Wave velocity and refractive index:

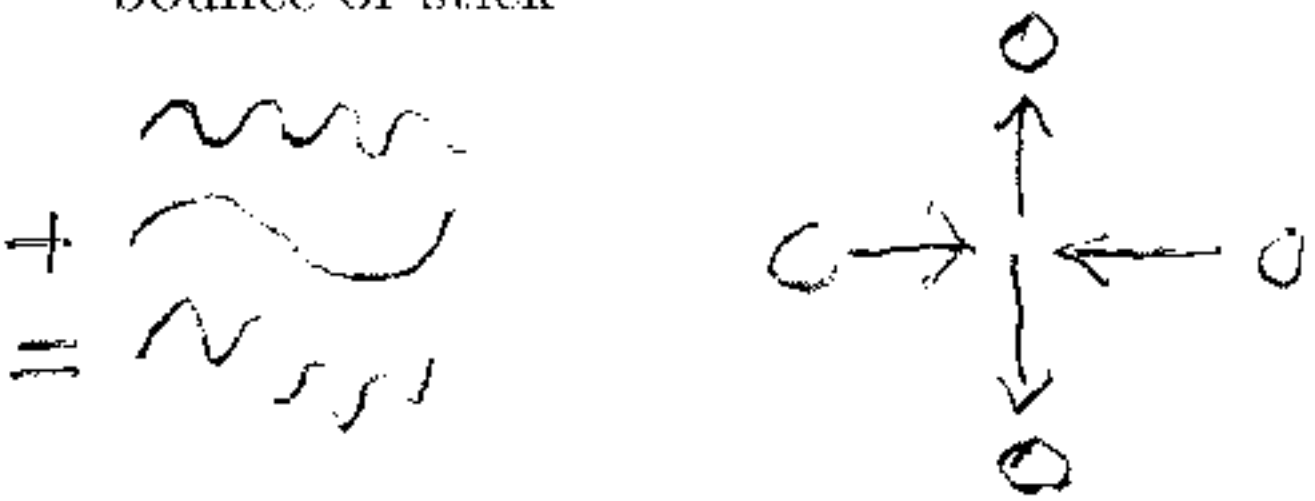
The refractive index for a wave of a certain type in different media gives the relationship between wave velocities in the different media, $n_1 c_1 = n_2 c_2$. For example, light waves travel in vacuum at a speed of $c = 3 \times 10^8$ m/s. Treating that as the standard, by definition the refractive index of empty space is $n = 1$. For any other medium i, the refractive index and the light speed in that medium are related by $n_i c_i = c$, so the bigger n_i the smaller the speed c_i. How does this connect the wavelengths of waves in different media? Answer: For a single, steady wave going from one medium to another, frequency f NEVER changes. Because $c = f\lambda$, this means $n_1 \lambda_1 = n_2 \lambda_2$. Therefore, the higher the refractive index the shorter the wavelength.

A sound wave in air has a wavelength of

340 m/s, and in water of 1500 m/s. For a wave with wavelength in air $\lambda = 0.17$ m, what is the frequency and wavelength in water? $f = c_{air}/\lambda_{air} = 2000$ Hz. Therefore 2000 Hz $= f_{water} = c_{water}/\lambda_{water}$, or $\lambda_{water} = 1500/2000 = 0.75$ m. This means $n_{water}/n_{air} = 0.227$.

Interference:

Waves add, not bounce – Particles bounce or stick



Standing waves: Different traveling waves add.

Beats: Waves add constructively, then cancel, then add again.

Interference and diffraction from slits: Two-slit interference: Maximum intensity for $d\sin\theta = n\lambda$.

Example: For green light, $\lambda = 0.55\mu$ [1 m = 10^6 μ.], and two slits separated by $d = 0.5$ cm, how far from the center peak is the 5th side peak, if the observation screen is 15 m away from the slits? Answer: $\Delta x = (5\lambda/d) \times 15$ m =0.825×10^{-2} cm.

Single-slit diffraction: Minimum intensity for $w\ \sin\theta = m\lambda$.

Example: What is angle of 2nd minimum for single slit of width 16.5 μ, $\lambda = 0.55\mu$? Answer: In small-angle approximation, $\theta \approx 2\lambda/d$=0.067 rad =$3.84^o$, which is just at the border of accuracy for the small-angle approximation. It's interesting to try with a calculator what happens if you use $\sin\theta$ in place of θ.

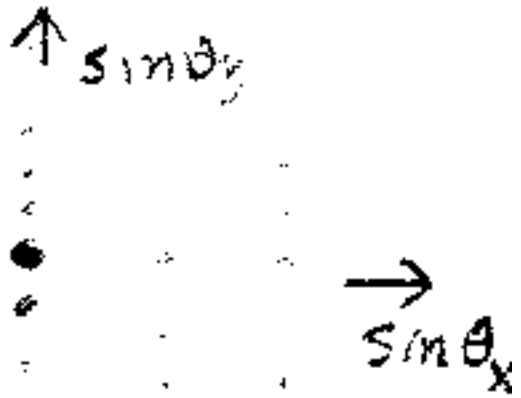
More than two slits, spacing a: Sharp maxima for $a\sin\theta = n\lambda$, suppressed by sin-

gle slit diffraction for $w\sin\theta = m\lambda$.

Example: Width $w = 16.5\mu$, with $\lambda = 0.55\mu$, and spacing $a = 99\mu$. What is angle spacing of sharp peaks? Wher do peaks first vanish? Answers: $\Delta\theta = \lambda/a = .00556$ rad, $\theta_{min} = \lambda/w = 0.0333$ rad, meaning the 6th side peak vanishes.

