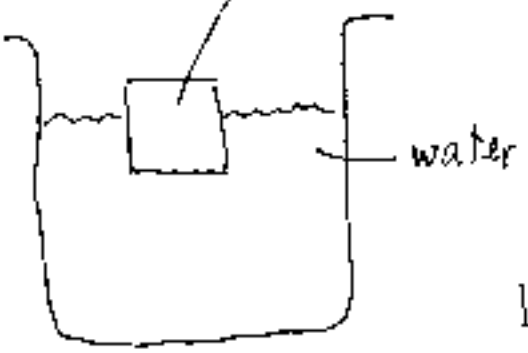
10/26/98

Ex.



$\rho_{wood} = 0.7 \times 10^3$ kg/m^3

What fraction (%) of wood is above water ?

Let m_{wood} = mass of wood

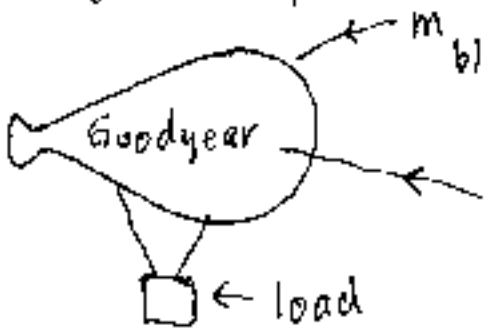
y = % above water

$$V_{wood} = \frac{m_w}{\rho_{wood}}$$

$$F_{buoy} = \rho_{water} \underbrace{(1-y)\frac{m_w}{\rho_{wood}}}_{\text{volume displaced}} \times g = m_w g = F_{down}$$

$$\therefore \quad 1 - y \approx \frac{\rho_w}{\rho_{water}} = \frac{.7 \times 10^3}{1 \times 10^3} = .7, \quad y = .3 = 30\%$$

Ex. Goodyear blimp



$m_{blimp} = 4250$ kg

5400 m^3 of helium gas

$\rho_{helium} = 0.178$ kg/m^3

Find $m_{load} = ?$

$$F_{buoy} = F_{down} = \left(m_{blimp} + m_{He} + m_{load}\right) g$$

F_{buoy}: $\rho_{air} \times 9.8 \times 5400 = 63509\ N$ (1.2 for ρ_{air})

m_{blimp}: 4250

m_{He}: 0.178×5400 (0.178 = ρ_{He}, 5400 = volume)

g: 9.8

$$\rightarrow m_{load} = 1263.3\ kg$$

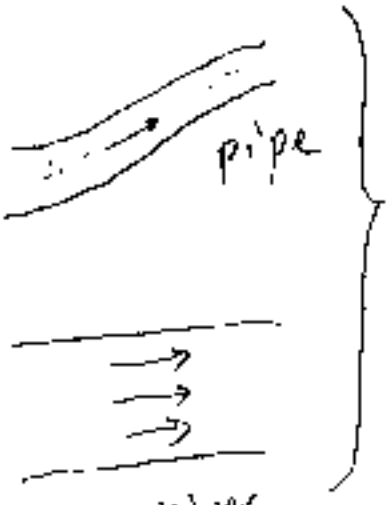
So far we have considered

water in a tank

.... swimming pool

..

where the fluid is not <u>flowing</u>



water in a pipe

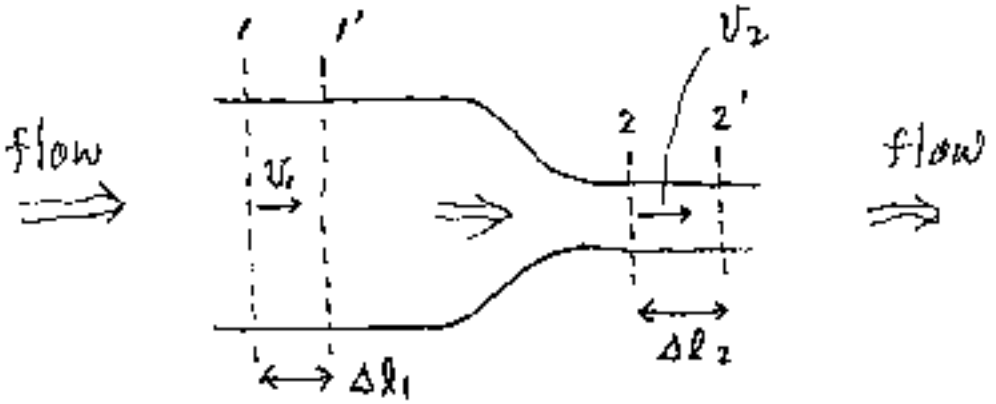
in a river

.....

