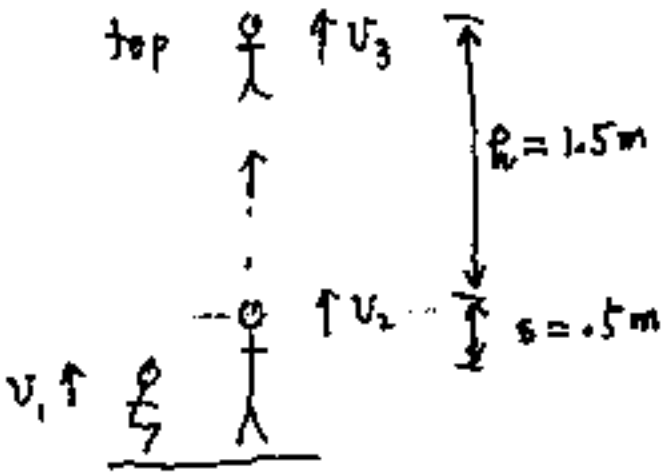
correction to (p.39a): Michael Jordan's legs are stronger!

redo the problem



$v_3^2 = v_2^2 + 2ah$

$0 \quad -9.8 \quad 1.5$

$\rightarrow v_2 = 5.42$ m

from ① to ②:

$v_2^2 = v_1^2 + 2as$

$5.42^2 \quad 0 \text{ (rest)} \quad .5$ m

$\rightarrow a = 29.4$ m/s²

this a is caused by F_{net}^{up}

Suppose m = 75 kg

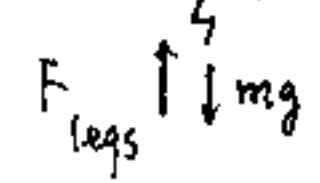
$F_{net}^{up} = ma$

$F_{legs} - mg$

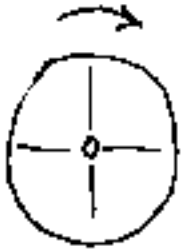
$\therefore F_{legs} = mg + ma$

$75 \times 9.8 \quad 75 \times 29.4$

$= 2940$ N



rotational motion

wheel rotates  at speed 85 rpm

↑ revolution per minute

how many degrees it rotates per second ?

one revolution = 360°

$$\frac{85 \times 360}{60} = 510 \text{ deg/sec}$$

how many radians per second :

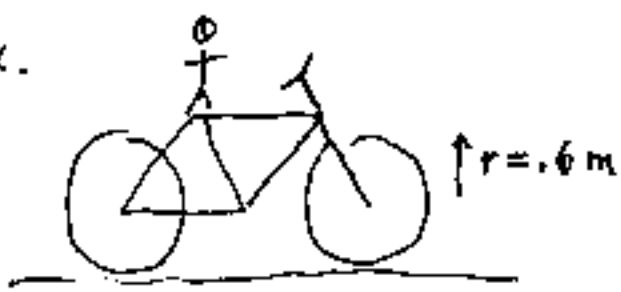
one revolution = 2π radians

$$\frac{85 \times 2\pi}{60} = 8.901 \text{ rad/sec}$$

angular velocity of wheel

$\omega = 85 \text{ rpm} = 510 \text{ deg/sec} = 8.901 \text{ rad/sec}$

| omega

Ex. 

$\Rightarrow$ $v = 20 \text{ km/hr}$

ω of bicycle wheel = ?

circumference of circle $= 2\pi r$

each revolution wheel advances $2\pi r$ distance

$$\therefore \omega_{\text{in rev per sec}} = \frac{20 \times 1000/3600}{2\pi \times .6} = 1.47 \text{ rev/sec}$$

$$= \underbrace{1.47 \times 2\pi}_{9.236} \text{ rad/s.}$$

---

linear motion with const acc	rotation motion with constant angular acc. α
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
$x_f = x_i + v_i t + \frac{1}{2} a t^2$	$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$
	rad / $\frac{\text{rad}}{\text{sec}}$ / $\frac{\text{rad}}{\text{sec}^2}$
$v_f^2 = v_i^2 + 2as$	$\omega_f^2 = \omega_i^2 + 2\alpha\theta$ ($\theta = \theta_f - \theta_i$)

