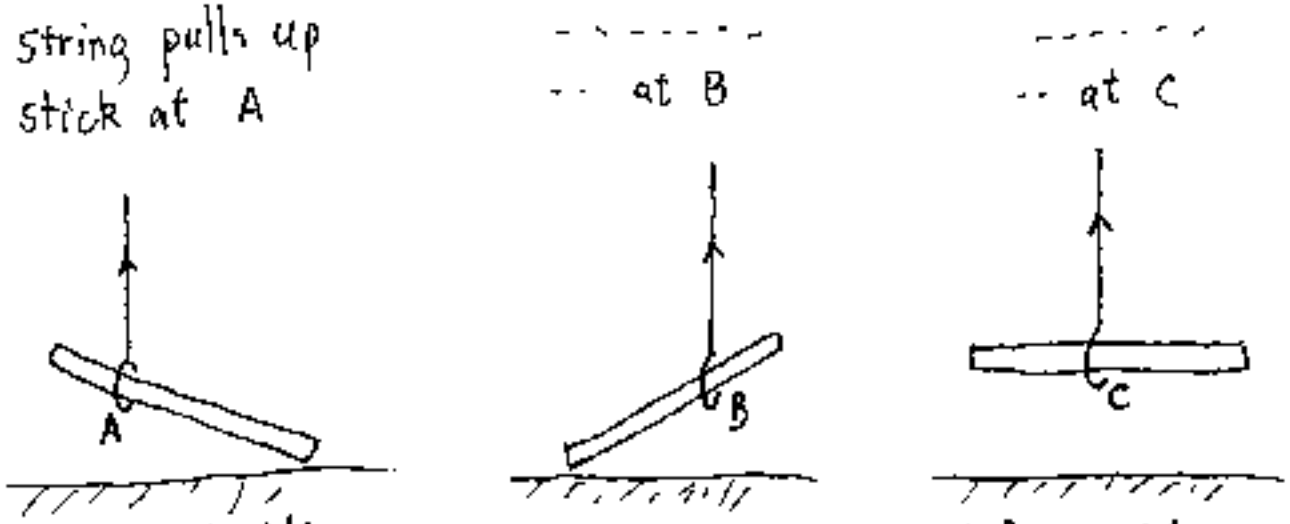
{ 10/19/98
121 }

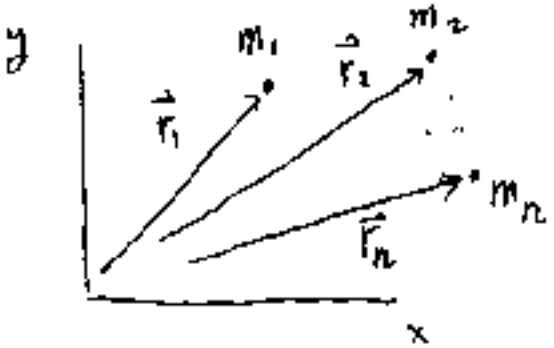
center of gravity (cg)

simple example,



"C" is center of gravity

For n masses,



M = total mass
$= m_1 + m_2 + \cdots + m_n$

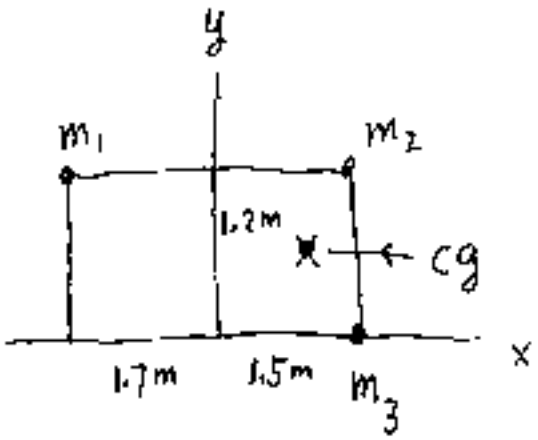
$\vec{R}$ = location of cg

$\vec{R}$ is defined as $\quad M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n$

$\vec{R} = (\bar{X}, \bar{Y}) \qquad M\bar{X} = m_1x_1 + m_2x_2 + \cdots + m_nx_n$

$\vec{r} = (x, y) \qquad M\bar{Y} = m_1y_1 + m_2y_2 + \cdots + m_ny_n$

Ex.



find cg of m_1, m_2 and m_3

↘ Center of gravity

$m_1 = 1$ kg , $m_2 = 2$ kg , $m_3 = 3$ kg , $M = 1+2+3 = 6$ kg

$$M\bar{X} = m_1x_1 + m_2x_2 + m_3x_3 = 1\times(-1.7) + 2\times 1.5 + 3\times 1.5$$