here fluid is flowing

<u>fluid dynamics</u> to study fluids in motion

Consider fluid flow in a pipe:



imagine boundaries 1 and 2 at time 0

at time Δt, they move to 1' and 2'

pipe cross-section area at 1 = A_1

" " " " 2 = A_2

Volume of flow passing 1 } must = { Volume of flow passing 2

$A_1 \underbrace{\Delta l_1}_{v_1 \Delta t} = A_2 \underbrace{\Delta l_2}_{v_2 \Delta t}$

$\therefore$ $\boxed{A_1 v_1 = A_2 v_2}$ ← this is the important "continuity equation"

Ex.

$A_1 = 3\ cm^2$ $\quad A_2 = 1\ cm^2$

$v_1 = 8\ m/s$ $\quad v_2 = ?$

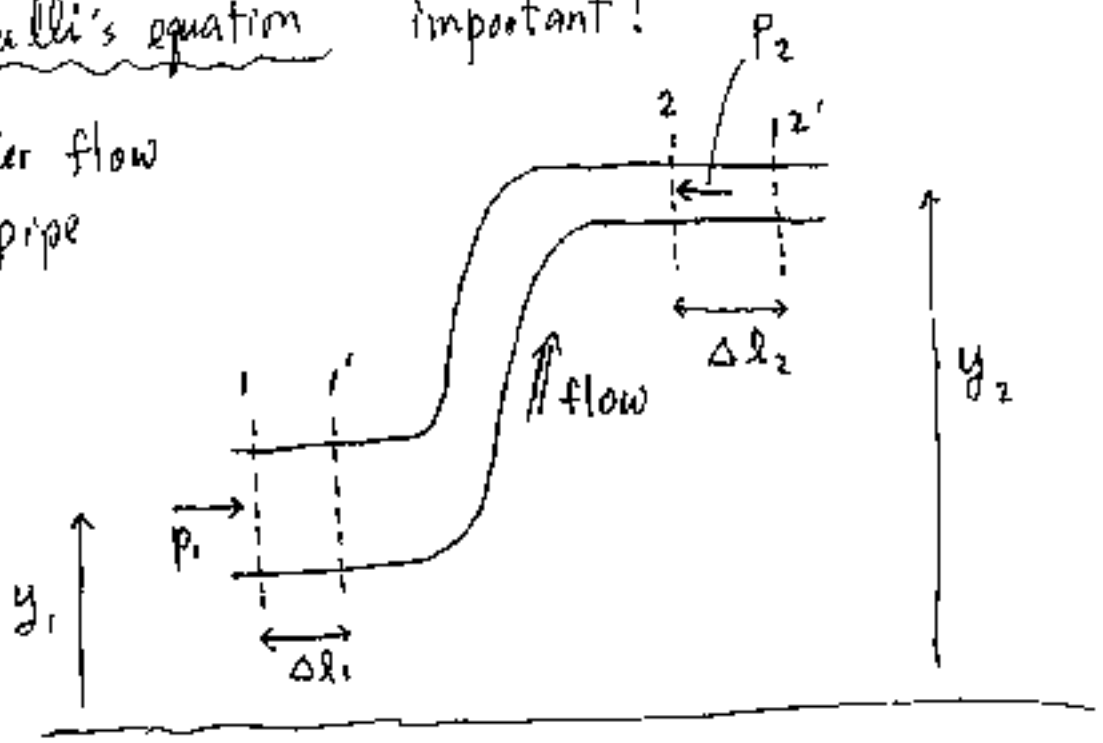
$A_1 v_1 = A_2 v_2$

3×10^{-4} $\quad 8$ $\quad 1 \times 10^{-4}$

$\therefore v_2 = 24\ m/s$

Bernoulli's equation important!

consider flow in a pipe



imagine boundaries 1 and 2

pressure p_1 > pressure p_2

$\Rightarrow$ 1 flows to 1', 2 to 2'

work done by p_1 and p_2

$$= F_1 \Delta l_1 - F_2 \Delta l_2 = p_1 A_1 \Delta l_1 - p_2 A_2 \Delta l_2$$

A_1 = pipe area at 1

A_2 = " " " 2

$$= (KE + PE)_{22'} - (KE + PE)_{11'}$$

$$= \frac{1}{2} m_{22'} v_2^2 + m_{22'} g y_2 - \frac{1}{2} m_{11'} v_1^2 - m_{11'} g y_1 \quad (*)$$

