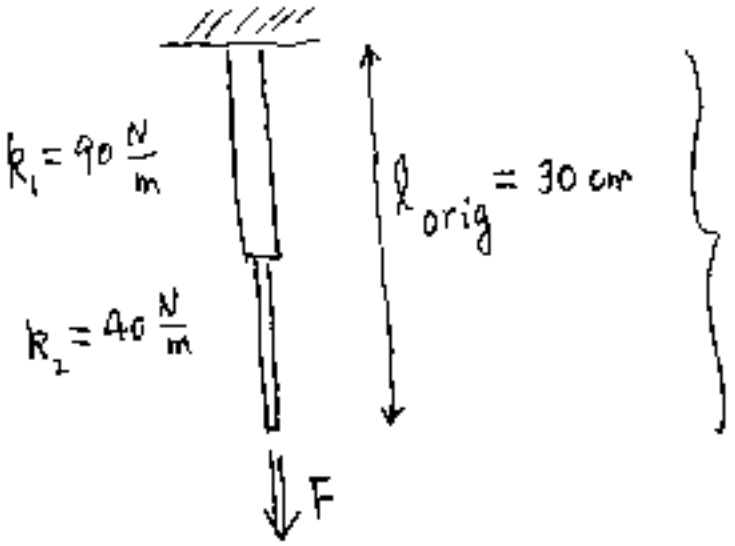
Ex. 2 rubber bands (11/2/98)

rubber bands "same" as spring



equivalent to one rubber band with k_{eff}, i.e.

find F = ? in order to stretch it to $l_{new} = 33$ cm

what is k_{eff}?

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{90} + \frac{1}{40} = \frac{130}{3600}$$

$$\rightarrow \quad k_{eff} = 27.69 \ N/m$$

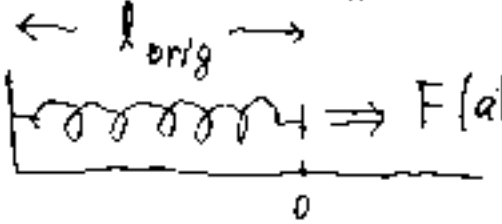
$$\Delta l = l_{new} - l_{orig} = 33 - 30 = 3 \text{ cm} = 0.03 \text{ m}$$

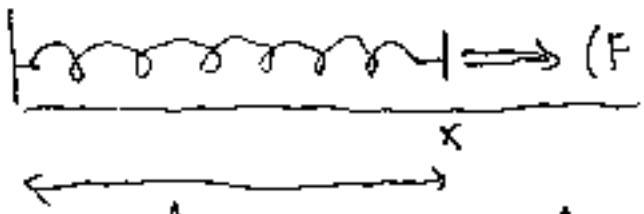
$$F = k_{eff} \, \Delta l = 27.69 \times 0.03 = 0.831 \ N$$

Check: rubber band-1 stretches $s_1 = \frac{0.831}{90} = 0.0092$

---------2 ----- $s_2 = \frac{0.831}{40} = \frac{0.0208}{0.03}$ (+ OK

Work done in stretching (or compressing) a spring :





$\ell_{new} - \ell_{orig} = x$

Work by F = force × distance

↖ it varies with x, $F = kx$, Hooke's law

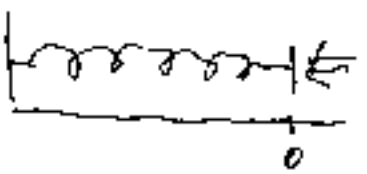
↑ linear relation, take average

$$= \frac{(F_{at\,0} + F_{at\,x})}{2} \cdot x$$

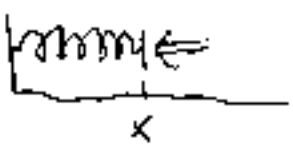
$$= \tfrac{1}{2}(0 + kx)x = \tfrac{1}{2}kx^2$$

∴ Work in stretching spring from 0 to x $= W = \frac{1}{2}kx^2$

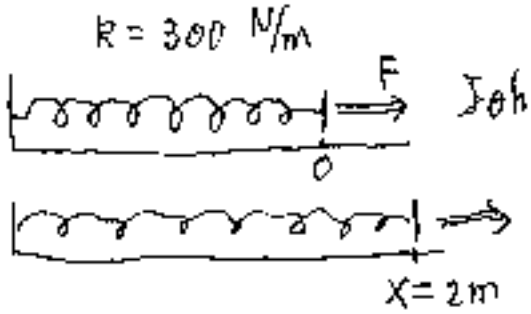
For compressing, $x < 0$, have same result, i.e.