note they are very similar

Ex. grinding wheel, $\omega_i = 0$ at $t_i = 0$

driven by electric motor, acc $\alpha = 2 \text{ rad/sec}^2$

at $t_f = 5$ sec, its angul velo $\omega = ?$ ($t = t_f - t_i = 5$)

$$= \omega_i + \alpha t$$

$$= 10 \text{ rad/sec}$$

same time, angular displacement

$$\theta_f = ? = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2 = 25 \text{ rad}$$

$$\theta_i = 0, \quad \omega_i = 0, \quad \frac{1}{2}\alpha = 1, \quad t^2 = 5^2$$

Ex.

flywheel, driven by gasoline engine,
angular acc is $\alpha = 1.5 \text{ rad/sec}^2$

initial angular velo. $\omega_i = 0$

$\omega_f = ?$ after it has rotated 15 revolutions

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

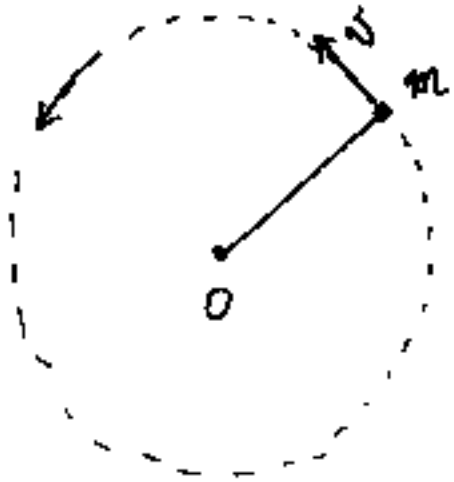
$\omega_i = 0$, $\alpha = 1.5$, $\theta = 15 \text{ rev} = 15 \times 2\pi \text{ radians}$

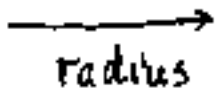
$$= 282.7$$

$$\omega_f = 16.8 \text{ rad/sec}$$

---

a particle in circular motion





key: centripetal force
centripetal acceleration

O = center of circle

mass m makes circular motion about O with constant angular velocity ω

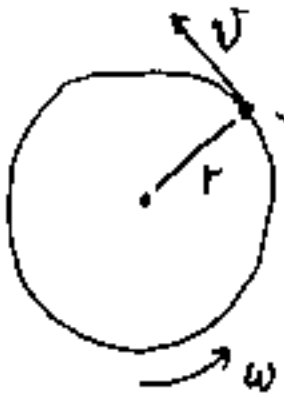
v is tangential velo. ($\perp$ radius)

T = period = time spent for one revolut

period $T = \frac{2\pi}{\omega}$, $v = \frac{\text{circumference}}{T}$

$= \frac{2\pi r}{2\pi/\omega} = \omega r$

Ex.



turntable

marble glued to its edge

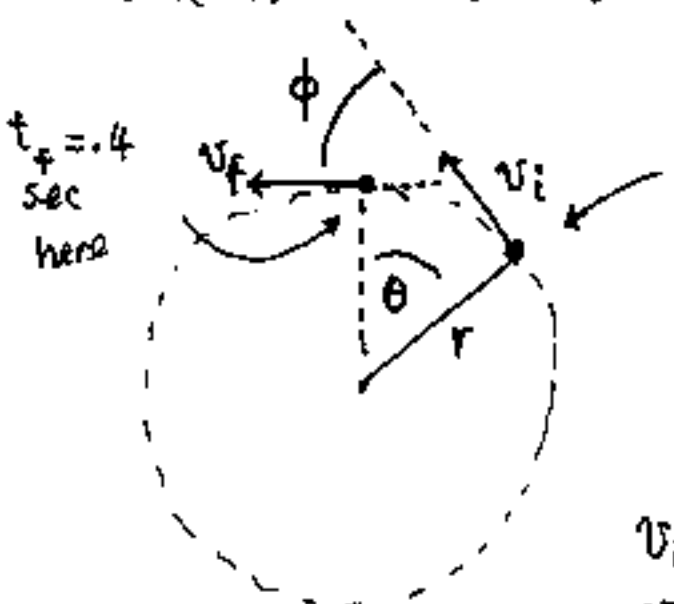
r = radius = 0.8 m

ω = constant = 1.5 rad/sec

v = tangential velo of marble $= ? = r\omega = 1.2$ m/sec

(.8, 1.5)