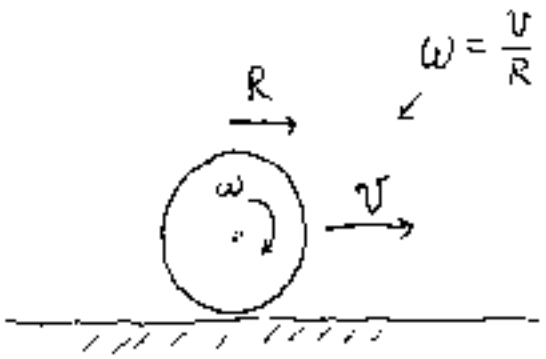
$$6\bar{X} = 0.967 \text{ m}$$

$$M\bar{Y} = m_1y_1 + m_2y_2 + m_3y_3 = 1\times 1.2 + 2\times 1.2 + 3\times 0$$

$$\bar{Y} = 0.6 \text{ m}$$

∴ cg is ✱ see figure

Ex.



uniform cylinder rolling with $v = 8$ m/sec

$R = .3$ m, $m = 20$ kg

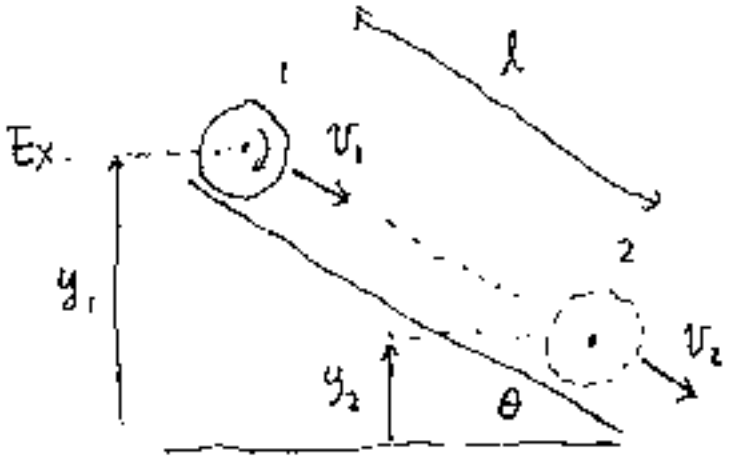
its kinetic energy = ?

can prove that

$$KE = KE_{cg} + KE_{\text{rotation about cg}}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = 960 \text{ J}$$

m: 20 ; v^2: 8^2 ; I: $\frac{1}{2}20\times.3^2$; ω^2: $\left(\frac{8}{.3}\right)^2$



uniform cylinder rolls (without slipping) down an incline

cylinder $m = 2.5$ kg
$R = .2$ m
$\ell = 3$ m , $\theta = 10^\circ$

initially cylinder at rest,
i.e. $v_1 = 0$ find $v_2 = ?$

y_1, y_2 are vertical positions for the cg of the cylinder

energy conservation

$$PE_1 + KE_1 = PE_2 + KE_2$$

$$mgy_1 \quad 0 \text{ at rest} \quad mgy_2 \quad \frac{1}{2} m v_2^2 + \frac{1}{2} I \omega_2^2$$

$$mg(y_1 - y_2) = \frac{1}{2} m v_2^2 + \frac{1}{2}\left(\frac{1}{2} m R^2\right) \frac{v_2^2}{R^2}$$

$$= \frac{3}{4} m v_2^2$$

$y_1 - y_2 = \ell \sin\theta$

ω_2 = angul. velo. of cylinder at 2

$$\omega_2 = \frac{v_2}{R}$$

$$\therefore \quad v_2^2 = \frac{4}{3} g \ell \sin\theta = 6.807$$

g, ℓ, 3, 10°

$$v_2 = 2.61 \text{ m/s}$$

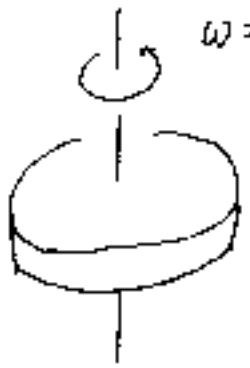
recall $\xrightarrow[m\ \ v]{}$ linear momentum $\vec{p} = m\vec{v}$

$$\vec{F}\,\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

rotating object has rotational momentum called angular momentum L

$L = I\omega$ with axis of rotation shown

Ex.

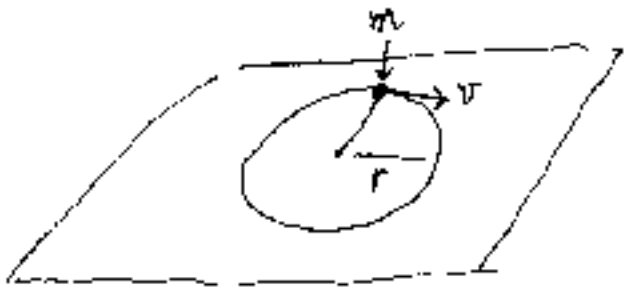


its angular momentum = ?

$L = I\omega = 2880 \ \frac{kg\cdot m^2}{sec}$

$I = \frac{1}{2} \times 80 \times 1.2^2$; $\omega = 50$

Ex. marble, attached to string, moves on a circular path.



