M-THEORY ON MANIFOLDS OF G_2 HOLONOMY:

THE FIRST 20 YEARS

SUPERGRAVITY @ 25

STONY BROOK, 2001

1982: HOLONOMY INTRODUCED INTO KALUZA-KLEIN THEORY. COUNTS NUMBER OF SUPER-SYMMETRIES N SURVIVING COMPACTIFICATION

Awada, Duff & Pope PRL 50 (1983) 294

FIRST NON-TRIVIAL EXAMPLE:

$(N=1, D=11)$ SUGRA ON SQUASHED S^7

$$\mathcal{H} = G_2 \Rightarrow (N=1, D=4)$$

1983: $(N=1, D=11)$ SUGRA ON K3

$$\mathcal{H} = SU(2) \Rightarrow N = \frac{1}{2} N_{max}$$

Duff, Nilsson, Pope PLB 129 (1983) 2043

2001: $D=11$ M-THEORY ON G_2 & K3 MANIFOLDS

D=11 SUPERGRAVITY

Cremmer, Julia & Scherk PLB 76 (1978) 409

FIELDS : $(g_{MN}, \psi_M, A_{MNP})$

SUSY : $\delta \psi_M = \tilde{D}_M \varepsilon$

$$\tilde{D}_M = D_M - \frac{i}{144}\left(\Gamma^{NPQR}{}_M + 8\Gamma^{PQR}\delta^N_M\right)F_{NPQR}$$

EQNS : $G_{MN} = T_{MN}(A)$

$$d^*F = F \wedge F$$

COSMOLOGICAL CONSTANT

Duff & van Nieuwenhuizen PLB 94 (1980) 179

$$F_{\mu\nu\rho\sigma} = 3m\,\varepsilon_{\mu\nu\rho\sigma}$$

4/7 SPLIT

Freund & Rubin PLB 97 (1980) 233

$R_{\mu\nu} = -12m^2 g_{\mu\nu}$ $\quad \tilde{D}_\mu = D_\mu + m\gamma_\mu\gamma_5$

$R_{mn} = 6m^2 g_{mn}$ $\quad \tilde{D}_m = D_m - \frac{1}{2}m\,\Gamma_m$

$\Rightarrow AdS_4 \times M^7$

HOLONOMY AND SUPERSYMMETRY +

NUMBER OF D=4 SUPERSYMMETRIES N

= NUMBER OF KILLING SPINORS η ON M^7

$$\tilde{D}_m \eta = 0$$

INTEGRABILITY CONDITION

$$[\tilde{D}_m, \tilde{D}_n]\eta = C_{mn}{}^{ab}\Gamma_{ab}\eta = 0$$

WEYL TENSOR

SO(7) GENERATORS

SUBSET OF SO(7) GENERATORS THAT LEAVE SPINOR INVARIANT YIELDS HOLONOMY GROUP

$$\mathcal{H} \subset SO(7)$$

SQUASHED S^7:

$$\mathcal{H} = G_2$$

$$8 \rightarrow 1 + 7$$

$$N = 1$$

NOTE ON TERMINOLOGY

SOME AUTHORS TAKE "HOLONOMY" TO REFER TO RIEMANNIAN CONNECTION D_M, THEN SQUASHED S^7 HAS $\mathcal{H} = SO(7)$ AND SAY THAT G_2 REFERS ONLY TO "WEAK HOLONOMY"

3-FORM A $\quad {}^*dA \sim A$ $\quad A_{abc} = \bar{\eta}\,\Gamma_{abc}\,\eta$

BUT

MATH: HOLONOMY REFERS NOT TO MANIFOLD M BUT CONNECTION ON M

PHYSICS: RELEVANT CONNECTION IS $\tilde{D}_M$ NOT, IN GENERAL, RIEMANNIAN ONE D_M

SO PREFER TO SAY $\mathcal{H} = G_2$ FOR SQUASHED S^7

N.B NOT RICCI FLAT

GET NON-ABELIAN GAUGE BOSONS $\nearrow$ $SO(5) \times SO(3)$

Englert PLB 119 (1982) 119

Bais, Nicolai & van N NPB 228 (1983) 333

Castellani, D'Auria, Fré & van N JMP 25 (1984) 3209

Warner PLB 128 (1983) 169

Bergshoeff, de Roo, de Wit & van N PLB 195 (1982) 97

RIGHT AND LEFT

LEFT-SQUASHED S^7 HAS $N=1$ BUT

RIGHT-SQUASHED S^7 HAS $N=0$

$(\equiv m \to -m)$

BUT $[\tilde{D}_m, \tilde{D}_n] = C_{mn}{}^{ab}\Gamma_{ab}$

CANNOT TELL THE DIFFERENCE!

