Matter Coupling in Supergravity

Sergio Ferrara

Stony Brook, 1st Dec. 01

Freedman, Van Nieuwenhuizen

Grisaru, Gibbons, Julia, De Wit, Kallosh, Fré

Cremmer, Girardello Scherk

Strominger, van Proeyen, D'Auria, Andrianopoli;

Ceresole, Castellani

Many important contributions from many others

West, Schwarz, Duff, Gates, Lindstrom

Warner, Sezgin, Rocek, Siegel, Sokatchev

Stelle, Howe, Brink, Townsend, Hull, Pope

Supergravity theories can be viewed as supersymmetric theories of gravitation.

Since gravity encompasses general covariance (diffeomorphisms), supergravity has general supercovariance (superspace diffeomorphisms)

$$\delta \psi_\mu^A = \frac{1}{\kappa} D_\mu \epsilon^A + \ldots$$

As much as general relativity a given supergravity theory may admit different "vacua" with some killing symmetries.

Vacua preserving some "rigid supersymmetry", if they preserve Lorentz invariance, fall into distinct classes

Super Poincaré or Super anti de Sitter

(this crucially depends on the space-time signature)

After the construction of (N=1, D=4) supergravity, in winter 1976, the first task on the menu was to obtain matter couplings. They were worked out (although not in full generality) as early as in summer 1976 and the first extended (N=2) supergravity in the fall of the same year. (At already that time, on the basis of supermultiplet reps. M. Gell-Mann remarked that there should have existed at most N=8 supergravity).

That result was closely related to the subsequent Nahm classification, implying that SuperPoincaré gravities should exist up to D=11 dimension and superconformal field theories up to D=6 (Anti de Sitter supergravity up to D=7)

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Locally Supersymmetric Maxwell-Einstein Theory

S. Ferrara* and J. Scherk
Laboratoire de Physique Théorique de l'Ecole Normale Supérieure,† Paris, France

and

P. van Nieuwenhuizen‡
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794
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The spin-2–spin-$\frac{3}{2}$ gauge multiplet for supergravity is coupled to the spin-1–spin-$\frac{1}{2}$ vector multiplet in a locally supersymmetric way.

The gauge action for local supersymmetry (supergravity) was constructed by Freedman, van Nieuwenhuizen, and Ferrara,[1] and contains the massless spin-2 and spin-$\frac{3}{2}$ gravitational and Rarita-Schwinger fields. Deser and Zumino[2] gave an alternative simpler derivation in which they stressed the importance of torsion in supergravity, and Nath and Arnowitt[3] showed that the theory of Refs. 1 and 2 can be obtained from their superspace approach by taking a singular limit. In this Letter we discuss the next step in supergravity theory: coupling to matter. We couple the spin-1–spin-$\frac{1}{2}$ vector multiplet of global supersymmetry[4] in a locally supersymmetric way to the supergauge action. The key is to start in lowest order of the gravitational constant by coupling the spin-$\frac{3}{2}$ field to the Noether current of global supersymmetry, and to restore local gauge invariance in higher orders by successively adding terms to action and transformation laws in a systematic way. This is thus the same approach as was followed successfully in Ref. 1. We work in second-order formalism without auxiliary fields, but our final results indicate the presence of other auxiliary fields in addition to torsion.

Before commenting on the derivation, we give our final results. The complete action is the sum of the supergauge action $\mathcal{L}^G$ and the matter action $\mathcal{L}^M=\hat{\mathcal{L}}^M+\mathcal{L}_4{}^M$ to be defined below. The latter contains a massless Abelian spin-1 vector field, A_μ (the photon), and a Majorana spin-$\frac{1}{2}$ field, λ, coupled to the spin-$\frac{3}{2}$ Majorana spinor, ψ_μ, and to the gravitational vierbein field, $e_{a\mu}$ (the graviton), by means of two terms: a coupling of ψ_μ to the Noether current of global supersymmetry which is linear in the gravitational constant κ, and a direct four-fermion interaction $\mathcal{L}_4{}^M$ of gravitational strength κ^2:

$$\hat{\mathcal{L}}^M=-\tfrac{1}{4}eg^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}-\tfrac{1}{2}e\bar{\lambda}\gamma^\mu D_\mu\lambda+\tfrac{1}{4}e\kappa\bar{\psi}_\mu\gamma^\alpha\gamma^\beta\gamma^\mu\lambda F_{\alpha\beta}, \tag{1}$$

$$\begin{aligned}\mathcal{L}_4{}^M=(e\kappa^2/8)[&-(\bar{\psi}\cdot\psi)(\bar{\lambda}\lambda)+(\bar{\psi}_\alpha\gamma_5\psi^\alpha)(\bar{\lambda}\gamma_5\lambda)+\tfrac{1}{4}(\bar{\psi}\cdot\gamma\gamma\cdot\psi)(\bar{\lambda}\lambda)+\tfrac{1}{4}(\bar{\psi}\cdot\gamma\gamma_5\gamma\cdot\psi)(\bar{\lambda}\gamma_5\lambda)\\&-\tfrac{1}{2}(\bar{\psi}\cdot\gamma\gamma_5\psi_\alpha)(\bar{\lambda}\gamma_5\gamma^\alpha\lambda)+\tfrac{1}{4}(\bar{\psi}_\alpha\gamma_5\gamma_\rho\psi^\alpha)(\bar{\lambda}\gamma_5\gamma^\rho\lambda)+\tfrac{3}{2}(\bar{\lambda}\lambda)(\bar{\lambda}\lambda)].\end{aligned} \tag{2}$$

The symbol $\bar{\psi}\cdot\psi$ stands for $\bar{\psi}_\alpha\psi^\alpha$ and $\gamma\cdot\psi$ for $\gamma^\alpha\psi_\alpha$.

The gravitational action is the same as in Refs. 1 and 2,

$$\mathcal{L}^G=-\tfrac{1}{2}\kappa^{-2}eR-\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma-(e\kappa^2/32)[(\bar{\psi}^b\gamma^a\psi^c)(\bar{\psi}_b\gamma_a\psi_c+2\bar{\psi}_a\gamma_b\psi_c)-4(\bar{\psi}_a\gamma\cdot\psi)^2]. \tag{3}$$

The total action $\mathcal{L}^G+\mathcal{L}^M$ is invariant under the following local supersymmetry transformations on the

Consistent Supergravity with Complex Spin-$\frac{3}{2}$ Gauge Fields

S. Ferrara* and P. van Nieuwenhuizen†
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794
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We construct a new theory of supergravity which is invariant under complex local supersymmetry transformations. This theory is obtained by coupling the massless spin-$(\frac{3}{2}, 1)$ multiplet of ordinary global supersymmetry to the spin-$(2, \frac{3}{2})$ gauge multiplet of previous supergravity.

Ordinary supergravity is formulated[1-3] in terms of two real gauge fields, the spin-2 vierbein field and the Majorana spin-$\frac{3}{2}$ Rarita-Schwinger[4] field. The latter is the gauge field belonging to local supersymmetry transformations with anticommuting real (Majorana) spinorial parameters. In this Letter we formulate a new theory of supergravity in terms of the real spin-2 and spin-1 vierbein and photon fields, and a complex spin-$\frac{3}{2}$ Rarita-Schwinger field. This theory is obtained by coupling the real spin-$(2, \frac{3}{2})$ gauge multiplet of supergravity to the real spin-$(\frac{3}{2}, 1)$ matter multiplet of global flat-space supersymmetry. The two real (Majorana) spin-$\frac{3}{2}$ fields are combined into the single complex (Dirac) spin-$\frac{3}{2}$ field. The resulting action turns out to have a complex local supersymmetry whose gauge field is this complex spin-$\frac{3}{2}$ field. This theory unifies gravitation and electromagnetism, the photon now belonging to the gauge multiplet of complex local supersymmetry, and constitutes the first example of a consistent interacting theory of complex spin-$\frac{3}{2}$ fields.