Work in compressing spring from 0 to x



$= W = \frac{1}{2}kx^2$

Ex, 

John pulls it from $x = 0$ to $x = 2m$

$$\text{Work}_{\text{by John}} = ? = \frac{1}{2} k x^2 = \frac{1}{2} \times 300 \times 2^2 = 600 \text{ J}$$

---

when you push a car, you change the KE of the car

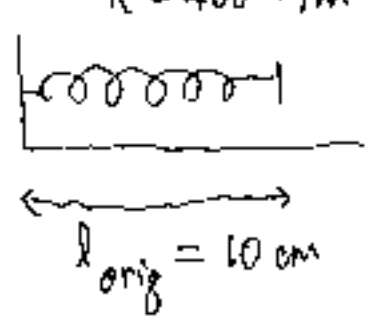
when you lift a brick, ... PE of the brick

what are you doing in stretching (compressing) a spring ?

Ans: Work in stretching or compressing a spring $\Big\} = PE_f^{\text{spring}} - PE_i^{\text{spring}}$

potential energy stored in spring $\Big\} PE = \frac{1}{2} k x^2$

$$x = (\ell_{new} - \ell_{orig})$$

Ex, 

$$PE^{\text{spring}}_{\ell = 15\text{ cm}} = ?$$

$$= \frac{1}{2} \times 400 \times (.15 - .10)^2$$

$$= 0.5 \text{ J}$$

For the same spring

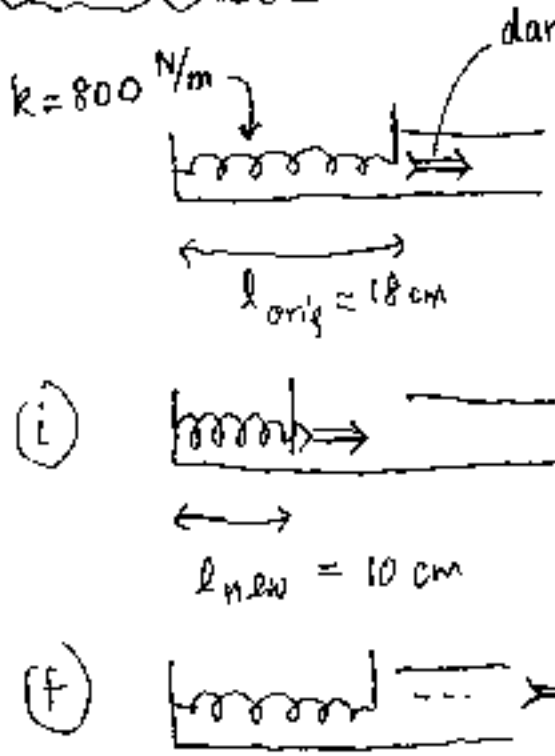
Work for stretching it from 15 cm to 20 cm $= ? = PE_{20} - PE_{15}$

PE_{20}: $\frac{1}{2} \times 400 \times (.2 - .10)^2$

PE_{15}: $\frac{1}{2} \times 400 \times (.15 - .10)^2$

$= 2 - .05 = 1.5\ J$

Ex. dart gun



$E_i = E_f$

$PE^i_{spring} + PE^i_{dart} + KE^i_{dart} = PE^f_{spr} + PE^f_{dart} + KE^f_{dart}$

$PE^i_{dart} = PE^f_{dart}$, ∵ same height

$KE^i_{dart} = 0$, $PE^f_{spring} = 0$

$\therefore \quad PE^i_{spring} = KE^f_{dart}$, $\frac{1}{2} k x^2 = \frac{1}{2} m v_d^2$

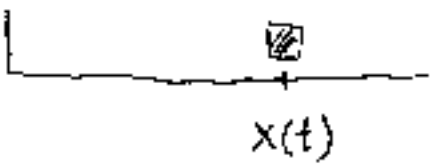
$k = 800$, $x = (.18 - .10)$, $m = 0.2$

$\Rightarrow v_d = 5.06\ m/s$

Suppose the motion of an object m is described by

$$x(t) = A\cos\omega t$$

(m) (m) (1/sec) (sec)

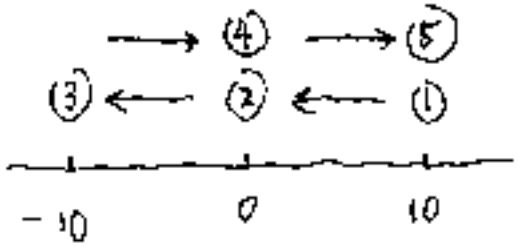


what it means ?

