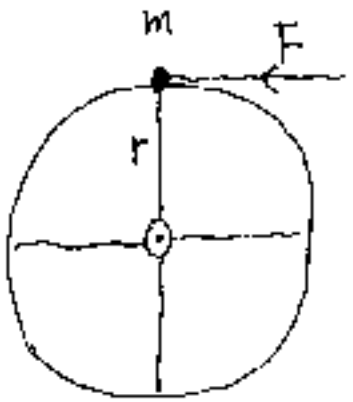
for rotation, torque makes things to accelerate

$\vec{\tau} = I\vec{\alpha}$

τ ← torque; I ← moment of inertia; α ← angular accel.

what is I ?

mass m on a wheel ↑ negligible mass



push m by force F

$$F = ma = mr\alpha$$

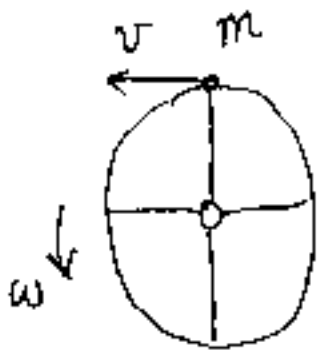
↑ tangential acc. ↓ angular acc.

multiply both sides by r:

$$\underbrace{Fr}_{\tau} = \underbrace{mr^2}_{I}\alpha$$

i.e. $\tau = I\alpha$, $I = mr^2$

similarly



kinetic energy of m is

$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2}\underbrace{mr^2}_{I}\omega^2$$

$\therefore$ rotational kinetic en. is

$$KE = \frac{1}{2} I\omega^2$$

$$I = mr^2$$

r = distance from m to axis of rotation

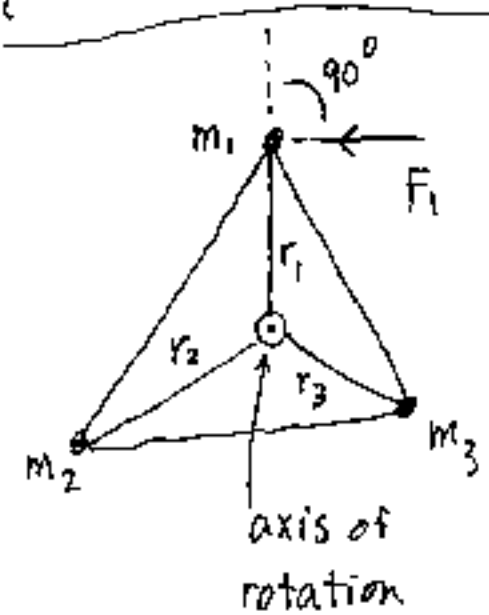
Hence for one "little" mass rotating, we have

$$\tau = I\alpha, \quad K_{rotation} = \frac{1}{2} I\omega^2, \quad I = mr^2$$

For many masses rotating together,

$$I = \sum_i m_i r_i^2 \quad [kg \cdot m^2]$$

Ex. 3 masses on a "massless" frame



$m_1 = 1.5$ kg
$m_2 = 2.5$
$m_3 = 3$

$r_1 = .5$ m
$r_2 = .8$
$r_3 = .4$

$$I_{total} = ? = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$ ← be sure to understand this's

$$= 1.5 \times .5^2 + 2.5 \times .8^2 + 3 \times .4^2$$

$$= 2.46 \text{ kg m}^2$$

$\alpha = ?$ when $F_1 = 30$ N is applied

$\tau_1 = I\alpha$

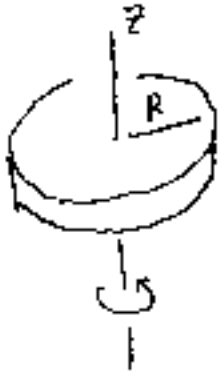
30×0.5 ↑ $F_1 \times r_1$; 2.46 ↑ I

$\therefore \alpha = 6.09 \frac{rad}{s^2}$

moment of inertia

see p. 261 of your book

disk about Z axis thru center



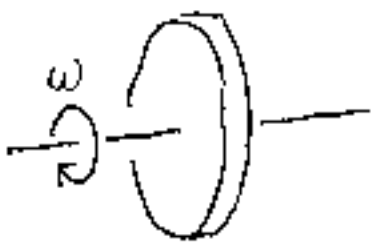
total mass m
radius R

$$I = \sum_{\text{all } i \text{ of disk}} m_i r_i^2$$

after some math, integration:

$$I = \frac{1}{2} m R^2$$

Ex.



