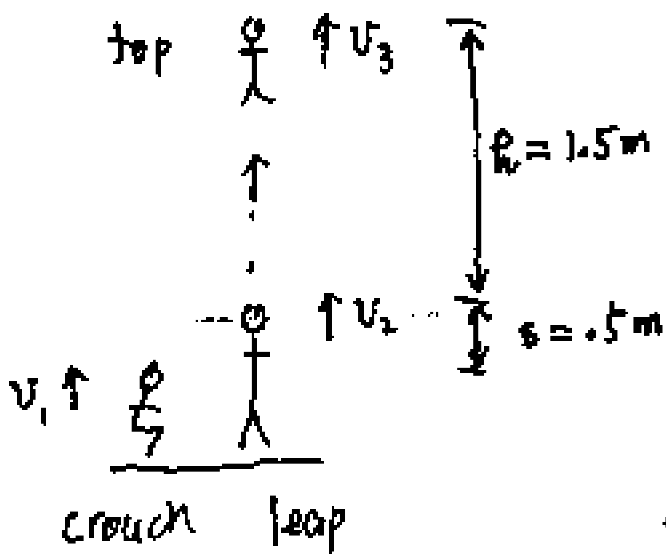
correction to p.39a: Michael Jordan's legs are stronger!

redo the problem



$$v_3^2 = v_2^2 + 2ah$$

$v_3 = 0$, $a = -9.8$, $h = 1.5$

$\rightarrow$ $v_2 = 5.42$ m

from ① to ②:

$$v_2^2 = v_1^2 + 2aS$$

$v_2^2 = 5.42^2$, $v_1 = 0$ rest, $S = .5$ m

$$\rightarrow a = 29.4 \ m/s^2$$

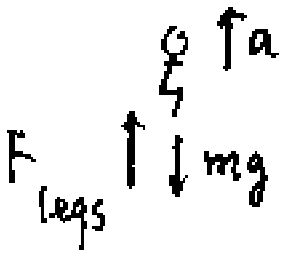
this a is caused by F_{net}^{up}

suppose $m = 75$ kg

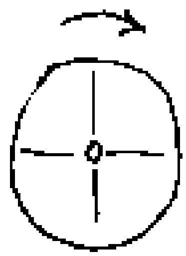
$$\underbrace{F_{net}^{up}}_{F_{legs} - mg} = ma$$

$$\therefore F_{legs} = \underbrace{mg}_{75 \times 9.8} + \underbrace{ma}_{75 \times 29.4}$$

$$= 2940 \ N$$



rotational motion

wheel rotates  at speed 85 rpm

rpm: revolution per minute

how many degrees it rotates per second?

one revolution = 360°

$$\frac{85 \times 360}{60} = 510 \text{ deg/sec}$$

how many radians per second?

one revolution = 2π radians

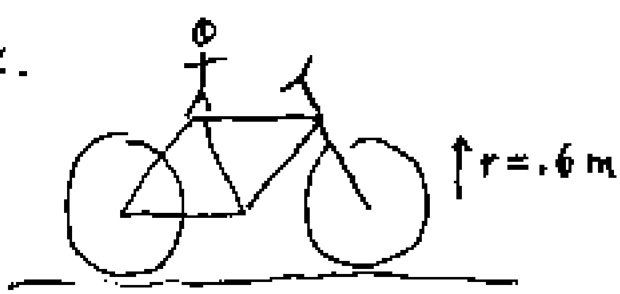
$$\frac{85 \times 2\pi}{60} = 8.901 \text{ rad/sec}$$

angular velocity of wheel

$$\omega = 85 \text{ rpm} = 510 \text{ deg/sec} = 8.901 \text{ rad/sec}$$

ω: omega

Ex.