Before discussing the derivation, we give the results. The complete action is the sum of the supergauge action $\mathcal{L}^G$ of ordinary supergravity, and the matter action $\mathcal{L}^M$. The former contains the vierbein field $e_{a\mu}$ (the graviton) and the Majorana Rarita-Schwinger field ψ_μ, while the latter describes a photon A_μ and a second Majorana spin-$\frac{3}{2}$ field φ_μ. The matter action couples, as expected, ψ_μ to the Noether current associated with the global supersymmetry invariance of the matter action, and contains, in addition, a four-fermion contact term $\mathcal{L}_4^M$.

$$\mathcal{L} = \mathcal{L}^G + \mathcal{L}^M, \quad \mathcal{L}^M = \hat{\mathcal{L}}^M + \mathcal{L}_4^M. \tag{1}$$

$$\hat{\mathcal{L}}^M = -\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\varphi}_\mu\gamma_5\gamma_\nu D_\rho\varphi_\sigma - \tfrac{1}{4}eg^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} + (\kappa/\sqrt{2})\bar{\psi}_\mu[eF^{\mu\nu} + \tfrac{1}{2}\gamma_5\tilde{F}^{\mu\nu}]\varphi_\nu, \tag{2}$$

$$\begin{aligned}\mathcal{L}_4^M = {} & -\tfrac{1}{4}\kappa^2(\bar{\psi}_\mu\varphi_\nu)[e(\bar{\psi}^\mu\varphi^\nu - \bar{\psi}^\nu\varphi^\mu) + \epsilon^{\mu\nu\rho\sigma}(\bar{\psi}_\rho\gamma_5\varphi_\sigma)] \\ & - (e\kappa^2/16)[(\bar{\psi}^b\gamma^a\psi^c)(\bar{\varphi}_b\gamma_a\varphi_c + 2\bar{\varphi}_a\gamma_b\varphi_c) - 4(\bar{\psi}^a\gamma\cdot\psi)(\bar{\varphi}_a\gamma\cdot\varphi)] \\ & - (e\kappa^2/32)[(\bar{\varphi}^b\gamma^a\varphi^c)(\bar{\varphi}_b\gamma_a\varphi_c + 2\bar{\varphi}_a\gamma_b\varphi_c) - 4(\bar{\varphi}_a\gamma\cdot\varphi)^2],\end{aligned} \tag{3}$$

$$\begin{aligned}\mathcal{L}^G = {} & -(2\kappa^2)^{-1}eR(e) - \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma \\ & - (e\kappa^2/32)[\bar{\psi}^b\gamma^a\psi^c)(\bar{\psi}_b\gamma_a\psi_c + 2\bar{\psi}_a\gamma_b\psi_c) - 4(\bar{\psi}_a\gamma\cdot\psi)^2].\end{aligned} \tag{4}$$

The action is invariant under the following transformation laws

$$\delta A_\mu = \sqrt{2}(\bar{\epsilon}\varphi_\mu), \quad \delta e^a{}_\mu = \kappa\bar{\epsilon}\gamma^a\psi_\mu, \tag{5}$$

$$\delta\varphi_\mu = -(1/\sqrt{2})[F_{\mu\nu}\gamma^\nu + \tfrac{1}{2}e\tilde{F}_{\mu\nu}\gamma^\nu\gamma_5]\epsilon + \tfrac{1}{2}\kappa(\bar{\psi}_\mu\varphi_\nu - \bar{\psi}_\nu\varphi_\mu)\gamma^\nu\epsilon + \tfrac{1}{4}\kappa e\epsilon_{\mu\nu\alpha\beta}(\bar{\psi}^\alpha\varphi^\beta)\gamma^\nu\gamma_5\epsilon, \tag{6}$$

$$\delta\psi_\mu = \delta\psi_\mu{}^G + \tfrac{1}{4}\kappa\sigma^{ab}\epsilon[2\bar{\varphi}_\mu\gamma_a\varphi_b + \bar{\varphi}_a\gamma_\mu\varphi_b], \tag{7}$$

$$\delta\psi_\mu{}^G = 2\kappa^{-1}D_\mu\epsilon + \tfrac{1}{4}\kappa\sigma^{ab}\epsilon[2\bar{\psi}_\mu\gamma_a\psi_b + \bar{\psi}_a\gamma_\mu\psi_b]. \tag{8}$$