$m_{22'} = \rho A_2 l_2$, $m_{11'} = \rho A_1 l_1$

Volume of flow passing 1 = Volume of flow passing 2

i.e. $A_1 l_1 = A_2 l_2$

Hence ⊛ becomes

$$p_1 - p_2 = \frac{1}{2}\rho v_2^2 + \rho g y_2 - \frac{1}{2}\rho v_1^2 - \rho g y_1$$

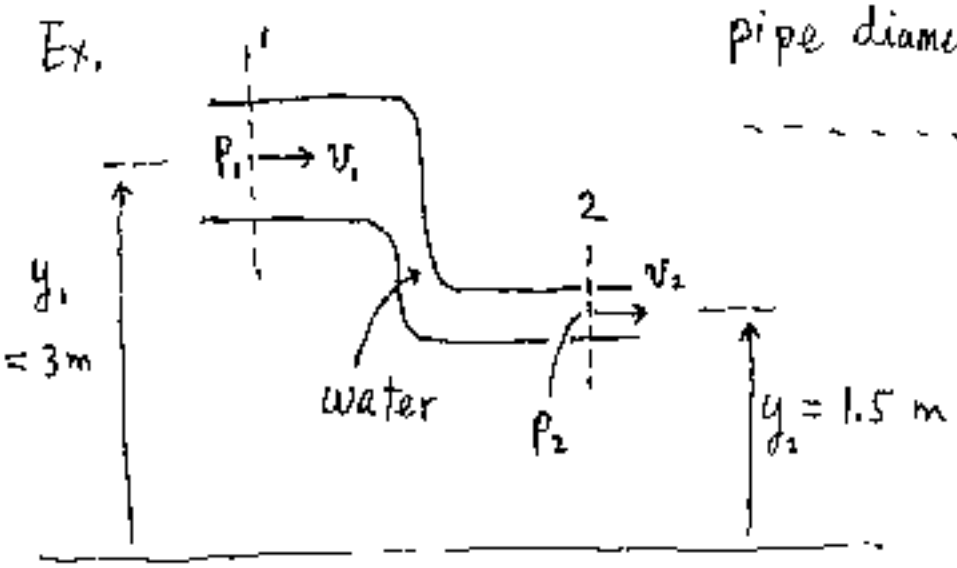
rearrang

$$\boxed{p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2}$$

Bernoulli's eq

p_a $\quad \frac{kg}{m^3}$ $\quad 9.8$ $\quad m$

Ex.



pipe diameter at 1 = d_1 = 4 cm

" " " 2 = d_2 = 2 cm

$p_1 = 15 \times 10^5$ Pa

$v_1 = 12$ m/s

find p_2, v_2

continuity eq: $A_1 v_1 = A_2 v_2$

$A_1 = \pi d_1^2$, $A_2 = \pi d_2^2$

$\pi d_1^2 = \pi (4 \times 10^{-2})^2$ $\quad = \pi (2 \times 10^{-2})^2$

$\therefore v_2 = 48$ m/s

now use Bernoulli's eq

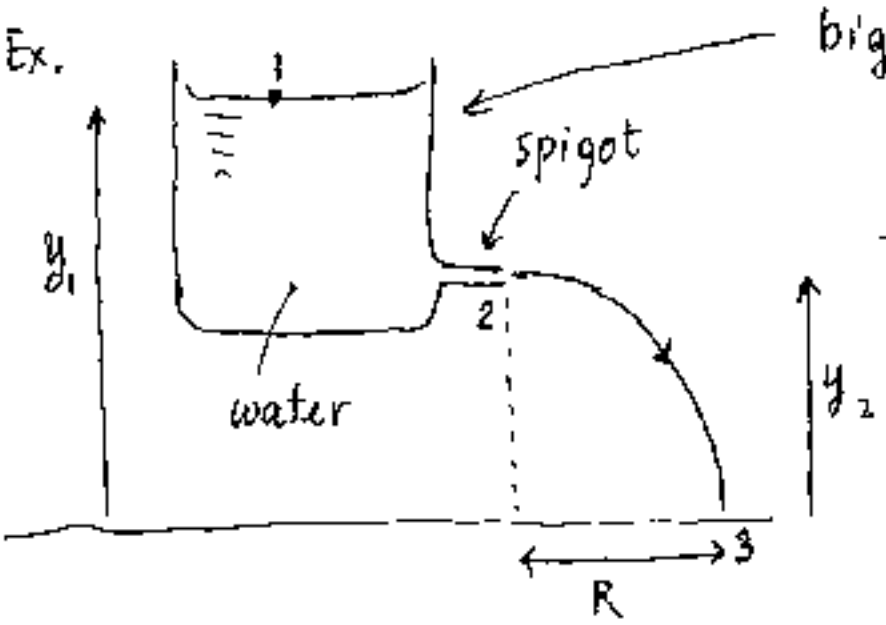
$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