look at it more carefully



$t_i = 0$ it is here

ϕ = angle between two tangential velo. $= ?$

$\theta = \omega(t_f - t_i) = 1.5 \times .4 = .6$ rad.

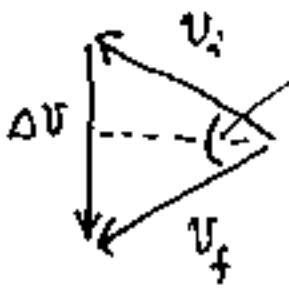
$v_i \perp$ radius at $t_i = 0$

$v_f \perp$ radius at $t_f = .4$

$\therefore \phi = \theta = .6$ rad

Now take t_f to be very close to t_i, i.e. $t_f = t_i + \Delta t$

$\Delta t \to 0$



$\Delta\phi = \underbrace{\Delta\theta}_{\text{angle between two radii}}$

magnitude of v_i = that of $v_f = v$

For very small angle $\Delta\phi$,

$$\Delta v = v\,\Delta\phi = v\,\Delta\theta$$

$$\text{acceleration } a_r = \frac{\Delta v}{\Delta t} = v\frac{\Delta\theta}{\Delta t} = v\omega$$

$$\downarrow = \frac{v}{r}$$

$$\therefore\ a_r = \frac{v^2}{r}$$

important

v = tangential velo.

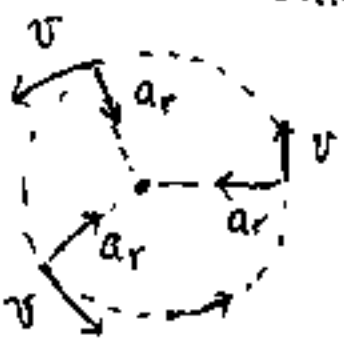
radius m

m/s^2 $\qquad$ m/s

note: a_r is always pointing inward to center of circle

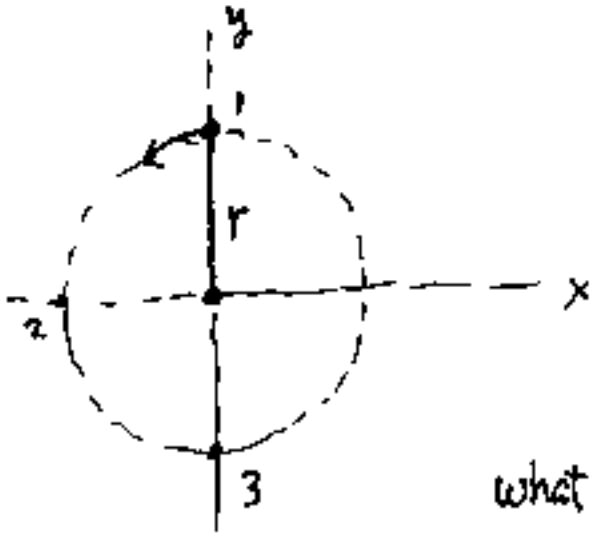
it is the acc. for tangential velo

.... called centripetal acc.



a_r alway $\perp$ to v.