$m = .2$ kg

$r = .4$ m, $v = 8$ m/s

$L = ? = I\omega = 0.64 \ \frac{kg\cdot m}{sec}$

$I = mr^2 = .2 \times .4^2$

$\omega = \frac{v}{r} = \frac{8}{.4}$

to change angular momentum L, need apply torque:

$$\Delta L = L_f - L_i = \tau \Delta t$$

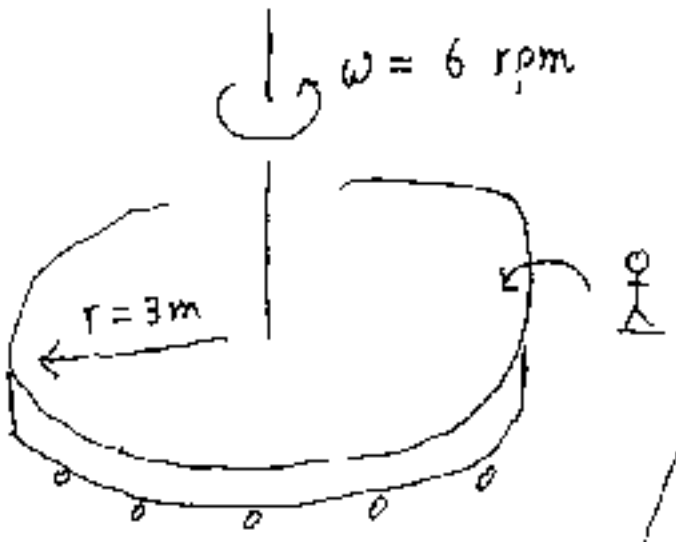
if $\tau = 0$ (i.e. no external applied to the system), we have

$$\Delta L = L_f - L_i = 0$$

i.e. $L_f = L_i$ ← conservation of angular momentum

Ex. merry-go-round (mgr)

$I_{mgr} = 4000\ kg\text{-}m^2$



boy ($m_b = 50$ kg) gently steps on the edge of mgr

$\omega_{mgr+b} = ?$

note: radial direction, so that boy give no torque to mgr

no torque ⇒ $L_i = L_f$

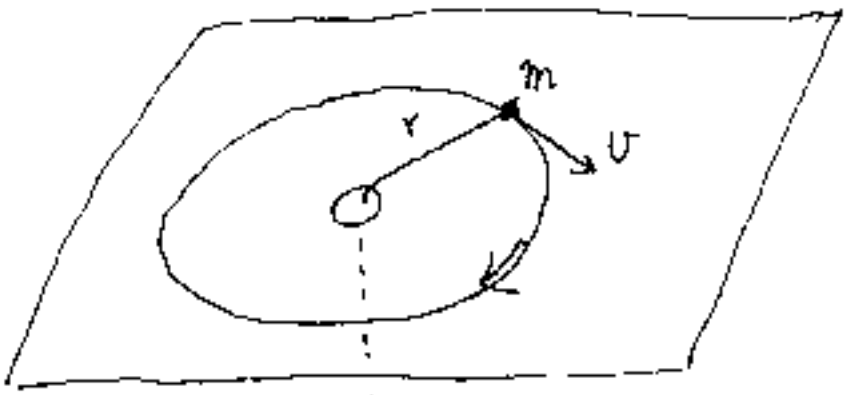
$L_i = 4000 \times \frac{6 \times 2\pi}{60}$

$L_f = (I_{mgr} + I_{boy}) \times \omega_f$

$I_{mgr} = 4000$, $I_{boy} = 50 \times 3^2$

$\therefore\ \omega_f = 0.56\ rad/s = 5.39\ rpm = \omega_{mgr+b}$

Ex. marble, circling (no friction) on table



$m = .2$ kg

$v = 5$ m/s

$r = .3$ m

string → held by force ↓ F

L_{marble}

$= I\omega = (mr^2)\frac{v}{r}$

$= 0.3$ kg-m^2/s (1)

suppose increase force, pull marble inward, smaller circle, with $r_{new} = 0.2$ m

$v_{new} = ?$

Force is anti $\parallel$ to r, $\therefore$ gives no torque to m

$\Rightarrow$ $L_i = L_f = (m r_{new}^2)\frac{v_{new}}{r_{new}} = m r_{new} v_{new}$

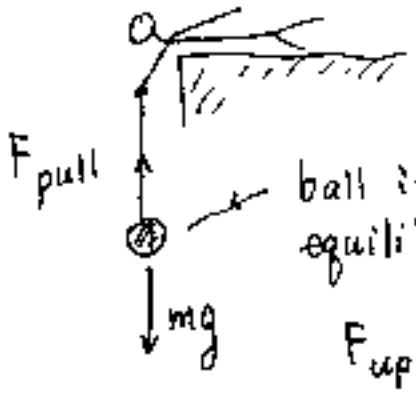
L_i: 0.3 by (1)