p_1: 15×10^5 — $\frac{1}{2}\rho v_1^2$: 1000, 12^2 → 72000 — $\rho g y_1$: 1000, 9.8, 3 → 29400

p_2 — $\frac{1}{2}\rho v_2^2$: 1000, 48^2 → 11.52×10^5 — $\rho g y_2$: 1000, 9.8, 1.5 → 14700

$\therefore\ p_2 \simeq 4.35\times10^5\ p_a$, much less than p_1

Ex.



big water tank

$A_1 = 4\ m^2$

↑ tank area at 1

$A_2 = 2\ cm^2$

spigot area

$y_1 = 5\ m$

$y_2 = 1.6\ m$

find R

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 \quad (1)$$

$p_1 = p_{atm}$ ← cancel → $p_2 = p_{atm}$

continuity: $A_1 v_1 = A_2 v_2$

4 — 2×10^{-4}

$\therefore\ v_2 = 2\times10^4\ v_1$

$v_2 >> v_1$

can drop $\frac{1}{2}\rho v_1^2$

Hence ① becomes

$$\rho g (y_1 - y_2) = \frac{1}{2} \rho v_2^2$$

$$v_2 = \sqrt{2g(y_1 - y_2)} = 8.16 \text{ m/s} = v_{\text{horizontal}}$$

$9.8 \quad 5 \quad 1.6$

range: $R = v_{\text{horizontal}} \times t_{23}$

$8.16 \quad .57$

$= 4.66 \text{ m}$

$$t_{23} = \sqrt{\frac{2y_2}{g}} \quad \begin{matrix} \leftarrow 1.6 \\ \leftarrow 9.8 \end{matrix}$$

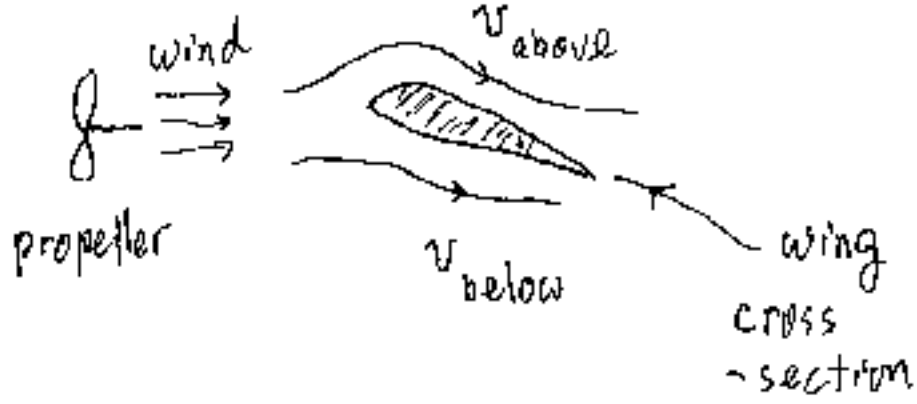
$= .57 \text{ sec}$

---

important applications of Bernoulli's eq:

Air plane

how it flies ?



Bernoulli →

$$\left(p + \rho g y + \frac{1}{2} \rho v^2\right)_{\text{above}} = \left(p + \rho g y + \frac{1}{2} \rho v^2\right)_{\text{below}}$$

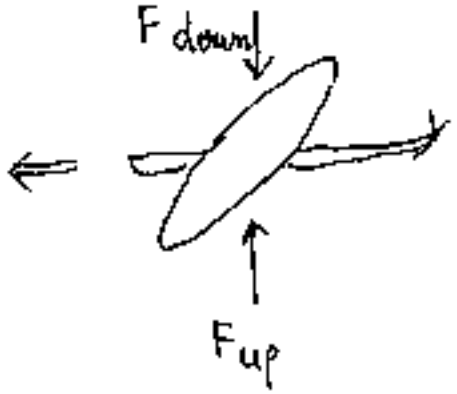
(1)