Suppose $A = 10$ m, $\omega = .5 \frac{1}{sec}$ then

	t	ωt	$\cos\omega t$	$x(t)$
①	0	0	1	10
②	$\frac{\pi}{2}/.5$	$\pi/2$	0	0
③	$\pi/.5$	π	-1	-10
④	$\frac{3\pi}{2}/.5$	$\frac{3\pi}{2}$	0	0
⑤	$2\pi/.5$	2π	1	10

see motion oscillates between $x=10$ and $x=-10$



⑤ and ① are same position

time for one cycle = t_{12341} = period = T

$\omega T = 2\pi \rightarrow T = \frac{2\pi}{\omega}$, ω = angular velocity

T: sec, ω: sec^{-1}

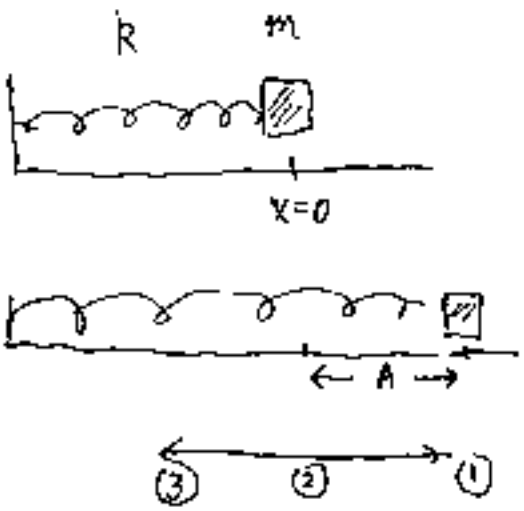
brief review:

$x(t) = A\cos\omega t$

- $x(t)$: position of an oscillating object, as a function of time [m]
- A: amplitude, $x_{max} = +A$, $x_{min} = -A$ [m]
- ω: angular velo. [$\frac{1}{sec}$]

period $T = \frac{2\pi}{\omega}$ [sec], frequency $f = \frac{1}{T}$ [Hertz] ie. numbers of oscillations per sec

simple oscillator



pull to point ①, let go.

Its motion is given by

oscillates ① → ② → ③ → ② → ① ...

$x(t) = A\cos\omega t$

$\omega = \sqrt{\frac{k}{m}}$ — k: N/m, m: kg, ω: 1/sec

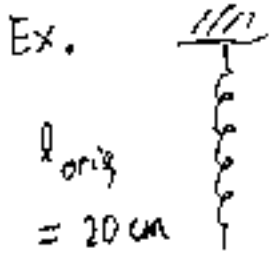
$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ (*) — T: sec

discussion at (*): $m \uparrow$ than $T \uparrow$; big m → slow oscillation → period longer

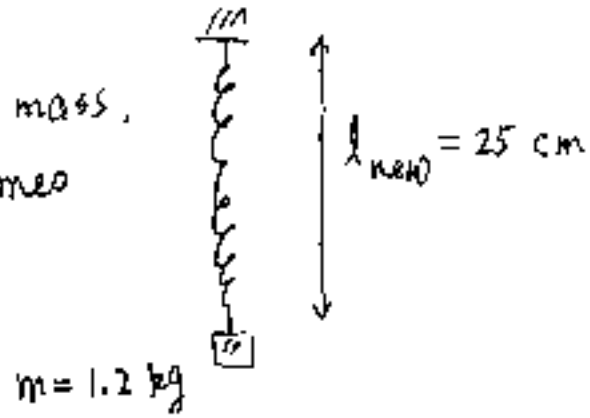
$k \uparrow$ then $T \downarrow$;

k big means strong spring → fast oscillation → period shorter

Ex.



