PHY 121/123 LECTURE NOTES – Week 13

Temperature, thermal properties of gases, and the molecular hypothesis

Temperature is a concept which has intuitive meaning for all of us, but when we want to make it precise we are forced to define it by the ways we measure it. The important thing is that a variety of different ways all give consistent results, making the concept as real to us as anything can be which we cannot touch, see, hear, taste, or smell. It's interesting to reflect on that statement: We cannot touch temperature, but by touching we can sense whether something is hotter or colder than we are. We cannot see temperature, but seeing something glow brightly gives a clue that it is hot. We cannot hear temperature, but the sound of a roaring fire again is a clue to heat, and the sound of crackling ice is a clue to

cold. As with touch, we cannot taste temperature, but our tongue certainly is sensitive to heat or cold. We cannot smell temperature, but a scorched or burning smell again is a clue to heat. In other words, we all use clues from our senses to deduce at least qualitative information about how hot or cold something is. During the next couple of weeks, among other things we shall be looking at more precise and quantitative ways to evaluate temperature.

The crucial point is to find things we can measure which either are proportional to temperature, or which change slightly in a known way proportional to a slight change in temperature. There are a lot of these, and we'll only look at a few. To start, let's introduce scales or units for quoting temperature. The first is the Fahrenheit scale, defined by making the freezing point of water 32 °F and the boiling point 212 °F. This

makes the difference between the two 180° in these units. The next is the Celsius or centigrade scale, with the freezing point 0 °C and the boiling point 100 °C. In these units the difference is 100°, which is behind the name 'centigrade'. From these you can work out the relationship between temperatures measured in the two units:

$$T\ ^\circ\mathrm{F} = 32 + (9/5)T\ ^\circ\mathrm{C}.$$

Now we introduce one more, the Kelvin or absolute scale, in which $T = 0$ corresponds to the lowest possible T. To make things a bit simpler, a change of one degree in the Kelvin scale is also a change of one degree in the Celsius scale:

$$T\ \mathrm{K} = T\ ^\circ\mathrm{C} + 273\ ,$$

so that absolute zero is -273 degrees on the Celsius scale.

Problem 1. What is 20 °C in °F? This is a practical issue if you go to a country where weather predictions are in Celsius degrees. Using the above formula, we get 32 + (9/5)20 = 68 °F, in other words, comfortable room temperature. Here is another version to think about: 98.6 is the normal Fahrenheit oral temperature of a healthy person. What is that in Celsius? Again, this is something one has to deal with when among people who are used to Celsius temperatures.

The next idea for us is one which begins to show why the Kelvin scale is useful and important. For a sufficiently dilute gas of particles, there is a simple relation among the pressure p, the volume V occupied by the gas, the number N of particles, and the temperature T:

$$pV = nRT \ .$$

Here the number of particles is $N = nN_A$,

where $N_A = 6.02 \times 10^{23}$ is the number of particles in a mole, so that n is the number of moles. Here $R = 8.31$ J/mol – K is the *gas constant.*

Problem 2. Suppose we have 3 moles of gas at standard (1 atm) pressure, and $T = 300$ K. What is this temperature in °F? This is a straightforward conversion, and the answer is 81 °F. Thus 300 is the 'round-number' value in K corresponding to room temperature, while 293 K corresponds to our standard room temperature of 20 °C.

Problem 3. For the above system, what is the volume occupied by the gas? Substitution into the gas law formula, with $p = 1.01 \times 10^5$ Pa, gives $V = 0.074$ m^3, or 74 ℓ, where this symbol stands for a liter, and 1 m^3 contains a thousand liters. One liter is just a bit larger than a quart. One mole of gas at standard (1 atm) pressure and (0 °C) temperature has a volume of 22.4 ℓ.

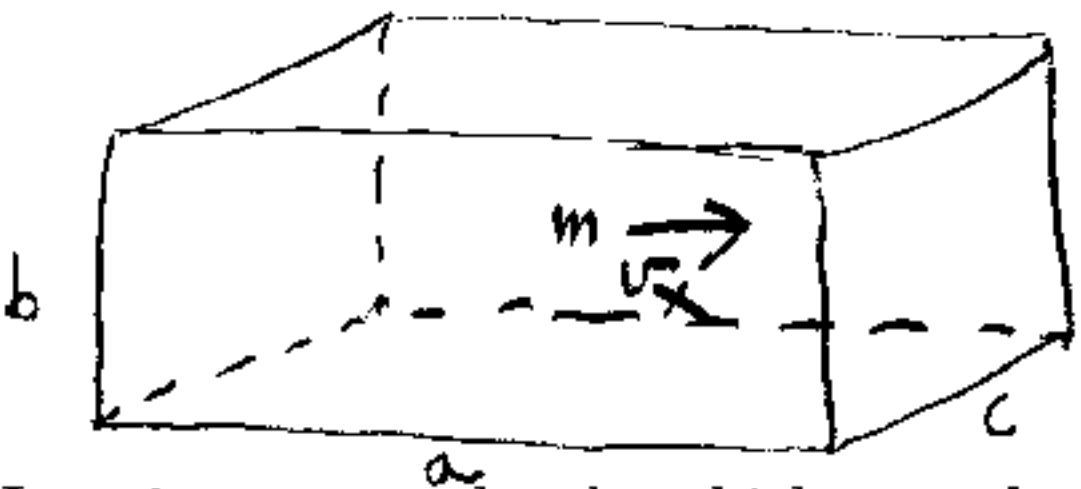
Problem 4. If, as the book says, one mole of carbon has a mass of 12.0 kg, what is the mass of one carbon atom? To find this, we just divide the mass of a mole by the number N_A of atoms in a mole, getting $m_C = 1.99 \times 10^{-26}$ kg. We haven't, and won't for a while, give a direct argument why N_A is the number of atoms or molecules in a mole, but it makes perfect sense that once this quantity has been defined, the mass of a mole of anything should be simply N_A times the mass of one of the particles. Similarly, the total energy in a mole should be just N_A times the average energy of one. We'll be using that soon.