Ex.



a ball, attached to string, is rotating at

$\omega = 100$ rev/min

$r = 70$ cm

what are the tangential velocities at points 1, 2 & 3 ?

magnitude of v $= r\omega = 0.7 \times \frac{100 \times 2\pi}{60} = 7.33$ m/s

what are a_r's ?

magnitude a_r $= \frac{v^2}{r} = \frac{7.33^2}{.7}$

$= 76.76$ m/s^2

directions are ↖ used here

---

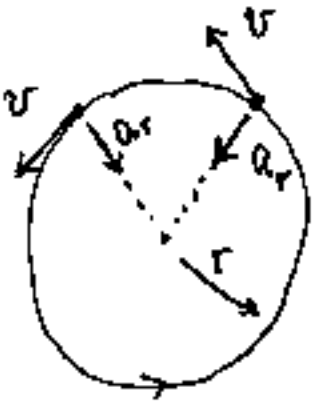
remember $F = ma$

here $F_r = ma_r$

i.e. we need a force to produce this a_r

10/8/99 R-7

short review:



an object moves on a circular path
its tangential velo. is v
note direction of v is changing

this direction-change is caused by a_r (centripetal acc., $\perp$ v, pointing inward to center of circle.)

magnitude: $a_r = \frac{v^2}{r}$

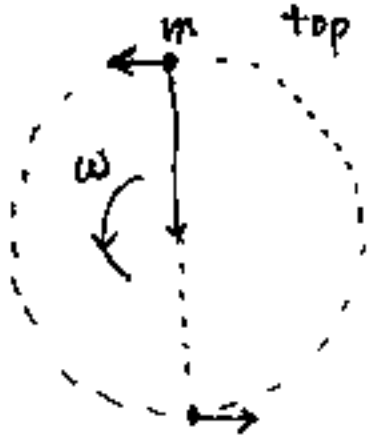
to produce this a_r, we need an "inward pulling force"

$$F_{centripetal} = m\frac{v^2}{r}$$

(from newton: $F_r = ma_r$)

Newton: kg, m, $(\frac{m}{s})^2$

Ex. rock, tied to string, swing in circle



m = .5 kg ω = 55 rpm

r = .75 m

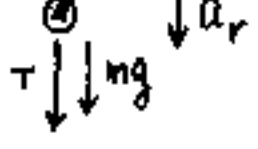
find $T_{top} = ?$ $T_{bot} = ?$

tension in string

1st find tangential velo. $v = ? = \omega r \approx 4.32$ m/s

$\omega = \frac{55 \times 2\pi}{60}$; $r = .75$

free-body top :

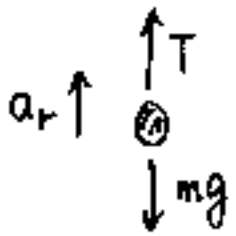


$$-T - mg = m\left(-\frac{v^2}{r}\right)$$

$$T = m\frac{v^2}{r} - mg \approx 7.54\ N = T_{top}$$

($m = .5$, $\frac{v^2}{r} = \frac{4.32^2}{.75}$, $g = 9.8$)

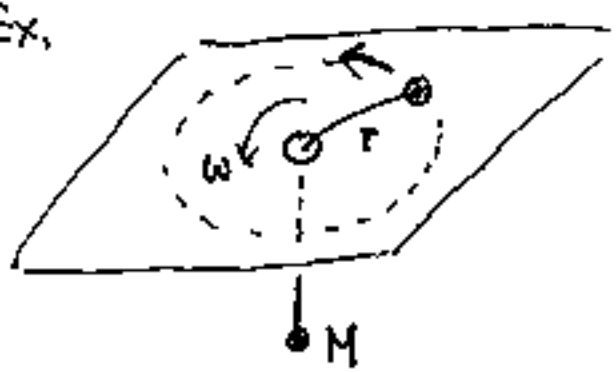
free-body bottom



$$T - mg = m\frac{v^2}{r}$$

$$T = mg + m\frac{v^2}{r} \approx 17.34\ N = T_{bottom}$$

Ex.