$m r_{new}$: .2 / .2

$v_{new} = 7.5$ m/s

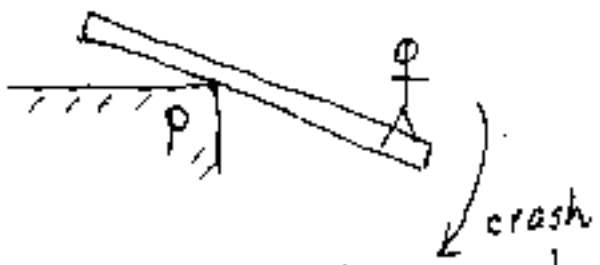
moves faster

Equilibrium



$F_{up} + F_{down} = 0$

$F_{pull} \quad -mg$



boy on plank

not in equilibrium

" in danger "

$\tau_P = \circlearrowright \neq 0$

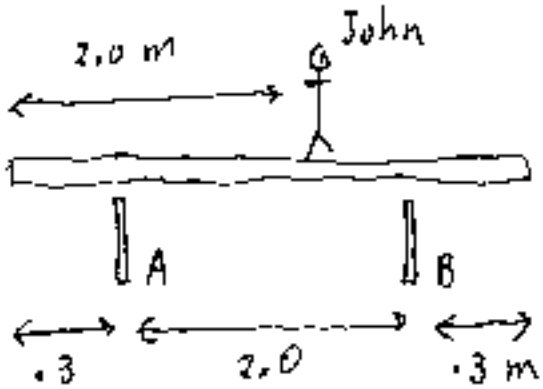
↑ torque about pivot P

any object in equilibrium, must have

(i) $\sum \vec{F} = 0$, ie. sum of all forces acting on it $= 0$

(ii) $\sum \vec{\tau} = 0$, ie. sum of all torques acting on it $= 0$

Ex.

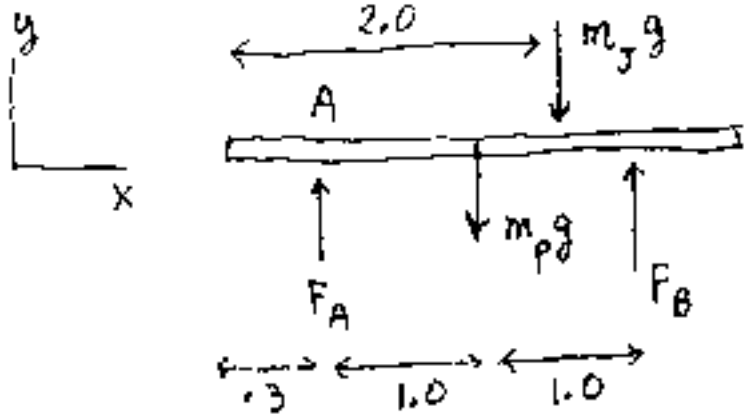


$m_{John} = 70$ kg

$m_{plank} = 20$ kg

find forces F_A, F_B on the stands A + B.

important first step, draw free body diagram



note $m_p g$ acting thru the cg of plank ↓ center

$\Sigma F_y = 0$: $F_A + F_B - m_J g - m_p g = 0$

(m_J: 1/70, m_p: 1/20)

$$F_A + F_B - 90 \times 9.8 = 0 \qquad (1)$$

$\Sigma \tau = 0$ about any pivot point:

take a convenient pivot point ⟶ take A

$\Sigma \tau_A = 0$ sign convention ↺ + torque

↻ − torque

$$-m_p g \times 1 - m_J g \times 1.7 + F_B \times 2 = 0 \qquad (2)$$

(m_p: 1/20, 9.8; m_J: 1/70, 9.8)

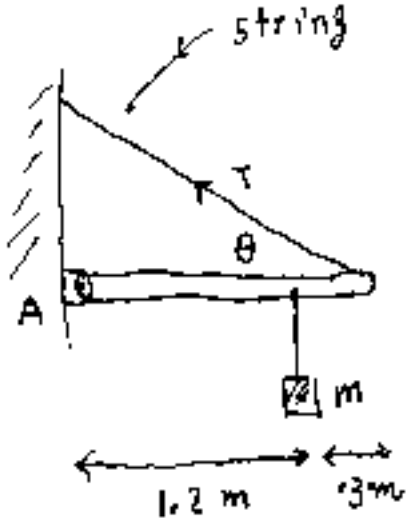
note F_A gives zero torque

be sure to understand the arms

$\tau = F_\perp \times$ arm

$\therefore$ $F_B = 69.5 \times 9.8$ N, put in (1) ⟶ $F_A = 20.5$ N

Ex.



$m = 4$ kg

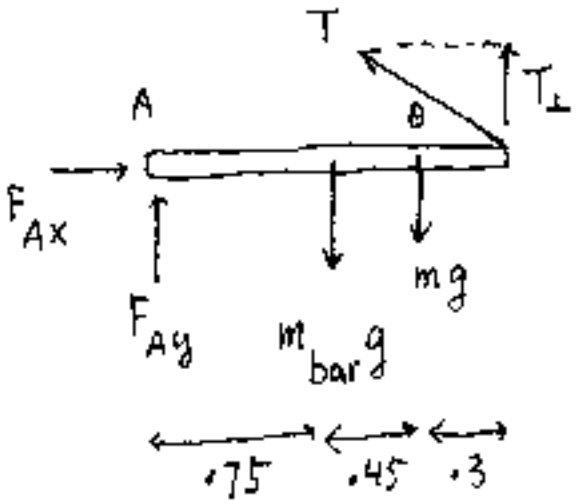
$m_{bar} = 10$ kg

$\theta = 30°$

find tension $T = ?$

supporting force from hinge A $= ?$

free-body diagram:



{ note F_{Ax}, F_{Ay} have zero torque about A

sum torques about A $\Big\} = 0$: $\quad 0 = 0 - 10 \times 9.8 \times .75 - 4 \times 9.8 \times 1.2 + T \sin 30 \times 1.5$

$\rightarrow T = 160.72$ N

$\Sigma F_x = 0$: $\quad F_{Ax} - T \cos 30 = 0$, $\therefore F_{Ax} = 139.18$ N

($T \uparrow 160.72$)

$\Sigma F_y = 0$: $\quad F_{Ay} - 10 \times 9.8 - 4 \times 9.8 + 160.7 \times \sin 30$

$\therefore F_{Ay} = 56.84$ N

Chap-9 mainly fluids

CAPA-8 due 10/30

density ρ [pronounced rho]

- some matter of mass m kg
- its volume is V m^3

then its density is $\rho = \frac{m}{V}$ $\left[\frac{kg}{m^3}\right]$

Substance	ρ $[kg/m^3]$
air (30°c)	1.16
water	1.00×10^3
iron	7.9×10^3
gold	19.3×10^3
⋮	⋮
nuclear matter	$\sim 10^{17}$

What is "nuclear matter"?

atom =

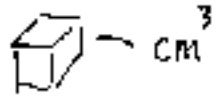
← electrons orbiting outside

nucleus (made of protons + neutrons)

↑

is made of nuclear matter

mass of one teaspoonful of nuclear matter $= ? \approx \underbrace{1\,cc}_{} \times \rho_{nucl\ matt}$



$= 10^{-8} \times 10^{17} = 10^{9}$ kg

one big oil tanker $\approx$ mass of 200 big tankers

≈ 50000 ton

$= 5 \times 10^{6}$ kg

Ex. gold sphere of $r = 0.12$ m, has mass 125 kg?
Is it pure gold?

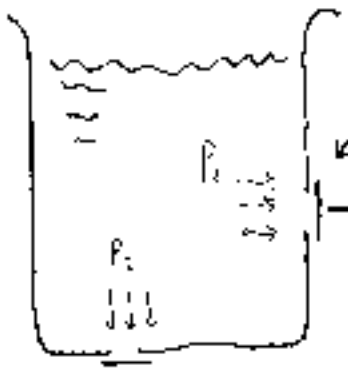
$$\rho = \frac{m}{V} = \frac{125}{\frac{4\pi}{3}(0.12)^3} = \frac{125}{.00723} = 17.28 \times 10^{3}\ kg/m^3$$

but $\rho_{gold} = 19.3 \times 10^{3}\ kg/m^3$, $\therefore$ fake

Wed

Pressure

tank of water



door-1 of area A_1

hold it with force F_1

pressure here is $p_1 = \frac{F_1}{A_1}$

door-2 A_2

F_2

$p_2 = \frac{F_2}{A}$

pressure $p = \frac{F_\perp}{A}$ $\frac{N}{m^2}$ or Pascal [Pa]

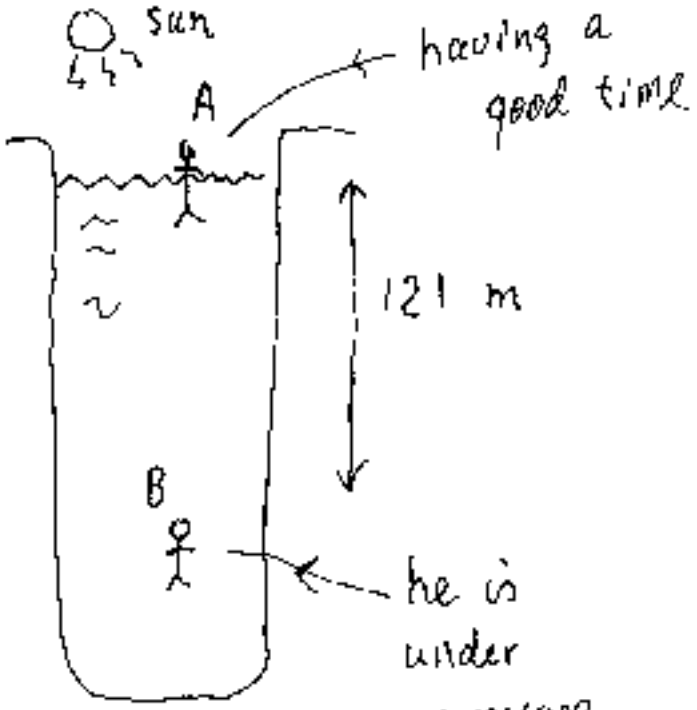
↖ $F_\perp$ is force perpendicular to area A