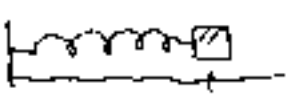
attach mass, becomes



When it oscillates up and down, $T = ?$

$F = ky$ $\quad 1.2 \times 9.8 = k \times (.25 - .20)$, $\therefore k = 235.2$ N/m

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \begin{matrix} \leftarrow 1.2 \\ \leftarrow 235.2 \end{matrix} \quad = 0.45 \text{ sec}$$



$x(t) = A\cos\omega t$ ← position of oscillator

velocity $\quad v(x) = \frac{dx}{dt} = -A\omega\sin\omega t$

acceleration $\quad a(t) = \frac{dv}{dt} = -A\omega^2\cos\omega t$

$= -\omega^2 x(t)$

$$KE_{oscillator} = \frac{1}{2} m v^2 = \frac{1}{2} m A^2 \omega^2 \sin^2 \omega t$$

$$PE_{spring} = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2 \omega t$$

$$E_{total} = KE_{osc} + PE_{spring}$$

$$= \frac{1}{2} A^2 m\omega^2 \sin^2 \omega t + \frac{1}{2} k A^2 \cos^2 \omega t$$

$m\omega^2 = k$, recall $\sin^2\theta + \cos^2\theta = 1$

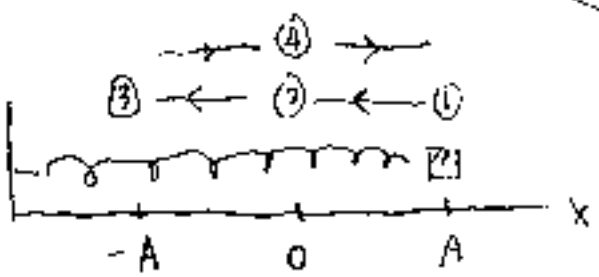
$$= \frac{1}{2} k A^2$$

∴ total energy

E (oscillator + spring) $= \frac{1}{2} k A^2$

all the time

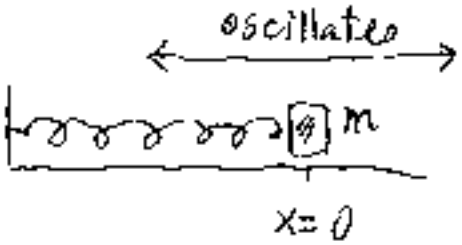
oscillates



position	PE	KE	velocity	acceleration	$\lvert x \rvert$
①	max, $\frac{kA^2}{2}$	0	0	max	max
②	0	max, $\frac{kA^2}{2}$	max	0	0
③	$\frac{kA^2}{2}$	0	0	max	max
⋮			↑	↑	
			$-A\omega \sin\omega t$	$-A\omega^2 \cos\omega t$	

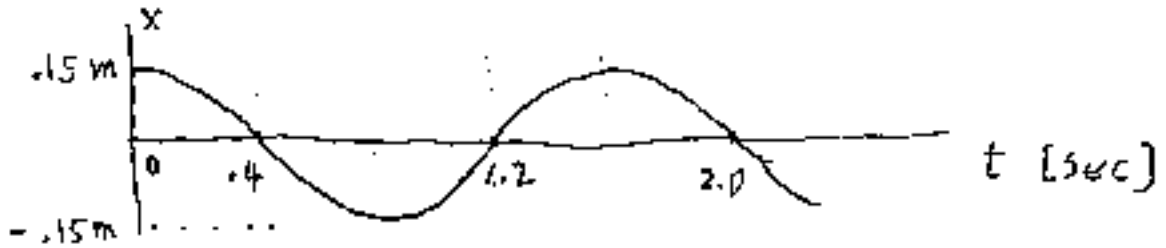
sum always $= \frac{1}{2} k A^2$

Ex.



$m = 0.2$ kg

Its motion is measured as



angular velocity $\omega = ?$ 1st node $\rightarrow$ $\omega \times .4 = \frac{\pi}{2}$ $\cos \underbrace{\omega t}_{\frac{\pi}{2}} = 0$

$\therefore \omega = 3.92 \text{ sec}^{-1}$

spring constant $k = ?$ $\omega = \sqrt{\frac{k}{m}}$ $\therefore k = 3.08$ N/m

3.92 .2

total energ $E = \frac{1}{2} k A^2 = \frac{1}{2} \times 3.08 \times .15^2 = 0.035$ J

Period $T = 2.0 - .4 = 1.6$ sec from figure

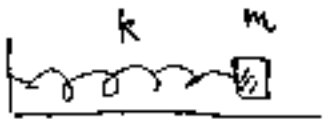
or $T = \frac{2\pi}{\omega} = \frac{2\pi}{3.92} = 1.6$ sec

at $t = 1.4$ sec, $v = ? = -A\omega \sin \underbrace{\omega t}_{3.92 \times 1.4 \times 57.3}$

