PHY 121/123 LECTURE NOTES – Week 14

Temperature, specific heat capacity, phases of matter, and forms and rates of heat transfer

This week we are concerned with more than the correlations between temperature and other quantities like pressure and volume. Now we want to know how much heat it takes to change the temperature of a system by a certain amount, and also how rapidly the required heat can be transferred. To begin, let us take note of some practical realities which show that our understanding of matter is not quite as good as the success of the ideal gas law suggested. The first point is that not all matter is in gas form. That statement includes us, so we can be grateful there is more to the world than gases, whether or not ideal. Nevertheless, the ideal gas gives a number of patterns

which are quite general, and still have important exceptions.

The first statement is that raising the temperature of a system in equilibrium ALWAYS requires the addition of energy. For the ideal gas this is obvious, because the internal energy, equal to the sum of the kinetic energies of the individual molecules, is proportional to the temperature, so that higher T requires more energy. The next statement is often true, but there can be exceptions: Adding energy makes a system expand. For example, most solids expand with a small rise in temperature, so that the length between points in the solid increases according to the rule

$$\Delta \ell = \alpha \ell \Delta T \ ,$$

where α is the coefficient of linear thermal expansion. When the length changes are small, the corresponding fractional changes

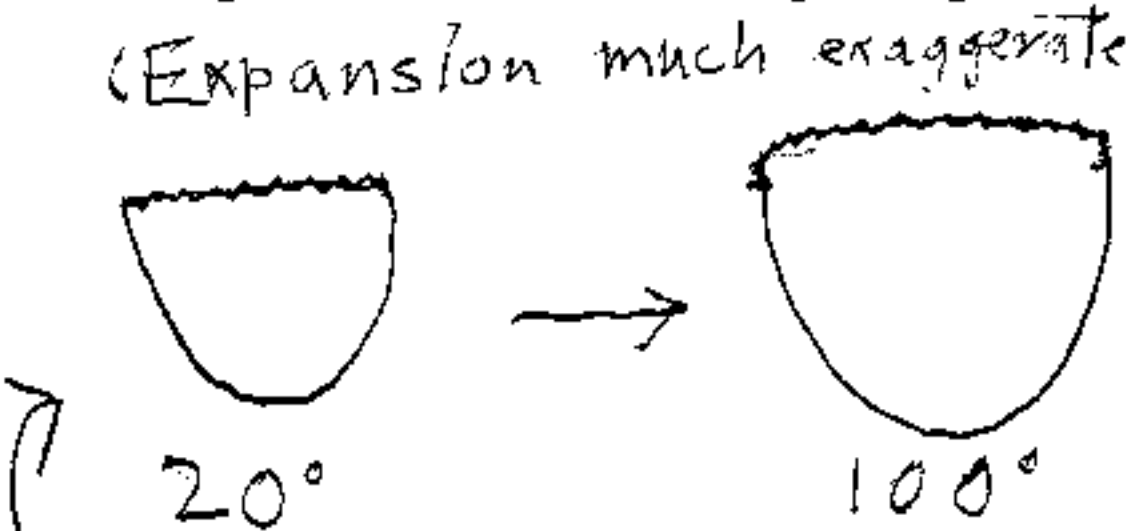
in area $\Delta A/A$ are twice as great, and in volume $\Delta V/V$ are three times as great, as the relative change in length $\Delta \ell/\ell$.

Dramatic volume-changing that goes in a different direction is the behavior of water as it is cooled towards and through the freezing point. Between 4 and 0 °C, lowering the temperature increases the volume of a given mass of water. This fact is behind the turnover which occurs in northern lakes every fall and spring. In the fall, the water is cooled by contact with the surroundings, and the water at the top reaches 4° first. At that point, being at maximum density, it trades places with the warmer, less dense water below. This warmer water now starts cooling, and cools right through 4° without moving down, because the water underneath is already at maximum density, and only will cool further after the water on top has gotten closer to 0°. In the spring, the

water at the top warms up to 4° first, then sinks, so that the new colder water on the top goes up through 4° later.