$\Longrightarrow$ $v = 20$ km/hr

ω of bicycle wheel = ?

circumference of circle $= 2\pi r$

each revolution wheel advances $2\pi r$ distance

$$\therefore \omega_{\substack{\text{in rev}\\ \text{per sec}}} = \frac{20 \times 1000/3600}{2\pi \times .6} = 1.47 \text{ rev/sec}$$

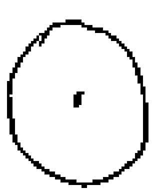
$$= \underbrace{1.47 \times 2\pi}_{9.236} \text{ rad/s.}$$

---

linear motion with const acc	rotation motion with constant angular acc. α
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
$x_f = x_i + v_i t + \frac{1}{2} a t^2$	$\underbrace{\theta_f}_{\text{rad}} = \underbrace{\theta_i}_{\text{rad}} + \underbrace{\omega_i}_{\text{rad/sec}} t + \frac{1}{2} \underbrace{\alpha}_{\text{rad/sec}^2} t^2$
$v_f^2 = v_i^2 + 2as$	$\omega_f^2 = \omega_i^2 + 2\alpha \underbrace{\theta}_{\theta_f - \theta_i}$

note they are very similar

Ex.



granding wheel, $\omega_i = 0$ at $t_i = 0$

driven by electric motor, acc $\alpha = 2$ rad/sec^2

at $t_f = 5$ sec, its angul velo $\omega = ?$

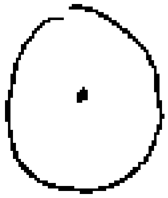
$$= \omega_i + \overset{2}{\alpha} \overset{5 = t_f - t_i}{t}$$

$$= 10 \text{ rad/sec}$$

same time, angular displacement

$$\theta_f = ? = \underset{0}{\theta_i} + \underset{0}{\omega_i} t + \frac{1}{2} \underset{2}{\alpha} \underset{5^2}{t^2} = 25 \text{ rad}$$

Ex.



flywheel, driven by gasoline engine,
angular acc is $\alpha = 1.5\ \mathrm{rad}/\mathrm{sec}^2$
initial angular velo. $\omega_i = 0$
$\omega_f = ?$ after it has rotated 15 revolutions

$$\omega_f^2 = \underbrace{\omega_i^2}_{=0} + 2\underbrace{\alpha}_{=1.5}\theta$$

$\theta = 15$ rev $= 15 \times 2\pi$ radians

$$= 282.7$$

$$\omega_f = 16.8\ \mathrm{rad/sec}$$

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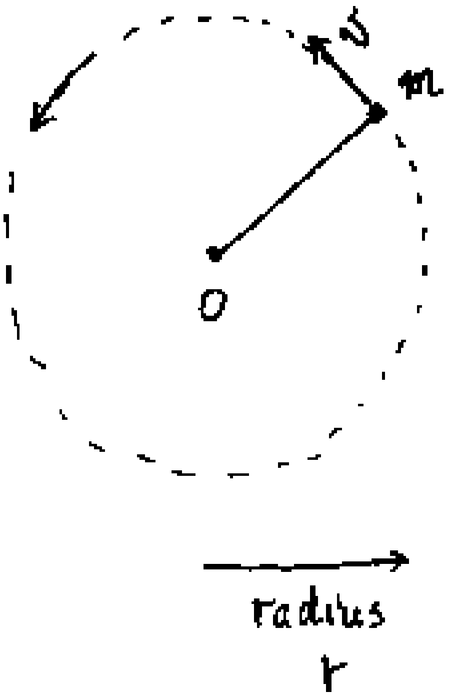
a particle in circular motion

key: centripetal force
centripetal acceleration



O = center of circle

mass m makes circular motion about O with constant angular velocity ω

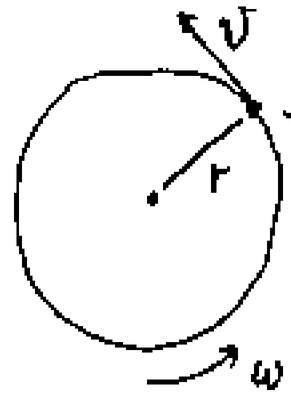
v is tangential velo. ($\perp$ radius)