.15 3.92 rad degree/rad

$= +.42$ m/s

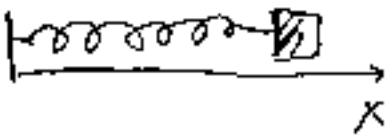
$v_{max} = ? = A\omega = .15 \times 3.92 = 0.588$ m/s

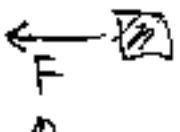
For oscillator 

we have used $x(t) = A\cos\omega t$, $\omega = \sqrt{k/m}$

We have not derived this result yet. Let us now derive it:

free body diagram





force from spring $= -kx$

if $F = ma$ is

$$-kx = ma = m\frac{d^2x}{dt^2} \quad \cdots (1)$$

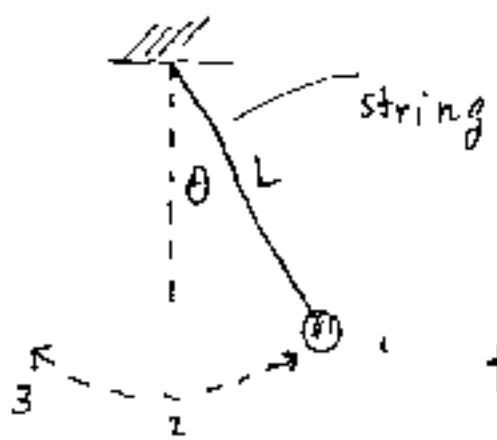
try $x = A\cos\omega t$, substitute into (1):

$$-kA\cos\omega t = -mA\omega^2\cos\omega t$$

Hence $x = A\cos\omega t$ is a solution of (1) provided

$k = m\omega^2$, i.e. $\omega = \sqrt{\frac{k}{m}}$ ← N/m, ← kg, ↑ 1/sec

Simple pendulum



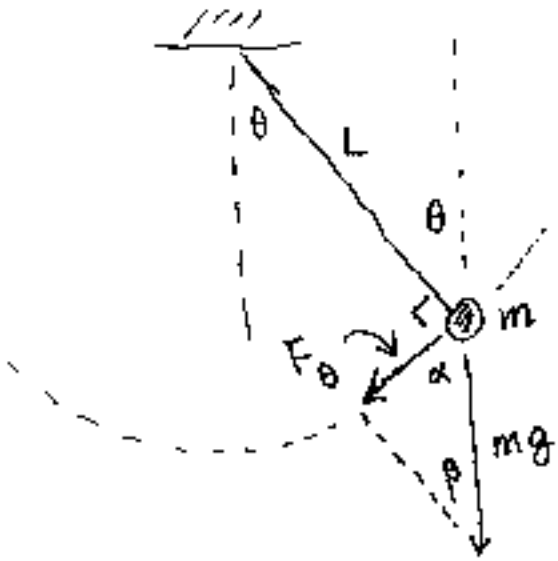
oscillates

1 → 2 → 3 → 2 → 1

→ repeat

t_{12321} = period = T

Let us study motion of pendulum



m moves in θ direction

F_θ = force in θ direction

$= -mg \sin\beta$

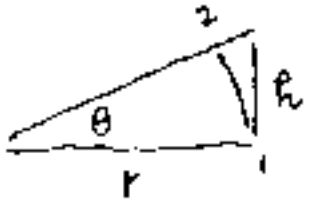
$\theta + \alpha = 90^\circ$

$\beta + \alpha = 90^\circ$

$\therefore \beta = \theta$

$F_\theta = -mg\sin\theta = m a_\theta$

$= m L \frac{d^2\theta}{dt^2}$ ①



$\sin\theta = \frac{h}{r}$

$\theta = \frac{\text{arc } \widehat{12}}{r}$

for small θ, $h = \widehat{12}$

$\Rightarrow \theta = \sin\theta$

Hence for small θ, ① becomes

$$-\frac{g}{L}\theta = \frac{d^2\theta}{dt^2}$$

same form as the eq.

$$-\frac{k}{m}x = \frac{d^2x}{dt^2}$$

for [illegible]

