Midterm 2 Solutions

1. $KE + PE = \text{constant}$

$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\omega^2\right)$

($r^2\omega^2 = v^2$)

$PE = mgh$

$KE_{initial} = 0$

$KE_{final} = mg(h_{initial} - h_{final})$

$= \frac{1}{2}mv_{final}^2 + \frac{1}{2}\left(\frac{2}{5}mv_{final}^2\right)$

$= \frac{7}{10}mv_{final}^2$

Yellow

$\frac{7}{10}v_{final}^2 = gh$

$v_{final} = \sqrt{\frac{10gh}{7}}$

$= \sqrt{\frac{10 \cdot (9.8) \cdot (1.7\,m)}{7}}$

(a) (10 pts) $= 4.88\ m/s$

White

[NOTE v_f less because some KE goes into rotation]

v_{final}

$= \sqrt{\frac{10 \cdot (9.8\frac{m}{s^2}) \cdot (2.1m)}{7}}$

$= 5.42\ \frac{m}{s}$

On the second ramp, the ball keeps going till all the KE goes back to PE, which means the original height.

(b) (10 pts) $h = 1.7\,m$ $\qquad h = 2.1\,m$

2(a) $\frac{r}{r_E} = x = \frac{3.85 \times 10^8 \text{ m}}{6.37 \times 10^6 \text{ m}}$

$= 60.4$ (5 points)

(b) $T^2/r^3 = \text{constant}$

$\frac{T_2^2}{r_2^2} = \frac{T_1^2}{r_1^2} \Rightarrow T_2 = T_1 \sqrt{\frac{r_2^3}{r_1^3}}$

$= T_1 \sqrt{x^3}$. $T_{moon} = 1.41 \text{ hr} \sqrt{(60.4)^3}$

$= 1.41 \text{ hr} \cdot 469 = 662 \text{ hr}$ (10 points)

<u>Alternate (b)</u>

$\frac{v^2}{r} = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r = g(r) = g\left(\frac{R_E}{r}\right)^2 = \frac{g}{x^2}$

$= \frac{9.8 \text{ m/s}^2}{(60.4)^2} = 2.69 \times 10^{-3} \frac{\text{m}}{\text{s}^2}$ = gravity of earth at radius of moon.

$T = 2\pi \sqrt{r/g(r)} = 2\pi \sqrt{\frac{3.85 \times 10^8 \text{ m}}{2.69 \times 10^{-3} \frac{\text{m}}{\text{s}^2}}}$

$= 2.38 \times 10^6$ s.

$= 2.38/3600 = 661$ hr.

[Difference between 2 is round off error]

c) $T = 662/24 = 27.6$ days

The moon is full about every 30 days, so this <u>is close.</u> (5 points)

<u>Not</u> expected on exam: Earth goes around sun so moon has to 'catch up' $T_{full\ moon}$ about 2 days longer than T.

Yellow

3. p_{txi}

$= 7.4 \frac{m}{s} \cdot 140g$

$= 1036\ g \frac{m}{s}$

$p_{tyi} = 0$

$p_{bxi} = -1.4 \frac{m}{s} \cdot 740g$

$= -1036\ g \frac{m}{s}$

$p_{byi} = 0$

$p_{txi} + p_{bxi} = 0$

(a) $p_{tyi} + p_{tyi} = 0$

(10 points)