$y_{\text{above}} \approx y_{\text{below}}$ cancel

Wing is specially shaped, so that

$$v_{\text{above}} > v_{\text{below}}$$

Thus from ① we have $p_{below} > p_{above}$



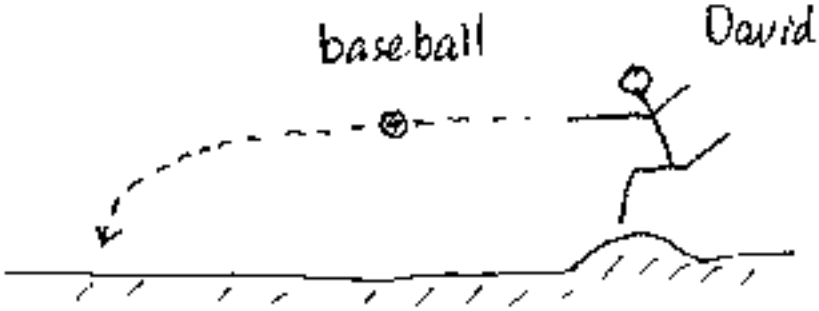
area of wing = A

$F_{net} = F_{up} - F_{down}$

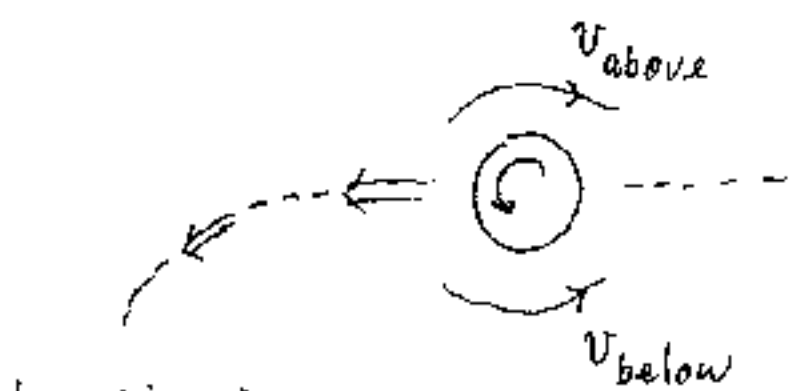
$= p_{below} \times A - p_{above} \times A$

airplane "floats" in air !

sinker ball



how he did it ? he puts top-spin ↻ as shown



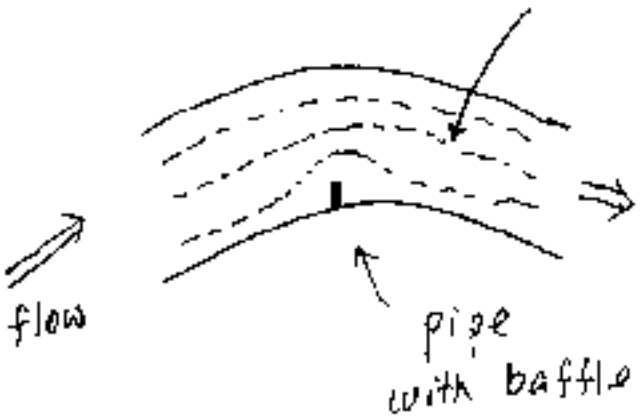
$v_{above} < v_{below}$

$\therefore p_{above} > p_{below} \Rightarrow$ ball curves down

ideal flow

here things are so nice
(flowers, nice beach, all A⁺'s ...)

sprinkle some ink to a flow, see beautiful streamlines.