ice hockey puck
$m = .15$ kg
circular motion
no friction

$\omega = 30$ rpm
$r = .5$ m
} steady

hanging mass $M = ?$

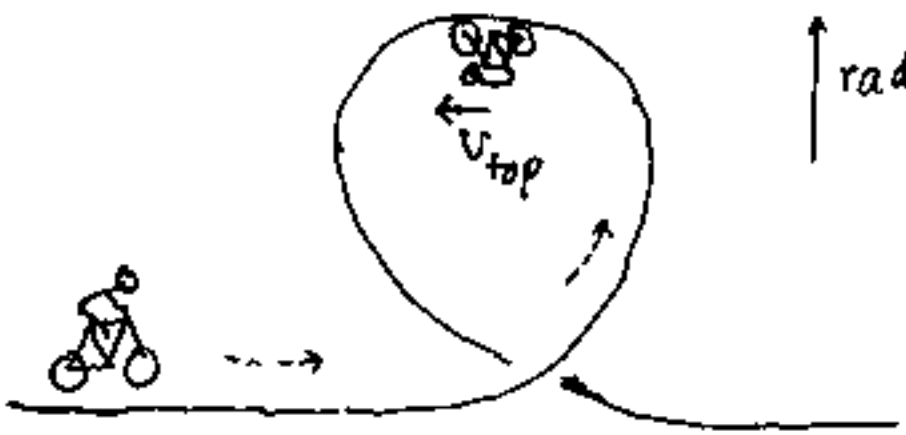
$$\text{tension in string} = Mg = F_{centrip} = m\frac{v^2}{r} \qquad 1.57^2$$

$g \approx 9.8$, $r = .5$, $.15$

$$v = \omega r \approx \frac{30 \times 2\pi}{60} \times .5 \approx 1.57 \text{ m/s}$$

$$\therefore M = 0.075 \text{ kg}$$

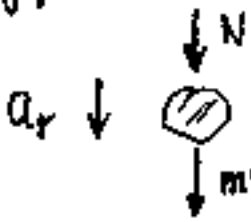
Ex. dare-devil motor cycle



m = motor cycle + dare devil

v_{top} = smallest velocity needed at top for safe ride (no crash)

free-body:



newton: $-N - mg = -m\frac{v_{top}^2}{R}$

$N > 0$ as long as tire in touch with ceiling

$N \approx 0$, where tire about to lose contact with ceiling → crash

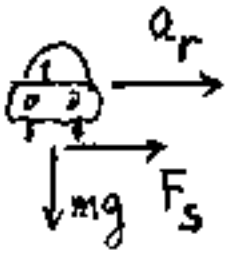
∴ smallest velo given by $N=0$, i.e. $v_{top}^2 \approx Rg = 30 \times 9.8$

$v_{top} = 17.15$ m/s

Ex. Car drives on circular road, which is flat.

How fast can it drive without skiding?

look at position (B)



F_s = static friction force

$\leq mg\mu_s$

$F = ma \rightarrow$ $F_s = m\frac{v^2}{r}$, i.e. $m\frac{v^2}{r} \leq mg\mu_s$

safe driving

if $\frac{mv^2}{r} > mg\mu_s$, "call 911"

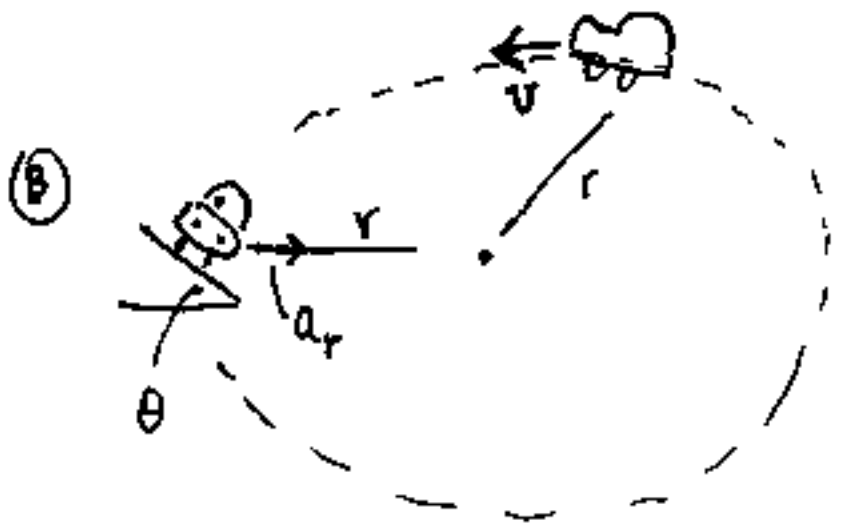
Suppose $r = 200$ m, $\mu_s = .15$, $v = 50$ km/hr
will car skid or not ?