T = period = time spent for one revolut.

period $T = \frac{2\pi}{\omega}$, $v = \frac{\text{circumference}}{T}$

$$= \frac{2\pi r}{2\pi/\omega} = \omega r$$

Ex.



turntable

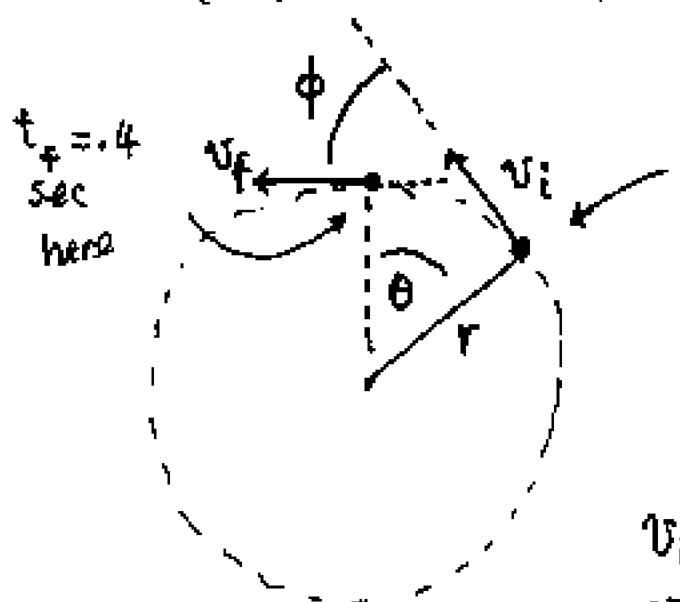
marble glued to its edge

r = radius = 0.8 m

ω = constant = 1.5 rad/sec

v = tangential velo of marble = ? $= r\,\omega = 1.2$ m/sec ($r = .8$, $\omega = 1.5$)

look at it more carefully



ϕ = angle between two tangential velo. = ?

$\theta = \omega(t_f - t_i) = 1.5 \times .4 = .6$ rad.

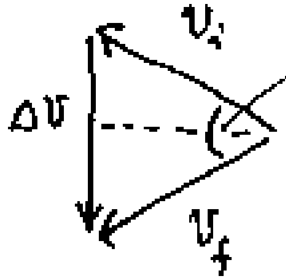
$v_i \perp$ radius at $t_i = 0$

$v_f \perp$ radius at $t_f = .4$

$\therefore\ \phi = \theta = .6$ rad

Now take t_f to be very close to t_i, i.e. $t_f = t_i + \Delta t$

$\Delta t \to 0$



$\Delta\phi = \underbrace{\Delta\theta}_{\text{angle between two radii}}$

magnitude of v_i = that of v_f = v

For very small angle $\Delta\phi$,

$$\Delta v = v\,\Delta\phi = v\,\Delta\theta$$

$$\text{acceleration } a_r = \frac{\Delta v}{\Delta t} = v\frac{\Delta\theta}{\Delta t} = v\omega$$