White

p_{txi}

$= 8.4 \frac{m}{s} \cdot 180g$

$= 1512\ g \frac{m}{s}$

$p_{tyi} = 0$

$p_{bxi} = -1.8 \frac{m}{s} \cdot 840g$

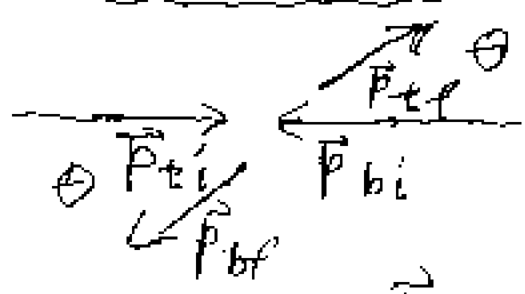
$= -1512\ g \frac{m}{s}$

$p_{byi} = 0$

$p_{txi} + p_{bxi} = 0$

$p_{tyi} + p_{byi} = 0$

$\boxed{\vec{P}_{total} = 0}$

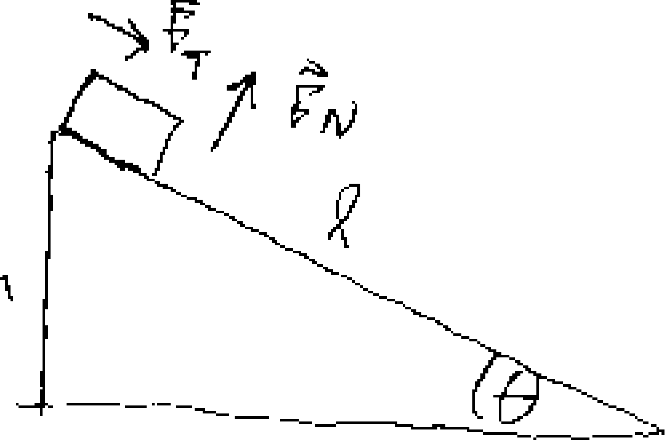


$\vec{p}_i = \vec{p}_f = 0$, so $\vec{p}_{bf}$ opposite $\vec{p}_{tf}$. Conservation of KE means $|\vec{p}_{bi}| = |\vec{p}_{bf}|$, $|\vec{p}_{ti}| = |\vec{p}_{tf}|$, so balls have same v before as after collision.

$v_{bxf} = -v_b \cos\theta$

$= -1.31 \frac{m}{s}$ $(-1.4 \frac{m}{s} \cos 21°)$

$= -1.62 \frac{m}{s}$ $(-1.8 \frac{m}{s} \cos 26°)$

4. 

$h = l\sin\theta$

Yellow — White

$F_N = mg\cos\theta$

$F_T = mg\sin\theta - F_{friction}$

$= mg\sin\theta - \mu mg\cos\theta$

(a) $|W_{friction}| = F_{friction} \cdot l$

$= F_{friction} \cdot \frac{h}{\sin\theta}$

$= \mu mgh\cot\theta$

$= (0.49)(2.3\,kg)(9.8\,\frac{m}{s^2})(1.7\,m)\cdot\cot 27°$

$= 36.9\,J$

[$(0.42)(1.9\,kg)(9.8\,\frac{m}{s^2})(1.4\,m)\cdot\cot 32°$

$= 17.5\,J$ — $17.5\,J$] (10 pts)