$$\frac{v^2}{r} = \frac{(50 \times 1000/3600)^2}{200} = \frac{192.9}{200} = 0.964$$

$$g\mu_s = 9.8 \times .15 = 1.47$$

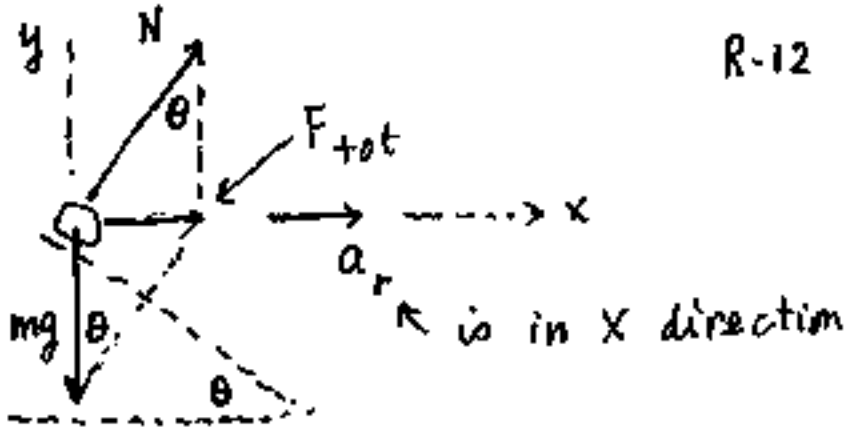
$\therefore$ $m\frac{v^2}{r} < mg\mu_s$. car drives safely without skiding.

Curves of highway is "banked", to prevent skiding



consider first the case of no friction betw road & tires

free-body diag.
at position (B)



↖ is in X direction

only N and mg act on car

N = normal force from road

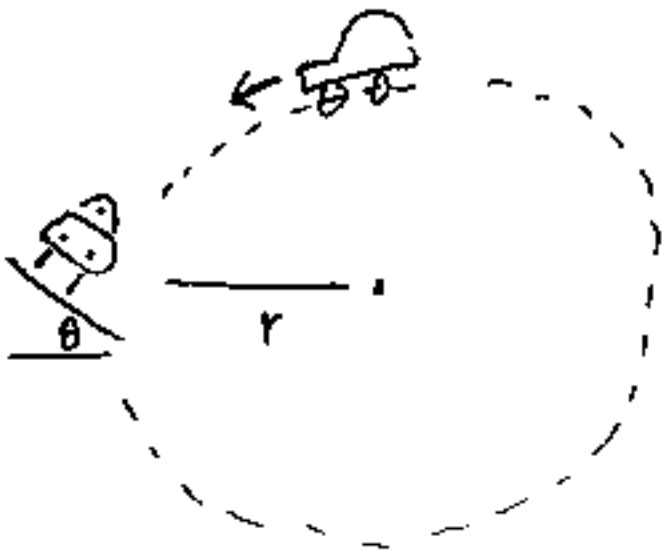
$\vec{F}_{tot} = \vec{N} + \vec{mg}$

... must in direction of $\vec{a}_r$ ...

y-balance : $N\cos\theta = mg$ ①

$(F=ma)_x$: $N\sin\theta = ma_r = m\frac{v^2}{r}$ ②

$\frac{②}{①}$, N cancels, $\boxed{\tan\theta = \frac{v^2}{rg}}$



suppose $v = 20\ m/s$

$r = 150\ m$

banking angle θ should = ?

$\tan\theta = \frac{20^2}{150 \times 9.8} = .272$

$\theta = 15.2^0$