Ex. super swimming pool

Mr. B is

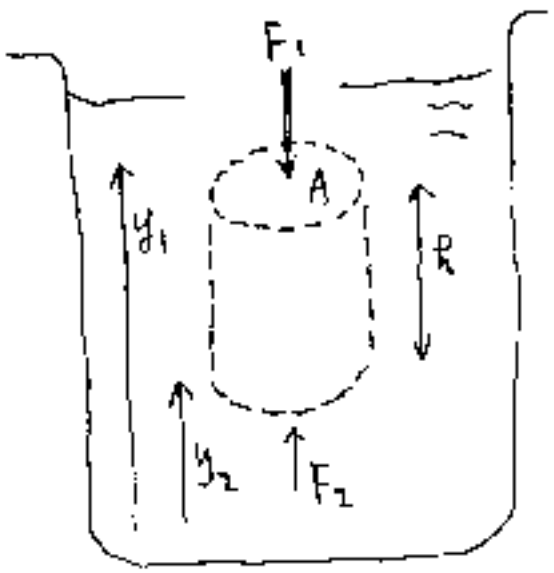
121 meter underneath water

Is pressure $p_B \begin{matrix} > \\ = \\ < \end{matrix} p_A$?



See "121" has a lot of pressure !

Pressure and depth



consider a cylindrical column of fluid of mass m

It is in equilibrium, hence

$$F_{up} = F_{down}$$

$$F_2 = mg + F_1$$

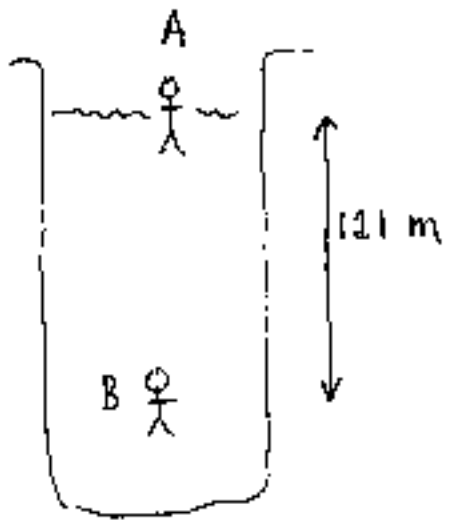
F_2 → $p_2 A$; mg → $\rho(y_1 - y_2)A$ (volume) ; F_1 → $p_1 A$

rearranging terms :

$$p_1 + \rho g y_1 = p_2 + \rho g y_2$$

or $$p_2 = p_1 + \rho g \underbrace{(y_1 - y_2)}_{h} = p_1 + \rho g h$$

Ex.



How much more pressure is Mr. B suffering than A ?

$$p_B - p_A = \rho g h$$

ρ: 1000, g: 9.8, h: 121

$$= 11.76 \times 10^5 \text{ Pascal}$$

B has a watch

circular crystal shield of $r = 1.5$ cm

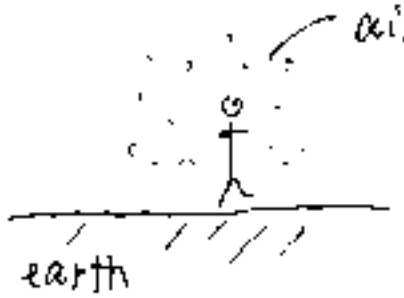
Force from water to shield

$$= ? = p_B \pi r^2 = 902.9 \text{ N}$$

p_B: $(11.76 \times 10^5 + P_A)$ (given as 1.013×10^5)

r^2: $(1.5 \times 10^{-2})^2$

(to discuss later)

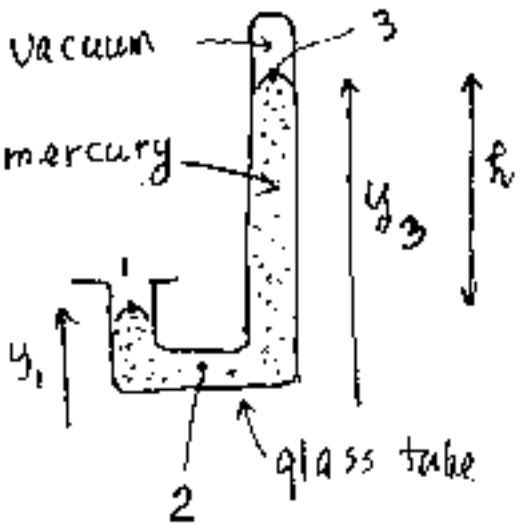


we are surrounded by "air", which exerts pressure on us

↑ air pressure or atmospheric pressure p_{atm}

how to measure p_{atm} ?

use barometer:



ρ = density of mercury
$= 13.595 \times 10^3$ kg/m^3

$$p_1 + \rho g y_1 = p_2 + \rho g y_2$$
$$= p_3 + \rho g y_3$$

note we use point-2 to make connection clearer

$$\rightarrow \quad p_1 = p_3 + \rho g (y_3 - y_2)$$

p_1: this is p_{atm}

p_3: this is 0, as vacuum has zero pressure

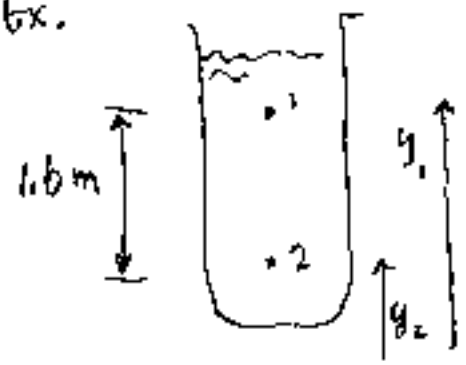
$(y_3 - y_2)$: h, measured as 0.76 m

$$\therefore \quad p_{atm} = 13.59 \times 10^3 \times 9.8 \times 0.75 = 1.013 \times 10^5 \text{ Pascal}$$
or N/m^2

gauge pressure (p_{gauge}) and absolute pressure (p)

$$p_{gauge} \equiv p - p_{atm}$$

Ex.



$p_{1\,gauge} = 9.2 \times 10^3$ Pa

$p_{2\,gauge} = 27.1 \times 10^3$ Pa

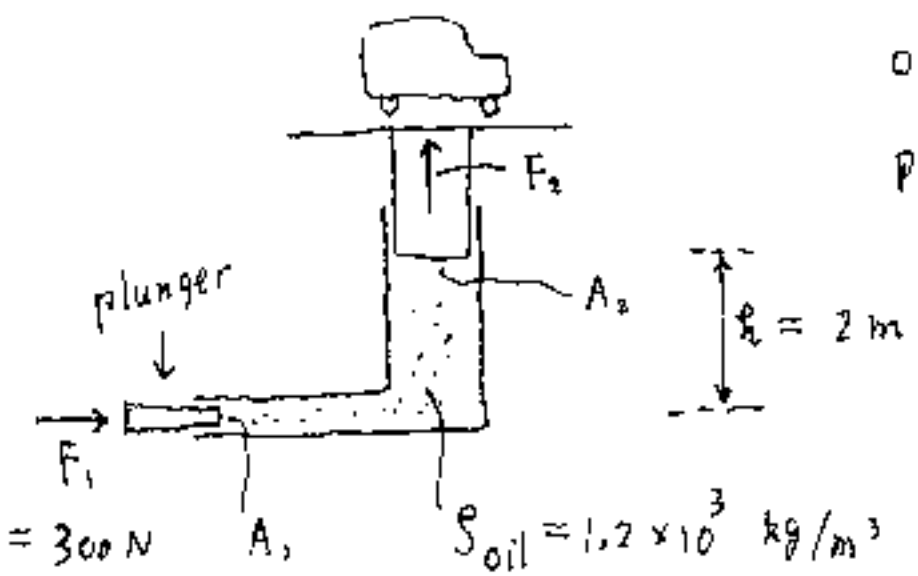
ρ of fluid = ?

$$p_1 + \rho g y_1 = p_2 + \rho g y_2$$

$$\rho \underbrace{g}_{9.8} \underbrace{(y_1 - y_2)}_{1.6} = \underbrace{p_{2\,gauge}}_{27.1 \times 10^3} + \underbrace{p_{atm}}_{\times} - \underbrace{p_{1\,gauge}}_{9.2 \times 10^3} - \underbrace{p_{atm}}_{\times}$$

$$\rho = 1.14 \times 10^3 \ kg/m^3$$

Ex. hydraulic lift



apply F_1 to plunger, oil pressure pushes platform up with force F_2

$A_1 = 4 \ cm^2$

$A_2 = 0.5 \ m^2$

$F_2 = ?$

$$p_1 = \frac{F_1}{A_1} = \frac{300}{4\times10^{-4}} = 75\times10^4 \text{ Pa}$$

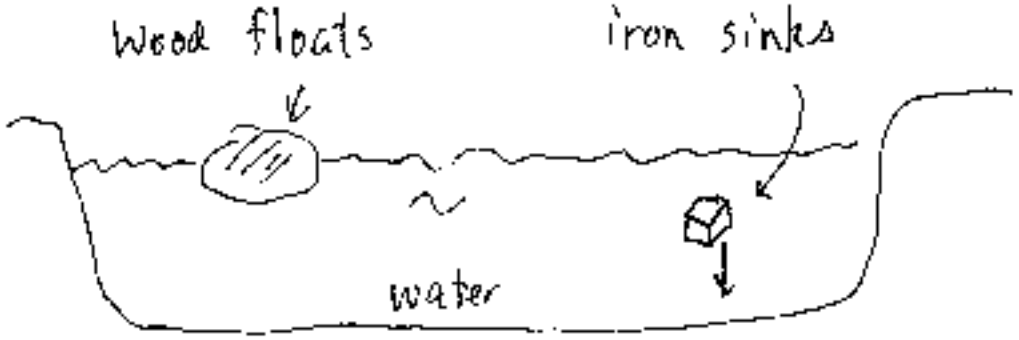
$$p_2 = p_1 - \rho g h = 75\times10^4 - 1.2\times10^3 \times 9.8 \times 2$$