(b) $mgh = 38.3\,J$ — $mgh = 26.1\,J$

$KE = 1.4\,J$ — $KE = 8.6\,J$

$v = \sqrt{2(KE)/m} = 1.1\,\frac{m}{s}$ — $v = \sqrt{2(KE)/m}$

(10 pts) — $= 3.0\,\frac{m}{s}$

5. $KE = \frac{1}{2} m v^2$

$\tau = 0 \Rightarrow$ $\boxed{r = l \text{ string}}$

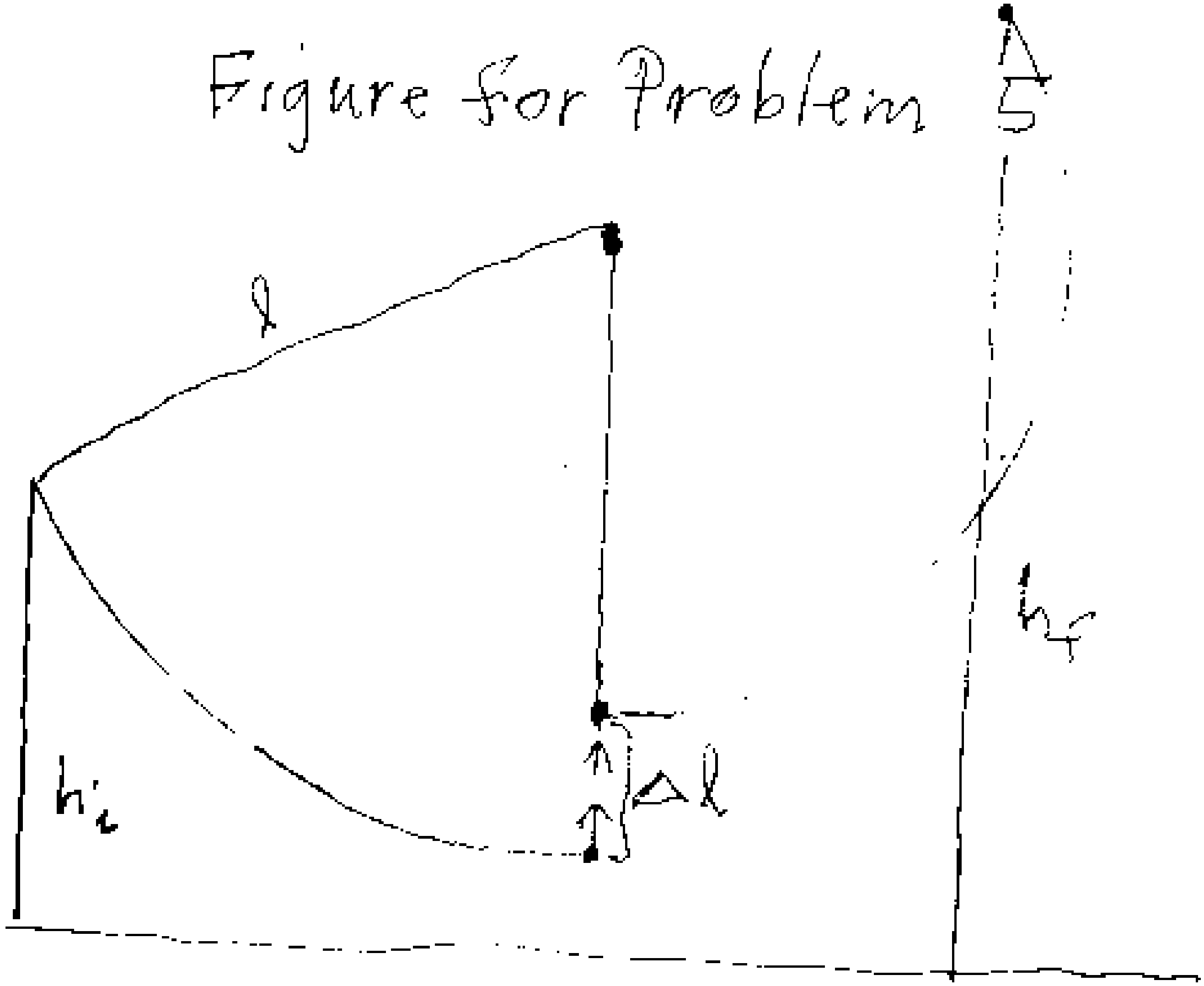
$L = mr^2\omega = mrv = \text{constant}$

$KE_1 = \frac{1}{2} m v_1^2$

$KE_2 = \frac{1}{2} m v_2^2 = \frac{1}{2} m \left(\frac{v_1 r_1}{r_2}\right)^2$

Yellow	White
$KE_1 = \frac{1}{2}(0.3\,kg)(2.3\,\frac{m}{s})^2$	$\frac{1}{2}(0.6\,kg)(2.8\,\frac{m}{s})^2$
$= .79\,J$	$2.35\,J$
$KE_2 = KE_1 \left(\frac{r_1}{r_2}\right)^2$	$KE_2 = KE_1 \left(\frac{r_1}{r_2}\right)^2$
$= 0.79 \cdot \left(\frac{0.9}{0.7}\right)^2$	$= 2.35 \left(\frac{0.7}{0.5}\right)^2$
$= 1.31\,J$	$= 4.61\,J$
(a) $\Delta KE = +0.52\,J$ (8pts)	$\Delta KE = +2.26\,J$
(b) $\Delta PE = mg\Delta h$ (4pts)	$\Delta PE = mg\Delta h$
$= 0.59\,J$	$= 1.18\,J$
(c) Extra height	
$mg\,h_{extra} = \Delta KE + \Delta PE$	$mg\,h_{extra} = 3.44\,J$
$= 1.11\,J$	
$h_{extra} = \frac{1.1\,J}{(0.3\,kg)(9.8\,m/s^2)} = 0.37\,m$	$h_{extra} = \frac{3.44\,J}{(0.6\,kg)(9.8\,\frac{m}{s^2})}$
	$= 0.59\,m$ (8pts)

Figure for Problem 5



Find $\Delta h = h_f - h_i > \Delta l$