Turnover in lakes is one thing, but water freezing is crucial to life as we know it. All substances shrink when they go from gas to liquid, and most shrink when they go from liquid to solid, but not water. The volume of a given mass of ice at 0° is 9 % larger than that of the same mass of water. This is the reason why ice stays on top of water as the temperature decreases. If water were more conventional and shrank when freezing, then the ice would sink, and pretty soon lakes and oceans in winter would be ice from bottom to top. In the following spring, only a thin layer of ice at the top would melt into water, and the vast bulk of the oceans would remain frozen. Therefore there would be no water for sea creatures to swim in, much less water vapor in the air to condense into rain,

and a very dry, cold world! This gives a background for the first problem, which actually involves a range above 4° where water does expand with increasing temperature.



Problem 1. A copper pot holds one liter of water at 20 °C. At 100 °C, how much more water by volume can it hold? How much more in mass? The linear coefficient of expansion for copper is $\alpha = 1.7 \times 10^{-}5$ /°C, so the fractional increase in volume is $3 \times 1.7 \times 10^{-}5 \times (100 - 20)$ l, yielding a volume increase of 4 ml. It might seem that therefore the mass of water which can be held in the pot increases, but that is not so, because the water expands more rapidly

than the copper. From the text, the volume expansion coefficient of water in this range is 20.7×10^{-5}, giving an increase in the volume of 1 l of water $20.7 \times 10^{-}5 \times (100 - 20)$ l, or 17 ml. Thus, 13 ml of water, with mass 13 g, will spill over the edge of the pot.

Problem 2. Let's turn to a different issue – how much heat does it take to change the temperature by a given amount? Again, at least over a small range of temperatures, a simple rule seems to work for a substance in its solid, liquid, or gas phase. The heat required to raise the temperature is $Q = cm\Delta T$, where c is the *specific heat capacity*, and m is the mass of the sample. Imagine an insulated container with 100 g of (liquid) water at 0 °C and some amount of water at 100 °C. If we would like the mixture to end up at 50 °C, what mass of water should we start with at the higher temperature? This is quite easy: The heat released

by the hot water has to be just enough to raise the temperature of the cold water by 50 °C: $-Q_H = cm(100-50)°\mathrm{C} = Q_L = c(100\mathrm{g})(50-0)°\mathrm{C}$. Canceling out common factors leaves $m = 100$ g. The mass of hot water cooled by 50 °C must be exactly equal to the mass of cold water heated by 50 °C. The minus sign on Q_H is to indicate that heat came out of the hot water. Positive Q for some system refers to heat put into it.

Problem 3. Let's complicate things by starting with 100 g of ice at 0 °C, and ask how much steam at 100 °C is needed if the mixture again is to end up at 50 °C. It takes heat to melt ice, a *latent heat* L_{melt} =334 J/g = 80 calorie/g. The word latent means 'hidden', and the reason is that one adds heat but the temperature doesn't rise, so the system did not get hotter. It also takes heat to vaporize water, $L_{\mathrm{vapor}} = 2260$ J/g = 540 cal/g. This time we also need to

know the value of the specific heat of water, $c_W = 4.19$ J/g $= 1$ cal/g. You can see that for water it is easier to work with calories! Once again we must insist that equal amounts of heat come out of the hot matter and go into the cold matter: $Q_H = (L_{\rm v} + c|\Delta T_H|)m = Q_L = (L_{\rm m} + c|\Delta T_L|)(100{\rm g})$, or $590m = 130(100)$ g. This gives $m = 22$ g.

Problem 4. Let's introduce a new substance, lead. Starting with 1 kg Pb metal at 327 °C (i.e., just not melted), mixed with 1 kg ice at 0 °C, how much liquid water will there be at the end? The key point in this problem is that water has an extraordinarily high specific heat. That of Pb is $c_{Pb} = 30.5$ cal/kg, compared to water with $c = 1000$ cal/kg. Thus the amount of water melted, in g, is $c_{Pb}(\Delta T_{Pb}/L_{\rm m} = 30.5(327)/80 = 125$ g. Note the assumption here, that some but not all ice melts, so that the final temperature had to be $0\,^{\circ}C$. What if the calculation

had indicated more melting than there was ice to start with? Then we know the final temperature will be higher, but we can calculate it by computing the leftover heat Q_ℓ and setting that equal to the heat required to raise both water and ice from 0 $^\circ C$ to the final temperature. Of course, the lead does not cool down to 0 and then go up, but the arithmetic is OK if we assume it does, because the down and up parts between the final temperature and 0 cancel out. This way, there is no waste in the work which started by assuming that not all ice melted.