$$\omega = \frac{v}{r}$$

$$\therefore \quad a_r = \frac{v^2}{r}$$

important

a_r: m/s²

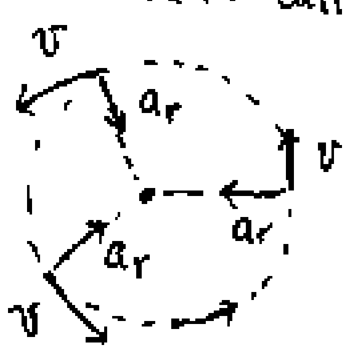
v = tangential velo. m/s

r: radius m

note: a_r is always pointing inward to center of circle

it is the acc. for tangential velo

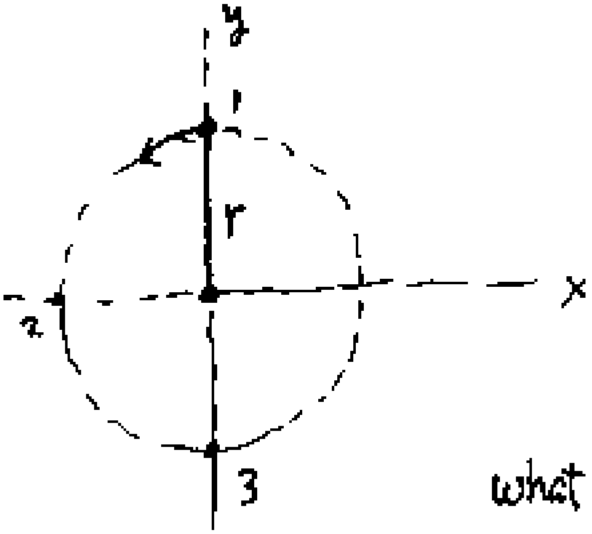
.... called centripetal acc.



a_r alway $\perp$ to v.

R-6

Ex.



a ball, attached to string, is rotating at

$\omega = 100$ rev/min

$r = 70$ cm

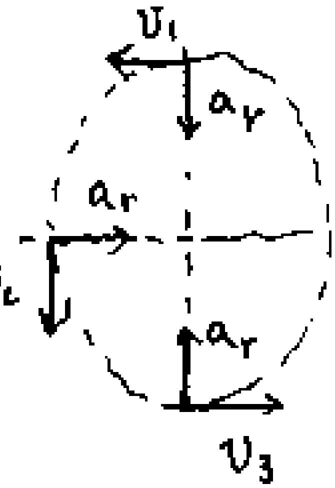
what are the tangential velocities at points 1, 2 & 3 ?

magnitude of v $= r\omega = 0.7 \times \frac{100 \times 2\pi}{60} = 7.33$ m/s

what are a_r's ?

magnitude a_r $= \frac{v^2}{r} = \frac{7.33^2}{.7}$

$= 76.76$ m/s^2



directions are

Wed here

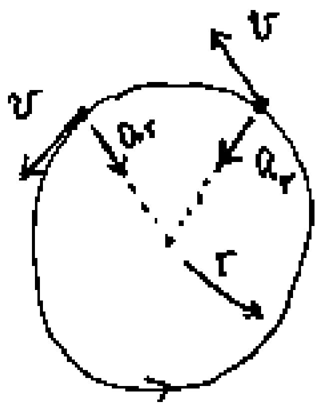
---

remember $F = ma$

here $F_r = m a_r$

i.e. we need a force to produce this a_r

short review:



an object moves on a circular path its tangential velo. is v

note direction of v is changing

this direction-change is caused by a_r (centripetal acc., $\perp v$, pointing inward to center of circle.)

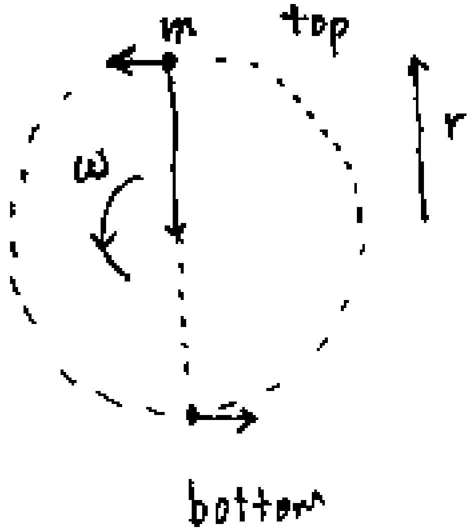
magnitude: $a_r = \frac{v^2}{r}$.

to produce this a_r, we need an "inward pulling force"

$$F_{centripetal} = m\frac{v^2}{r} \quad \left(\text{from newton: } F_r = m a_r\right)$$