uniform disk, $m = 4$ kg
$R = .3$ m
$\omega = 75$ rad/s

$$KE = ? = I\omega^2 = 1012.5 \text{ Joules}$$

$\frac{1}{2} \times 4 \times .3^2$ $\quad$ 75^2

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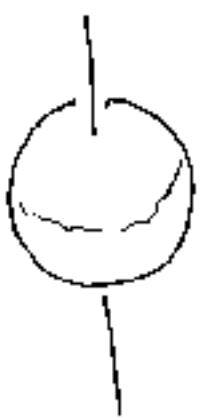
solid sphere about axis thru center

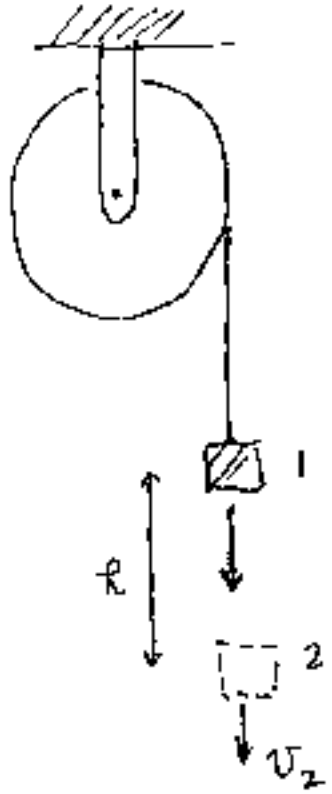


mass m
radius R

its $I = \frac{2}{5} m R^2$

Don't worry, formulas for Is will be given

Ex. pulley, block, string (no mass)



pulley = uniform disk
$M = 4$ kg, $R = .2$ m

block: $m = 1.5$ kg

released from rest

$v_{block} = ?$ at $R = 1.8$ m

i.e. $v_2 = ?$

loss of PE = gain in KE

$mgR = KE_f - KE_i$, $KE_i = 0$ as initially at rest

$= KE_{pulley,\ rotate} + KE_{block}$

1.5, 9.8, 1.8

$KE_{pulley,\ rotate} = \frac{1}{2} I \omega^2$

$KE_{block} = \frac{1}{2} m v_2^2$

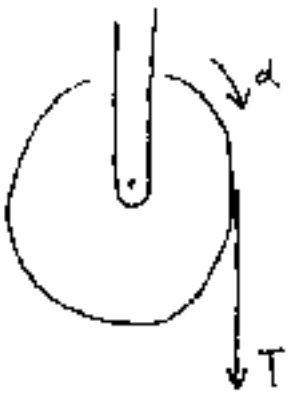
$= \frac{1}{2}\left(\frac{1}{2} MR^2\right)\left(\frac{v_2}{R}\right)^2 = \frac{1}{4} M v_2^2$

$26.46 = \left(\frac{1}{4}M + \frac{1}{2}m\right) v_2^2$

4, 1.5

$\Rightarrow v_2 = 3.89$ m/s

Let us use a different method:

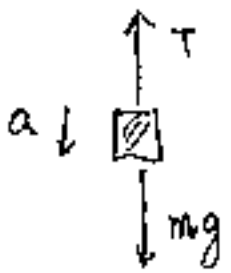


for pulley

torque $\tau = I\alpha$

$$TR = \left(\frac{1}{2}MR^2\right)\frac{a}{R}$$

$$T = \frac{1}{2}Ma \qquad (1)$$



$$-mg + T = -ma \qquad (2)$$

subst (1) into (2)

$$\left(\frac{1}{2}M + m\right)a = mg$$

$$a = \frac{m}{m + M/2}\,g = \frac{1.5}{1.5 + 2} \times 9.8 = 4.2\ m/s^2$$

$$v_2^2 = v_1^2 + 2as = 15.12, \quad v_2 = 3.89\ m/s \checkmark$$

($v_1^2 = 0$, $a = -4.2$, $s = -1.8$)

same as before

tension $T = \frac{1}{2}Ma = \frac{1}{2} \times 4 \times 4.2 = 8.4\ N$

10/16/98 midterm-2 on 11/12/98

mainly energy, momentum, rotation, fluid, simple-harmonic-motion

emphasize CAPA-5 → CAPA-9, lectures