Now we come to a question which faces all of us all the time, in our lives in general and in thinking about science. How do we know what to believe? To start, there is no 100% reliable way. Many people have been executed for crimes they did not commit,

and others who were guilty escaped conviction. Magicians can show us things we know are not happening, like sawing a person in half, but still that is what we see, so seeing is not necessarily believing. So, how do we convince ourselves that matter is made of atoms and molecules, as suggested in ancient times and more concretely two centuries ago? The process is similar to that used in diagnosis of disease: We make a guess or hypothesis, itself stimulated by initial observations, and then we deduce consequences if that hypothesis were right. If enough of these deductions work, we begin to have confidence that we have the correct picture. One beautiful deduction from the atomic-molecular picture is the ideal gas law, which was first found by laboratory measurements. Suppose the idea is right, and a gas consists of lots and lots of molecules flying around: How will the pres-

sure, volume, average kinetic energy and amount of gas be related? It turns out that the physics of the first part of this semester is enough to answer that question and deduce the ideal gas law.

Consider a rectangular volume, sides $\Delta x = a$, $\Delta y = b$, and $\Delta z = c$.



Imagine one molecule which may be moving in any direction, but has a certain velocity v_x in the x direction. If it bounces elastically off the right wall, it gives an impulse $\Delta p = 2mv_x$ to the wall. Then it goes to the left wall, and hits the right wall again after a time $\Delta t = 2a/v_x$. The average force it exerts on the right wall is

$F = \Delta p/\Delta t = mv_x^2/a$. Now, if there are N molecules in the box, each with its own velocity in the x direction, then the total force on the right wall is $Nm < v_x^2 >_{av} /a$, and the total pressure on the wall is the force divided by the area, or $Nm < v_x^2 >_{av} /abc = Nm < v_x^2 >_{av} /V$, where V is the volume of the box. Now with all the bouncing off each other and the walls, the averages $< v_x^2 >_{av}$ and $< v_y^2 >_{av}$ and $< v_z^2 >_{av}$ had better all be the same, but $v_x^2 + v_y^2 + v_z^2 = v^2$, so $m < v_x^2 >_{av} = 2/3 < K.E. >_{av}$, and this gives

$$pV = nN_A(2/3) < K.E. >_{av} \quad .$$

As often happens with a successful insight about nature, we have learned two things at once: First, the ideal gas law follows directly from the notion of a gas as a collection of molecules bouncing off each other. Secondly, temperature may be inter-

preted as giving the average kinetic energy of each molecule,

$$KE = (3/2)kT \ ,$$

where the *Boltzmann constant* is defined by $k = R/N_A$. This means the total kinetic energy of an ideal gas, which is the number of molecules multiplied by the average kinetic energy of one molecule, is simply

$$U = (3/2)nRT \ ,$$

where U also is sometimes called the *internal energy* of the gas.

With this matching of the microscopic and the visible world under our belts, we are ready for a few illustrative problems to pin down the concepts.

Problem 5. What is the typical speed of a gas molecule at temperature 0 K? Because the average kinetic energy is given by $(1/2)mv^2 = 3k_BT/2$, at zero temperature the average kinetic energy is zero, but as kinetic energy can never be negative this means that all molecules must have precisely $v = 0$. They all are absolutely still!

Problem 6. What is the average energy of a molecule at room temperature, T=20°C? This gives $KE = (3/2)(1.38 \times 10^{-23})(293\ \mathrm{K}) = 6.07 \times 10^{-21}$ J.

Problem 7. What is the typical speed of a water molecule in air at that temperature? We can use the 'visible' quantities, mole weight (g) and the gas constant R, a this is just multiplying both sides of the equation in # 6 by Avogadro's number N_A. Using SI units throughout gives $(1/2)(18 \times 10^{-3}kg)v^2 = (3/2)(8.31)(293)$, $v^2 = 406000$ $\mathrm{m}^2/\mathrm{s}^2$ or $v = 637$ m/s.

Problem 8. What is the typical distance between neighboring molecules in air at standard temperature and pressure? Volume/molecule is $V = 22.4 \times 10^{-3}\mathrm{m}^3/6.02 \times 10^{23} = 37.2 \times 10^{-27}\ \mathrm{m}^3 = (3.34 \times 10^{-9}\mathrm{m})^3$.