$F_{centripetal}$: Newton; m: kg; r: m; v^2: $\left(\frac{m}{s}\right)^2$

Ex. rock, tied to string, swing in circle



$m = .5$ kg $\quad \omega = 55$ rpm

$r = .75$ m

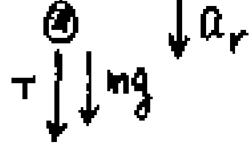
find $T_{top} = ?$ $\quad T_{bot} = ?$

tension in string

1st find tangential velo. $v = ? = \omega r \approx 4.32 \text{ m/s}$

$\omega = \frac{55 \times 2\pi}{60}$, $r = .75$

free-body top :

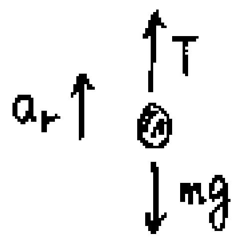


$$-T - mg = m\left(-\frac{v^2}{r}\right)$$

$$T = m\frac{v^2}{r} - mg \approx 7.54 \text{ N} = T_{top}$$

$m = .5$, $\frac{v^2}{r} = \frac{4.32^2}{.75}$, $m = .5$, $g = 9.8$

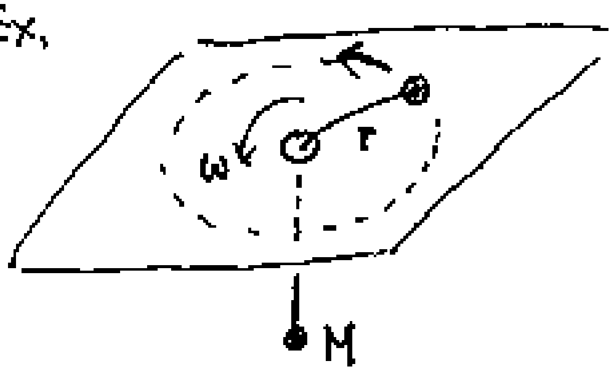
free-body bottom



$$T - mg = m\frac{v^2}{r}$$

$$T = mg + m\frac{v^2}{r} \approx 17.34 \text{ N} = T_{bottom}$$

Ex,



ice hockey puck

$m = .15$ kg

circular motion

no friction

$$\left.\begin{aligned} \omega &= 30 \text{ rpm} \\ r &= .5 \text{ m} \end{aligned}\right\} \text{ steady}$$

hanging mass $M = ?$

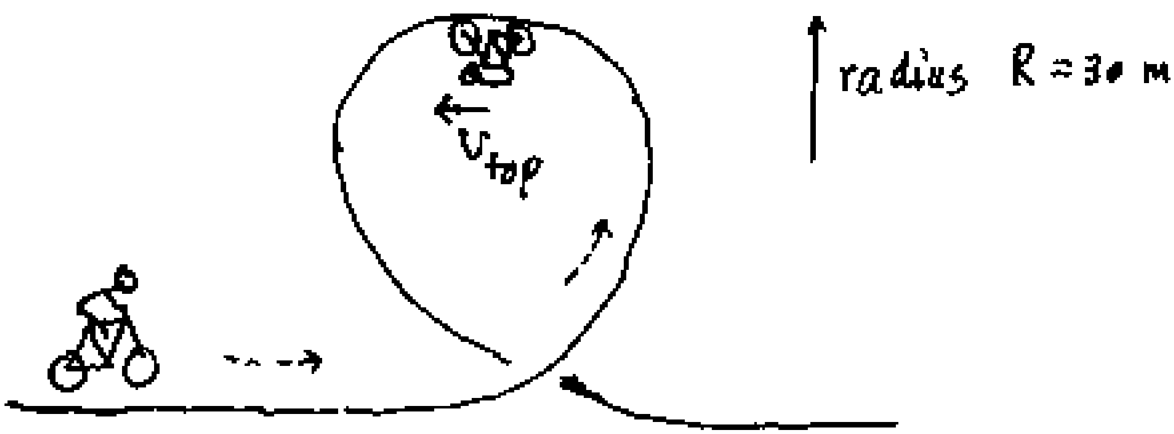
$$\text{tension in string} = Mg = F_{centrip} = m\frac{v^2}{r}$$