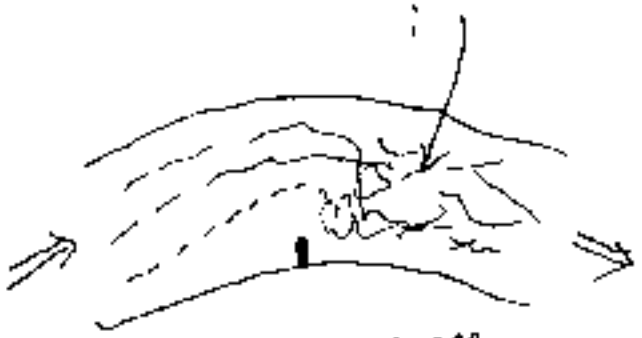


called laminar flow

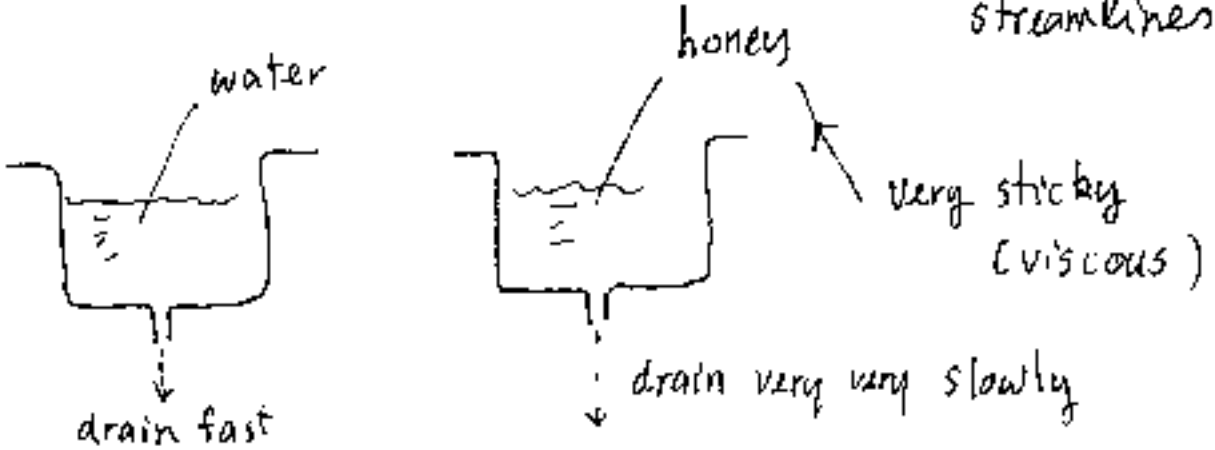
real-life flow

here things are disgusting!
(exams, income tax ...)

flow has friction viscosity turbulence



no more orderly, beautiful streamlines



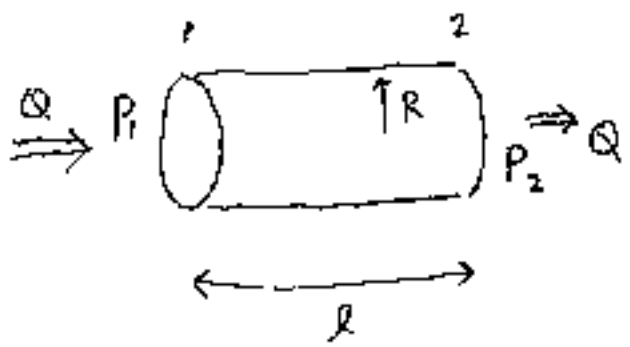
$\eta_{water} \ll \eta_{honey}$

η (eta) ≡ viscosity

↑ unit is [Pascal-sec]

Wed

Poiseulli's equation he was a medical doctor, studying blood flow in blood vessels.



consider a segment of artery

$P_1 > P_2$, blood flows from 1 to 2

Q = rate of flow [m^3/sec]

expect $(P_1 - P_2)$ larger → Q larger

R longer → Q more

he found

$$Q = \frac{\pi R^4}{8 \eta \ell} (P_1 - P_2)$$

m^3/sec $Pa\text{-}sec$ m Pa

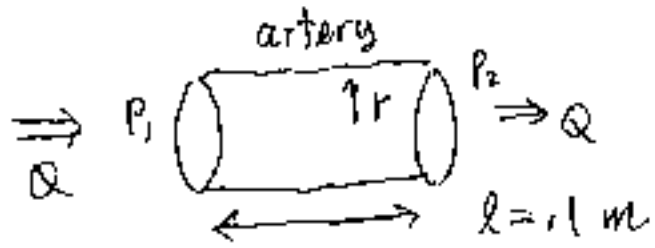
note: R^4

area $A = \pi R^2$

material	η (viscosity) at 20°C (room temp)
oil (motor)	0.9
water	1.0×10^{-3}
blood	3.0×10^{-3}

Ex. high blood pressure

healthy person

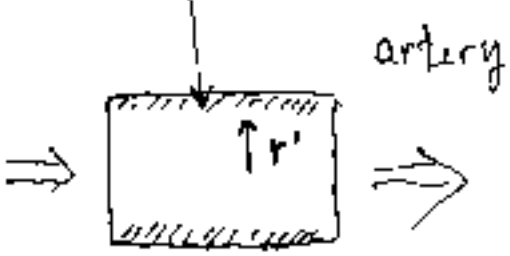


needs $Q = 10^{-6}$ m³/sec

$\Delta p = p_1 - p_2 = 2.1$ Pa

$r = .4 \times 10^{-3}$ m

clogged vein



$r' = .35 \times 10^{-3}$ m

needs same Q

$\Delta p'_{clog} = ?$

$$Q = \frac{\pi r^4}{8 \eta \ell} \Delta p = \frac{\pi r'^4}{8 \eta \ell} \Delta p'$$

$$\therefore \Delta p' = \left(\frac{.4}{.35}\right)^4 \times 2.1 = 3.58 \text{ Pa}$$