Problem 5. Here is another aspect of changes in a substance with heating and cooling: Because the volume of a given mass changes, if the system is under pressure then work is done by the system on its surroundings as it expands, and by the surroundings on the system as it contracts. Thus energy is required to do work $W = p\Delta V$ as wa-

ter freezes and consequently expands, e.g., at $p = 1$ atm $= 1.01 \times 10^5$ Pa. For 1 kg H_2O, how does this compare to the latent heat of melting? As the volume of the water is essentially 1 l, and the increase in volume is 8.83%, we have $\Delta V p = 8.83 \times 10^{-5}$ m$^3 \cdot 1.01 \times 10^5$ Pa $= 8.92$ J $= 2.13$ cal. This of course is quite small compared to the 80 kcal of heat released when 1 kg of water freezes, but it implies that increasing the pressure actually would *decrease* the melting temperature. Usually, with materials which expand when they melt, increasing the pressure raises the melting point.

Problem 6. Now let's discuss rates of heating and cooling associated with specific forms of heat transfer. Suppose an insulated silver wire, $A = 1$mm^2 across and $\ell = 1$ m long, connects two otherwise insulated water samples, one at 0 °C, and the other at 100 °C. How long does it take for

the hot water to drop and the cold water to rise by 1 °C? The rate of heat transfer is $dQ/dt = k_T A \Delta T/\ell$. Therefore all we need to know is the conductivity of water, $k_T = 406$ W/m·°C, giving $dQ/dt = 406(10^{-6}\text{m}^2/1\text{ m})\ 100\ ^\circ\text{C} = 4.06 \times 10^{-2}$ W = .0097 cal/s. To change the water temperature by 1 degree requires transfer of 1 cal, which takes 103 s, or 1 min, 43 s.

Problem 7. On the weather news in winter, one frequently hears of the 'wind-chill factor', which makes the air feel colder than the actual air temperature. Why is this? In still air, we lose heat to the air from skin, especially exposed skin, by conduction of the heat from the skin to the adjacent air. Because the specific heat capacity of the layer of air near the skin is quite small, this air rapidly gets heated to nearly the skin temperature, and because the heat conductivity of air also is small, the cooling of the

layer by heat transfer to more distant layers is slow. As a result, the dominant cooling effect is convection: As it warms, the air next to the skin becomes less dense, and therefore rises, to be replaced by cooler air. If there is a wind, then the air next to the skin is constantly changed, and so is much closer to the surrounding air temperature. Therefore, the rate of heat loss from the skin is much greater, and it feels colder. In other words, the gentle 'wind' generated by convection is replaced by a much faster wind, and so much faster cooling. In summer, one hears about the temperature-humidity index, which often is higher than the air temperature: One of the body's most efficient cooling mechanisms is bringing warm water to the surface by sweating, and then allowing this water to evaporate, taking away the enormous latent heat of vaporization. If the humidity is quite high, the air is already

close to 'full' of water vapor, and hence cannot accept evaporating water from our skin. With this cooling route blocked off, we are warmer, just as if we were in dry air at a much higher temperature.

Problem 8. The third method of heat transfer, besides conduction in which heat travels through material and convection in which heat is carried by moving material, is radiation in which heat is carried by light, or more generally by electromagnetic rays, whether visible to our eyes or not. The term radiation also is used for very high-energy submicroscopic particles, including electromagnetic X rays and gamma rays, but the amount of such radiation required to transfer appreciable heat is much greater than the amount required to harm or kill a person by ripping apart individual cells, and therefore is not interesting in connection with most practical heat transfer processes. Only

in a well-shielded nuclear power reactor, or in a nuclear bomb, is heat transfer an important aspect of radiation. Now consider a human-sized dummy out in space, with a temperature of 80 °F. How fast does this body lose power to radiation? Let's assume an efficiency of absorbing (and therefore of emitting) electromagnetic radiation $e = 0.5$, and a body surface area $A = 1.5$ m^2. Then the power loss is given by the difference between the power radiated by the body and the amount absorbed from the environment, according to the Stefan-Boltzmann law, $P = e\sigma A(T^4 - T_e^4)$, with the Stefan-Boltzmann constant $\sigma = 5.67\times10^{-8}$ W/m^2·K^4. Using $T = 300$ K and $T_e = 3$ K, we get $P = 344$ W, compared to the typical value 100 W generated by a person with normal food intake. Clearly, our dummy needs insulation to keep from freezing very fast!