$$= 72.6\times10^4 \text{ Pa}$$

$$F_2 = p_2 A_2 = 72.6\times10^4 \times 0.5 = 3.63\times10^5 \text{ N}$$

↑ big enough to lift huge truck

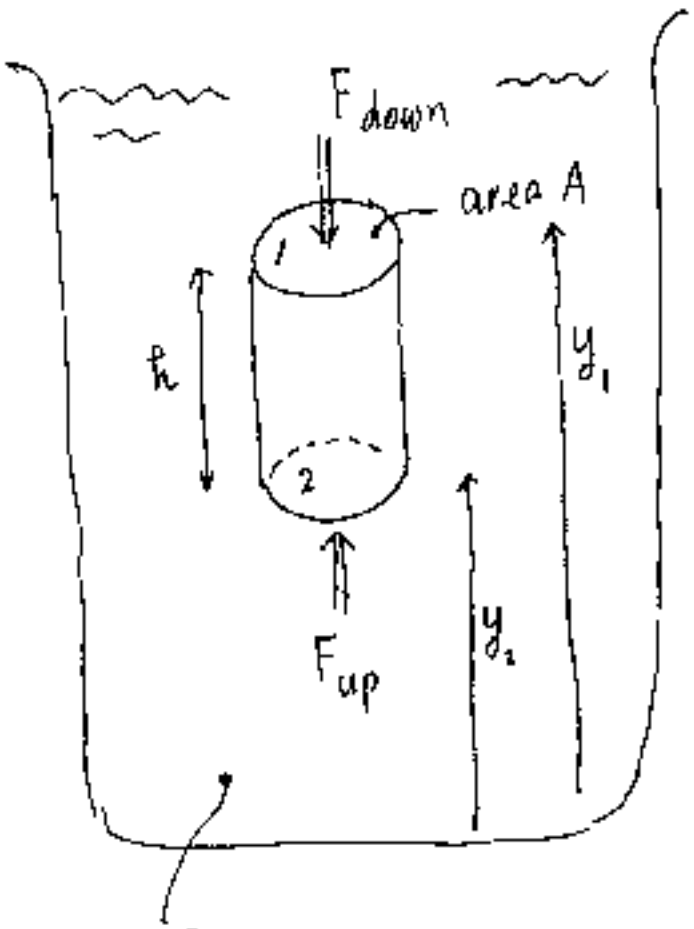
buoyant force



buoyant force from water $> (mg)_{wood}$

buoyant force from water $< (mg)_{iron}$

How to calculate buoyant force ?



Consider a cylinder immersed in a fluid

sideward forces all cancel

$$F_{buoyant} = F_{up} - F_{down}$$

$F_{up} = p_2 A$, $F_{down} = p_1 A$

$$F_{buoy} = \underbrace{(p_2 - p_1)}_{\rho_{fluid} g h} A$$

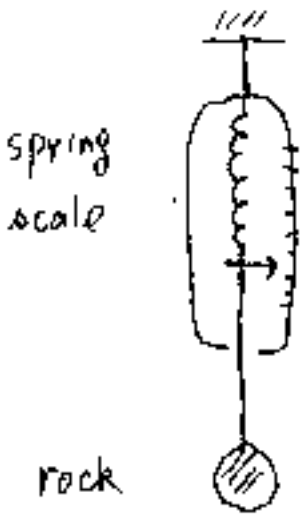
i.e. $F_{buoy} = \rho_{fluid}\, g \underbrace{hA}_{\text{this is volume of cylinder}}$

in general

$$F_{buoy} = \rho_{fluid}\, g \underbrace{V_{displ}}_{\text{volume of fluid displaced by the object}}$$

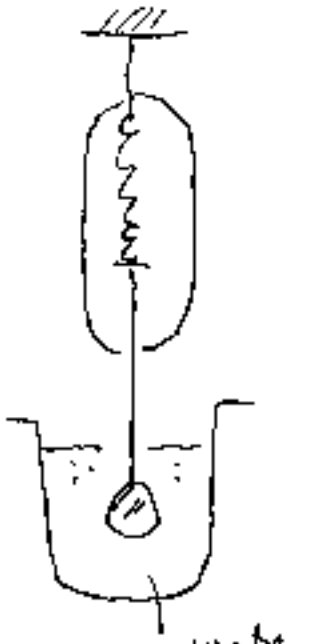
F_{buoy}: on an object

Ex. rock in air



scale reads $W_{rock}^{air} = 20\ N$

rock in water



scale now reads

$W_{rock}^{water} = 15\ N$

Find $\rho_{rock} = ?$

buoyant force

$$W_{rock}^{water} = 15 = \underbrace{m_{rock} g}_{20} - \rho_{water} V_{rock} g$$

ρ_{water}: 1000; g: 9.8

$$\therefore V_{rock} = 0.51 \times 10^{-3}\ m^3$$

$$\rightarrow \rho_{rock} = \frac{m_{rock}}{V} = \frac{20/9.8}{0.51 \times 10^{-3}} = 4.0 \times 10^{3}\ kg/m^3$$