$g = 9.8$, $m = .15$, $v^2 = 1.57^2$, $r = .5$

$$v = \omega r = \frac{30 \times 2\pi}{60} \times .5 = 1.57 \text{ m/s}$$

$$\therefore M = 0.075 \text{ kg}$$

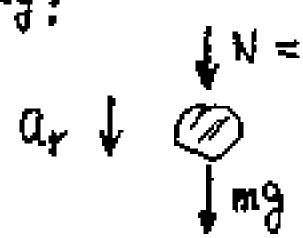
Ex. dare-devil motor cycle



m = motor cycle + dare devil

v_{top} = smallest velocity needed at top for safe ride (no crash)

free-body:



newton: $-N - mg = -m \frac{v_{top}^2}{R}$

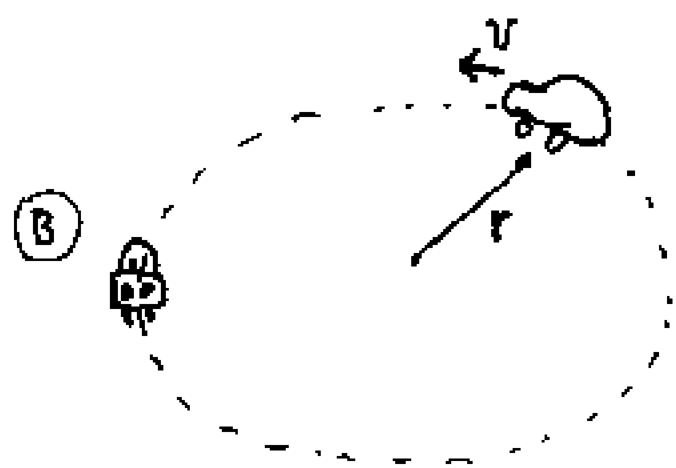
$N > 0$ as long as tire in touch with ceiling

$N \approx 0$, where tire about to lose contact with ceiling $\rightarrow$ crash

$\therefore$ smallest velo given by $N = 0$, i.e. $v_{top}^2 = Rg = 30 \times 9.8$

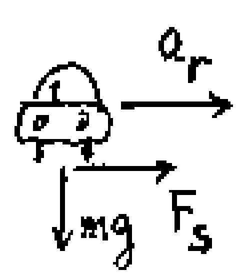
$v_{top} = 17.15$ m/s

Ex. Car drives on circular road, which is flat.



How fast can it drive without skiding?

look at position (B)



F_s = static friction force

$\leq mg\mu_s$

$F = ma \rightarrow \quad F_s = m\frac{v^2}{r}$, i.e. $m\frac{v^2}{r} \leq mg\mu_s$

safe driving

if $\frac{mv^2}{r} > mg\mu_s$. "call 911"

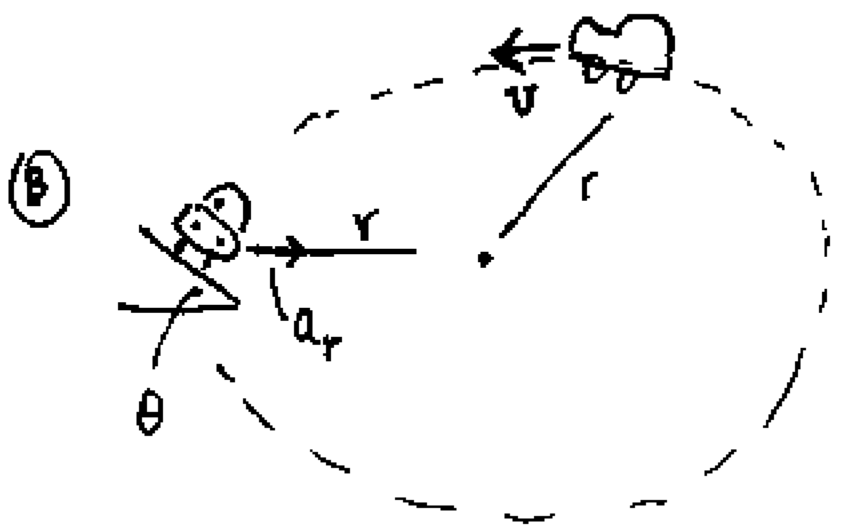
suppose $r = 200$ m , $\mu_s = .15$, $v = 50$ km/hr
will car skid or not ?