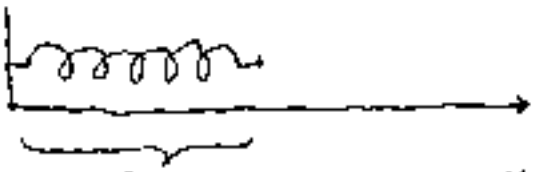
↑ much bigger than 2.1

heart has to work much harder → high blood pressure

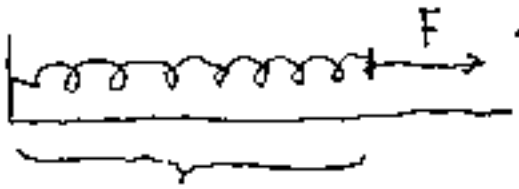
Chap. 10 mainly spring and SHM

SHM: simple, harmonic, motion

Spring



$l_{original}$ ← neither stretched nor compressed



← apply a pulling force F

l_{new}

$$l_{new} - l_{original} = \text{stretch} \equiv x$$

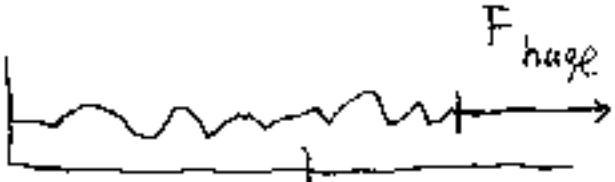
suppose we try

F [N]	0	2	4	8	0
x [m]	0	.01	.02	.04	0

↑ when force removed, spring resumes original length

⇒ spring is elastic

But when F is too big, spring is ruined,



in this case, spring will not shrink back

to its original shape, because we have exceeded its

elastic limit

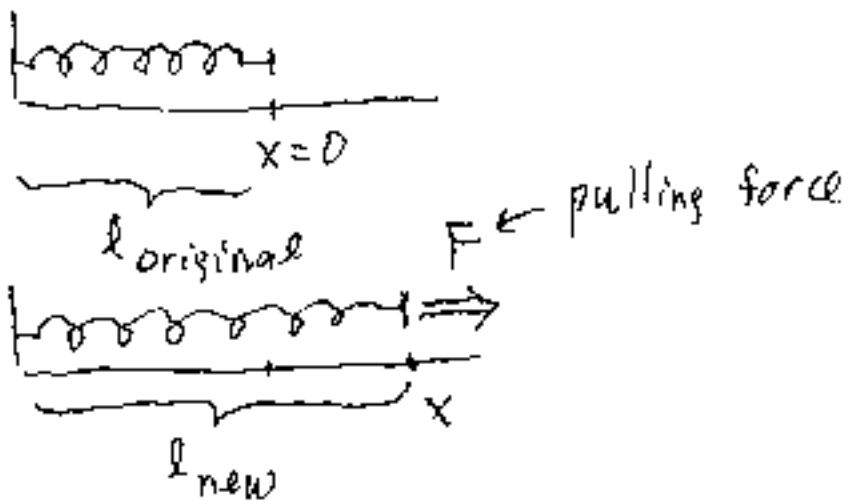
For moderate force (i.e. within elastic limit), we have

$F = kx$ Hooke's law

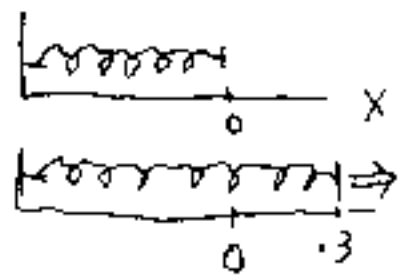
F = applied force

$x = \ell_{new} - \ell_{original}$

k: N, spring constant, N/m



Ex. $k = 5$ N/m, $x = .3$



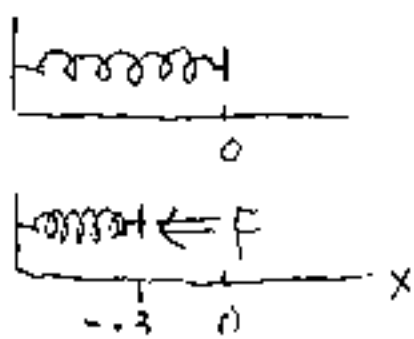
$F = ?$

$F = 5 \times .3 = 1.5$ N

direction in ⟹

Ex. $k = 8$ N/m, $x = -.3$

note signs



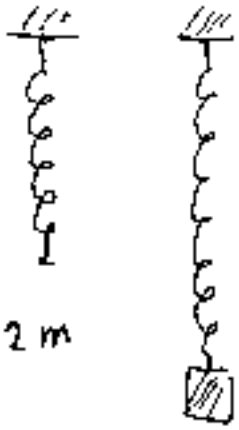
$F = kx = 8 \times (-.3)$

$= -2.4$ N

direction in ⟸

See $F = kx$ is applicable to both stretching and compressing of spring

Ex.