WHAT IS HOLONOMY ARGUMENT?

Castellani et al:

$$\tilde{D}_m = \partial_m + \underbrace{\omega_m{}^{ab}\Gamma_{ab} \pm \frac{i}{2} m \Gamma_a}_{SO(7) \to SO(8) \text{ CONNECTION}}$$

GENERALIZED $\mathcal{H}_{GEN} \subset SO(8)$

LEFT $\mathcal{H} = SO(7)^-$ $\quad 8 \to 1+7 \quad N=1$

RIGHT $\mathcal{H} = SO(7)^+$ $\quad 8 \to 8 \quad N=0$

BRANE APPLICATIONS

1991 : D=11 SUGRA ADMITS 2-BRANE SOLUTION

S^7

$$ds^2 = H^{2/3} dx^\mu dx_\mu + H^{-1/3}\left(dr^2 + r^2 d\Omega_7^2\right)$$

$$H = 1 + \frac{a^2}{r^6} \qquad N = \frac{1}{2} N_{max}$$

Duff & Stelle PLB 253 (1991) 113

1993 : NEAR-HORIZON GEOMETRY $AdS_4 \times S^7$

Gibbons & Townsend PRL 71 (1993) 5223

1996 : P-BRANES WITH FEWER SUSIES $< N_{max}/2$

$d\bar{s}^2$ BY REPLACING S^7 WITH M^7 $N < 8$

Duff, Lu, Pope & Sezgin PLB 371 (1996) 206

$\rightarrow AdS_4 \times M^7$

TRANSVERSE METRIC ASYMPTOTIC TO CONE $dr^2 + r^2 d\bar{\Omega}_7^2$ BUT NO CONICAL SINGULARITY AT $r = 0$, SINCE $H^{-1/3} r^2 \rightarrow$ constant

2001 : M-THEORY ON SINGULAR G_2 MANIFOLDS

AGAIN GET $N=1$ BUT NOW
CAN HAVE CHIRAL FERMIONS
AND REALISTIC GAUGE GROUPS
(AND RICCI FLAT)

Joyce

Townsend & Papadopoulos

Acharya

Atiyah, Maldacena, Vafa

Atiyah, Witten

Cvetic, Shiu, Uranga

Cvetic, Gibbons, Lu, Pope

Cvetic, Gibbons, Liu, Lu, Pope

⋮

1983 K3 COMPACTIFICATION

Duff, Nilsson & Pope* PLB 129 (1983) 39

$\mathcal{H} = SU(2) \Rightarrow N = \frac{1}{2} N_{max}$

58 parameters, no explicit metric, self-dual

Massless spectrum depends on TOPOLOGY rather than GEOMETRY

* Betti numbers

$b_0 = 1 \quad b_0 = 0 \quad b_2 = 22 \quad b_3 = 0 \quad b_4 = 1$

Euler number $\chi = \sum_{p=0}^{4} (-1)^p b_p = 24$

* Index theorems

* " we do not know any solutions with

$\mathcal{H} = SU(3)$ "

N=2 ON K3 SAME SUSY AS

N=1 ON T^4 EXAMPLE:

1986 D=11 SUGRA ON $K3 \times T^{n-3}$ &

HETEROTIC STRING ON T^n

NOT ONLY HAVE SAME SUSY BUT

SAME MODULI SPACE

$$\mathcal{M} = \frac{SO(16+n,n)}{SO(16+n) \times SO(n)}$$

Duff & Nilsson PLB 175 (1986) 417

1995 Hull & Townsend NPB 438 (1995) 109

Witten NPB 443 (1995) 85

THEY ARE DUAL!

1995 M-THEORY 5-BRANE WORLDVOLUME ON K3

$\equiv$ HETEROTIC STRING WORLDSHEET

Townsend PLB 373 (1996) 68

Harvey & Strominger NPB 449 (1995) 535

Schwarz