In these equations D_μ is the gravitational covariant derivative (without torsion), $\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, $\gamma\cdot\psi = \gamma^a\psi_a$, $e = \det(e^a{}_\mu)$, ϵ is a real space-time-dependent Majorana spinor, while Greek (Latin) indices consistently denote world (local Lorentz) tensors. We use the positive metric $g_{\mu\nu} = g^{\mu\nu} = \delta_\mu{}^\nu = (+,+,+,+)$ in flat space (the Pauli metric) and our gamma matrices are Hermitian with squares equal to one ($\gamma_5{}^2 = \gamma_a{}^2 = 1$ with $a = 1, 4$).

As in previous matter-supergravity couplings,[5-8] the bilinear term in the transformation law of the

Many supergravity theories, depending on the non-trivial dimension, on their gauging, and of their "matter" content, may or may not have supersymmetric vacua.

If not, the "super Higgs" mechanism must take place

$$\ldots\ N_A^{\,i}\,\bar{\chi}_i\,\gamma^\mu\,\psi_\mu^{\,A}\ \ldots$$

The physical situation would correspond to a supergravity theory which has a Minkowski vacuum with broken supersymmetry (vanishing cosmological constant)

Volkov, Soroka
Volkov, Akulov
Deser, Zumino
Polony
Cremmer, Julia, Scherk
S.F., van Nieuwenhuizen
Girardello

No-scale supergravities

Cremmer, S.F.,
Kounnas, Nanopoulos
Ellis

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SPONTANEOUS SYMMETRY BREAKING AND HIGGS EFFECT IN SUPERGRAVITY WITHOUT COSMOLOGICAL CONSTANT

E. CREMMER, B. JULIA and J. SCHERK
*Laboratoire de Physique Théorique Ecole Normale Supérieure *, Paris, France*

S. FERRARA
CERN, Geneva, Switzerland

L. GIRARDELLO
Istituto di Fisica dell'Università, Milano,
INFN, Sezione di Milano, Italia

P. van NIEUWENHUIZEN **
Instituut Lorentz, Leiden, The Netherlands

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The super Higgs effect is studied in the $(2, \frac{3}{2}) + (\frac{1}{2}, 0^+, 0^-)$ model. The most general action is obtained using the recently developed tensor calculus: it contains an arbitrary function of *two* variables $\mathcal{G}(A, B)$, A and B being the 0^+ scalar and 0^- pseudoscalar fields of the matter system. The conditions are given which $\mathcal{G}$ must satisfy in order that both the gravitino ψ_μ becomes massive and no cosmological term is induced. Explicit examples are given, a class of them leading to the mass formula $m_A^2 + m_B^2 = 4m_\psi^2$.

1. Introduction

Supersymmetry *** assigns equal masses to bosons and fermions in the same multiplet. Such a degeneracy is not observed in Nature. It is thus important to break supersymmetry, either explicitly or spontaneously. Below we consider spontaneous breaking of supersymmetry in the spin $(2, \frac{3}{2})$ + spin $(\frac{1}{2}, 0^+, 0^-)$

* Laboratoire propre du C.N.R.S. associé à l'Ecole Normale Supérieure et à l'Université de Paris-Sud.
** On leave from the Institute for Theoretical Physics, State University of New York, Stony Brook.
*** For reviews on supersymmetry, see, for instance, ref. [1].