$$\frac{v^2}{r} = \frac{(50 \times 1000/3600)^2}{200} = \frac{192.9}{200} = 0.964$$

$$g\mu_s = 9.8 \times .15 = 1.47$$

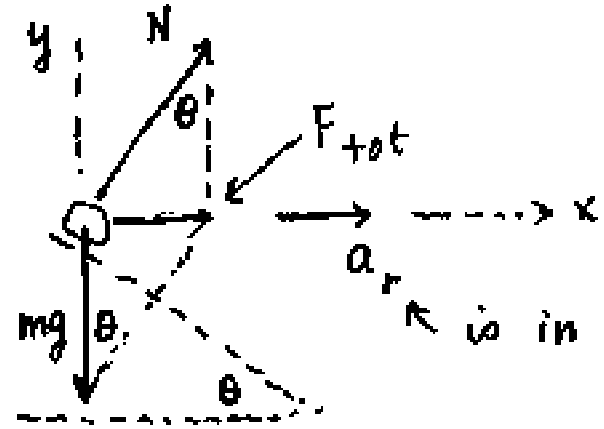
$\therefore \; m\frac{v^2}{r} < mg\mu_s$. car drives safely without skiding .

Curves of highway is "banked", to prevent skiding



consider first the case of no friction
betw road & tires

free-body diag.
at position (B)



a_r is in x direction

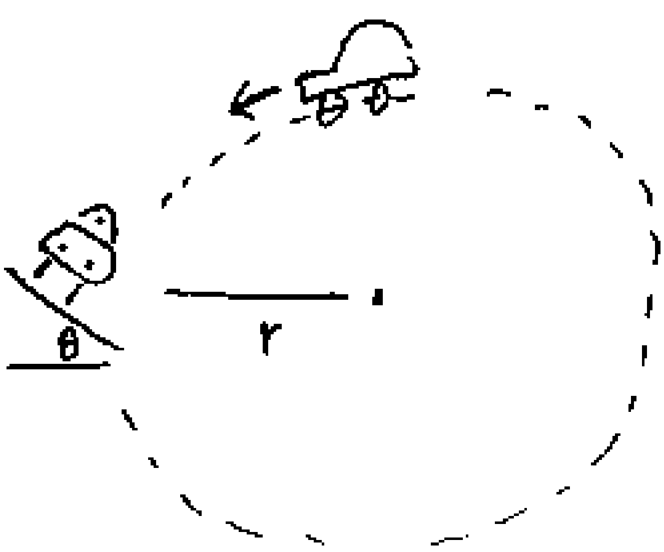
only N and mg
act on car
N = normal
force from
road

$\vec{F}_{tot} = \vec{N} + \vec{mg}$
... must in direction of $\vec{a}_r$

y-balance: $N \cos\theta = mg$ ①

$(F=ma)_x$: $N \sin\theta = m a_r = m \frac{v^2}{r}$ ②

$\frac{②}{①}$, N cancels, $\boxed{\tan\theta = \frac{v^2}{rg}}$



suppose $v = 20$ m/s
$r = 150$ m
banking angle θ
should = ?

$\tan\theta = \frac{20^2}{150 \times 9.8} = .272$

$\theta = 15.2^\circ$