Diffraction from a hole:

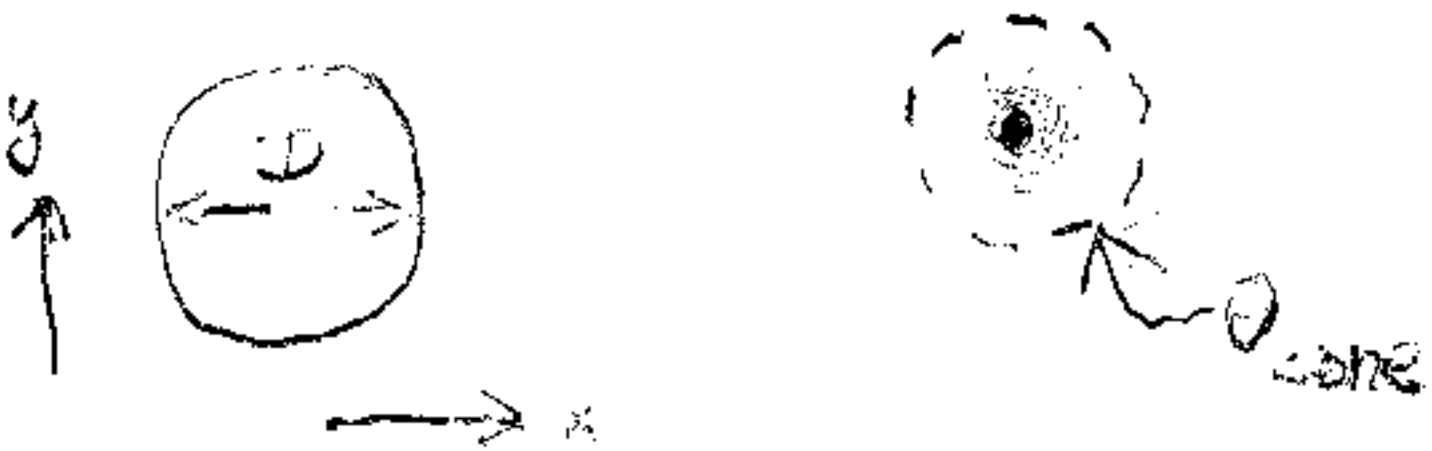
1)Rectangular:
$a\ \sin\theta_x = m_x\lambda_x, b\ \sin\theta_y = m_y\lambda_y.$



2)Circular:

Parallel rays smeared out to θ_{cone}:

$D\sin\theta_{cone} = 1.22\lambda$. Diffraction limit on resolution of lens.



Diffraction of x-rays from crystal planes,

spacing a:

Maxima for scattering angles

2θ, with $2a\sin\theta = n\lambda$. Note: If $\theta_{scatt} \equiv 2\theta$, then at small angles get $a\theta_{scatt} = n\lambda$, same as the slit diffraction formulas!

When diffraction angles get big: Maximum number of diffraction peaks: $a\sin\theta = n\lambda$. Because $\sin\theta \leq 1, n_{max} \leq a/\lambda$. Typical: $a = 1.7\mu$, $\lambda = 0.6\mu$ implies $n < 3$.

Standing waves in more than one dimension: Example: Square drumhead. Edges of drum fabric held down, with stretching tension $T/L = 2$ N/cm, Mass density $M/L^2 = 0.5$ kg/m^2, giving wave speed $c = \sqrt{(T/L)/(M/L^2)} = 20$ m/s. Measuring x and y from one corner, the waves are $h(x, y, t) = A \cos 2\pi f t \sin 2\pi x/\lambda_x \sin 2\pi y/\lambda_y$, where h has to vanish for $x = L$ and $y = L$. This gives $2L = (N_x + 1)\lambda_x = (N_y + 1)\lambda_y$, N_x, N_y numbers of interior nodes in x and y directions.

For frequency f, need wavelength λ of traveling wave in standing wave. For minimum, $\lambda_x = \lambda_y = 2L$. What is λ? Picture shows $\lambda_x = \sqrt{2}\lambda$ so $\lambda = \sqrt{2}L$, or $f = c/\sqrt{2}L = 20$ m/s/(1.41)0.1 m $= 14.1$ Hz.

General case: $1/\lambda^2 = 1/\lambda_x^2 + 1/\lambda_y^2 + \ldots$.

Finally, we come to the puzzler from the end of last week: For a pipe open at both ends, what is the lowest frequency f_0? Answer: This time the amplitude of the standing wave, Δx, when plotted versus position x along the pipe, must come in horizontally, or flat, at either end. This is quite obvious for the analogue case of a string attached to sliders which can move freely up and down at both ends, but equally true for the pipe. A solution to these conditions is an amplitude which is the same for all x, just a straight horizontal line in the plot. This clearly implies infinite wavelength, and therefore $f_0 = 0$.

The summary of allowed frequencies for a pipe or string depending on the conditions at the ends is

closed-closed: $f_N = (N+1)/2L$, where N is the number of nodes or zeros in the amplitude at points between the ends.

open-closed: $f_N = (N + 1/2)/2L$.

open-open: $f_N = N/2L$.

If you listened to the frequencies, and were sensitive even to $f = 0$, you could tell what kinds of ends the pipe or string had. It's possible to do almost as well for a drumhead with some unknown shape, but you would need a much more sensitive ear!