$\therefore$ solution is

$\theta = \theta_{max} \cos \omega t$

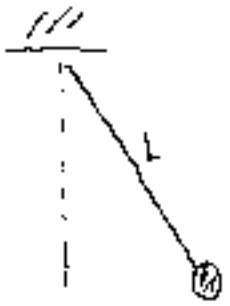
$\omega = \sqrt{\frac{g}{L}}$, period T is $T = 2\pi\sqrt{\frac{L}{g}}$ ← m ← m/sec^2

15-15 a

Phys-121 Exam-2 Seating

Thursday 11/12/98 8:30pm

Lab. Sections	Room
6, 7, 12, 13, 16, 17, 18	$Javits-100$
1, 3, 4, 9, 11, 14	$ESS-001$
8, 10, 19, 20	$Light-Eng-102$
2, 5, 15	$Light-Eng-152$

Ex. 

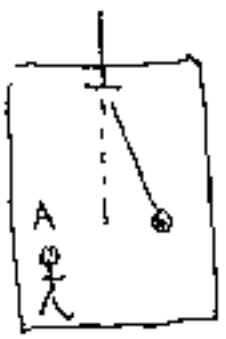
its period is measured as 1.5 sec

$L = ?$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$L = \frac{T^2 g}{(2\pi)^2} = \frac{1.5^2 \times 9.8}{(2\pi)^2} = 0.559 \text{ m}$$

Same pendulum put in elevator:



$a_{elevator} = 3\ m/s^2$ ↑

T_A = period as measured by Mr. A

$= ?$

$$T_A = 2\pi\sqrt{\frac{L}{g_{eff}}}$$

$L = 0.559$

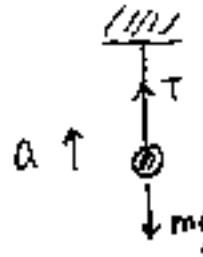
$g_{eff} = 12.8$

$\therefore T_A = 1.31$ sec

shorter than 1.5 sec

i.e. pendulum now swings faster.

when elevator accelerates upward



$T - mg = ma$

$T = m(g+a)$

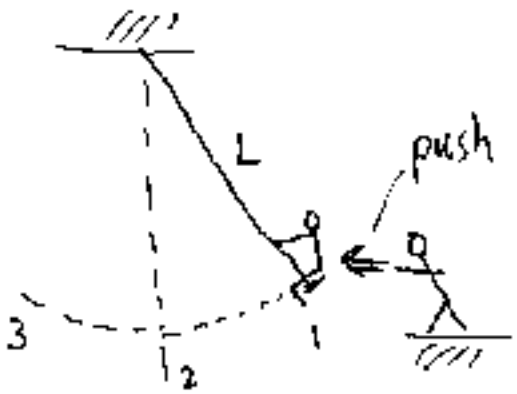
$= m g_{eff}$

tension T bigger than mg,

here $g_{eff} = 9.8 + 3 = 12.8$

Resonance

swing



when pushing a swing, you must push at the right time

swing motion: 1 2 3 2 1 2 3 2 1 ⋯→

T_{swing}

push (back & forth): ↑ push, back ↑ push, back ↑ push

T_{push}

$$T_{swing} = 2\pi\sqrt{\frac{L}{g}}$$

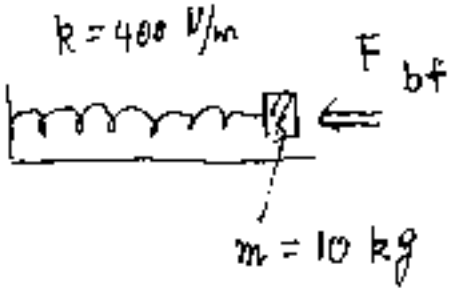
To have the child swing higher and higher, the push must match the motion of the swing, i.e. we need

$$T_{push} = T_{swing} \quad \text{or} \quad f_{push} = f_{swing}$$

$$f = \frac{1}{T}$$

When $f_{push} = f_{swing}$, we say <u>resonance</u> is achieved.

Ex.



push the oscillator with a back-and-forth force F_{bf}

its back-and-forth frequency is f_{bf}

what f_{bf} should one use in order to have resonance?

first find f_{spring} (when the spring is by itself)

$$\omega_{spring} = \sqrt{\frac{k}{m}} = \sqrt{\frac{400}{10}} = 6,32 \ sec^{-1}$$

$$f_{spring} = \frac{\omega_{sp}}{2\pi} = \frac{6.32}{2\pi} = 1.007 \ Hz$$

∴ to have resonance, should use $f_{bf} = 1.007$ Hz.