$k = 150$ N/m

$l_{orig} = 1.2$ m

mass m is hung

$m = 4$ kg

find $l_{new} = ?$

$F = ky$

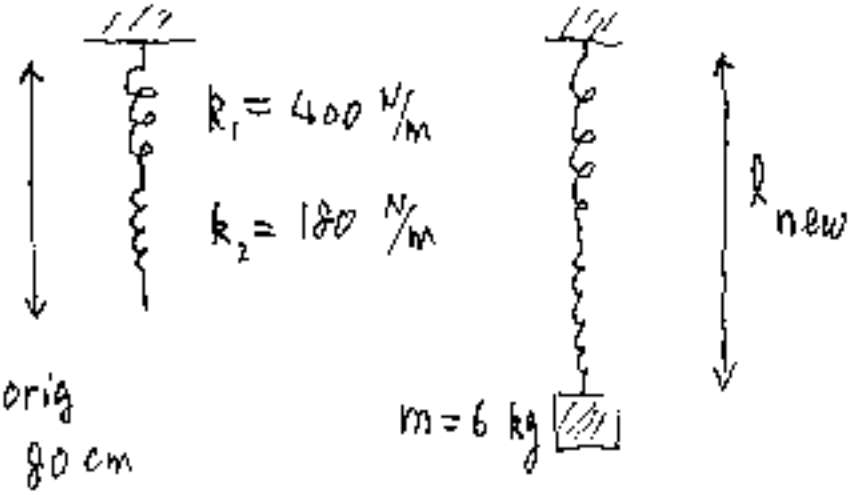
4×9.8 $\quad$ 150

$\therefore y = 0.26$ m

$l_{new} = l_{orig} + y$

$= 1.2 + .26 = 1.46$ m

Ex. two springs :



Find $l_{new} = ?$

Note spring-1 and spring-2 are subject to same force mg

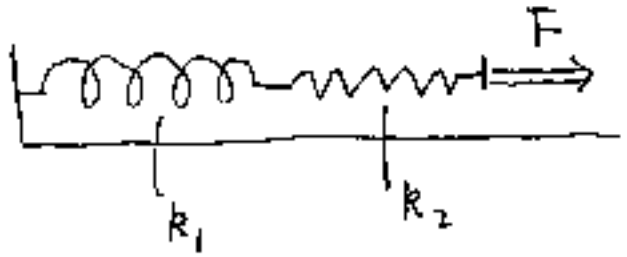
$\therefore \quad F = mg = 6 \times 9.8 = k_1 y_1 \quad (k_1 = 400) \quad \rightarrow y_1 = .147$ m

also $\quad F = mg = 6 \times 9.8 = k_2 y_2 \quad (k_2 = 180) \quad \rightarrow y_2 = .327$ m

$$l_{new} = l_{orig} + y_1 + y_2 = .8 + .147 + .327 = 1.274\ m$$

---

2 spring in general:



x_1 = stretch of spring-1

x_2 = ------ spring-2

Spring-1 is subject to force F, $\therefore \quad F = k_1 x_1$ -- (1)

Spring-2 is also ----------, $\therefore \quad F = k_2 x_2$ -- (2)

$$(1),(2) \Rightarrow (x_1 + x_2) = \frac{1}{k_1} F + \frac{1}{k_2} F = \left(\frac{1}{k_1} + \frac{1}{k_2}\right) F$$

$$= \frac{F}{\left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}} \quad \cdots (3)$$

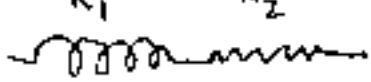
rename $x_{tot} = x_1 + x_2$, $\quad k_{tot} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$

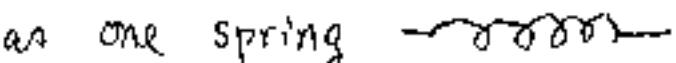
or $\frac{1}{k_{tot}} = \frac{1}{k_1} + \frac{1}{k_2}$

(3) becomes $F = k_{tot}\, x_{tot}$

k_{tot} is also called $k_{effective}$

Summary



is same as one spring  with k_{eff}