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NATURALLY VANISHING COSMOLOGICAL CONSTANT IN $N = 1$ SUPERGRAVITY

E. CREMMER
Ecole Normale Supérieure, Paris, France

and

S. FERRARA, C. KOUNNAS and D.V. NANOPOULOS
CERN, Geneva, Switzerland

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For $N = 1$ supergravity theories we show that the choice of a particular class of Einstein spaces for the Kähler manifold of the hidden sector leads to a vanishing cosmological constant without unnatural fine tuning. The total scalar potential from the hidden and physical sector is positive definite. The resulting low energy softly broken global supersymmetry for the matter fields is thus the same as in the case of factorized superpotential models with a flat Kähler metric.

The almost vanishing value of the cosmological constant is an old, interesting, but unsolved problem in gravity and more recently in supergravity theories. The experimental upper limit on that quantity is found to be extremely small ($\Lambda \sim 10$–50 GeV^4). However, no convincing theoretical or syymetry argument is known to explain the almost vanishing of the cosmological constant. It is the purpose of the current paper to study that problem in the context of $N = 1$ local supersymmetric theories.

Some successful phenomenological implications seem to follow from supersymmetric theories [1]. For instance, the stability under radiative corrections of the well-known hierarchy between the two mass scales (M_X, M_W) appearing in grand unified theories has a natural explanation in supersymmetric theories due to the non-renormalization theorems [2]. The splitting between the doublet and triplet parts of 5 Higgs in SU(5) theory can be also understood in the framework of supergravity theories [3]. Furthermore, the M_X, M_W hierarchy is naturally realized, assuming a radiative breaking of the $SU(2) \times U(1)$ subgroup [4,5]. It turns out that the radiative breaking is the simplest mechanism which does not destabilize the doublet-triplet hierarchy when the $SU(2) \times U(1)$ is embedded into a grand unification group [6]. Moreover, another serious hierarchy problem seems to find its natural explanation in the framework of a supersymmetric theory, such as the smallness of non-perturbative effects in QCD which are parametrized by θ_{QCD} ($\theta_{QCD} < 10^{-9}$!) [7].

Although the GUT hierarchy problems may be naturally solved in supergravity theories, it is not possible to shed any light on the cosmological constant problem, at least in the case where the minimal super-Higgs mechanism [8] [‡1] is assumed. In this work, we focus our attention on that specific problem. We will show that if a suitable non-minimal structure is introduced for the hidden sector (chiral supermultiplets which are related to the supersymmetry breaking and decouple in the flat limit), a vanishing value for the cosmological constant is automatically ensured. To this non-minimal structure of the hidden sector there corresponds a particular class of non-trivial Kähler manifolds which are actually Einstein spaces. Hence no fine-tuning of the value of the cosmological constant is necessary. In fact, the total potential (from the hidden sector and the usual matter sector) turns out to be positive definite and its value at the minimum is identically equal to zero.

In order to explain the main idea, let us first consider only one chiral multiplet (z, χ) coupled to super-

[‡1] For a discussion of Kähler invariance, see also ref. [9].

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During that time (the second half of 76),
it was realized that coupling to
matter multiplets encompassing spin 0
bosons were much harder to obtain
due to the possible non-polynomial nature
of the interactions.

The nonpolynomiality was soon understood,
in a geometrical setting, in terms
of non-linear σ models, where the
nature of the target-space depends on
the number of supersymmetries (N-extended
Supergravity) and on the space-time dimension D

For example $N=2$ non-linear σ-models
coupled to gravity encode
"quaternionic" geometry (Hypermultiplets)
and "special" Hodge-Kähler geometry
(vector multiplets) with a wide class
of applications in string and M-theory
(De Wit, van Holten, van Proeyen, Strominger, Castellani et al)

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MATTER COUPLINGS IN SUPERGRAVITY THEORY

S. FERRARA *, F. GLIOZZI and J. SCHERK
Laboratoire de Physique Théorique de l'Ecole Normale Supérieure, Paris, France **

P. VAN NIEUWENHUIZEN ***
ITP SUNY at Stony Brook, LI NY 11794, USA

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The Abelian (Maxwell) and non-Abelian (Yang-Mills) spin-1 – spin-$\frac{1}{2}$ vector multiplets of global symmetry are coupled to the spin-2 – spin-$\frac{3}{2}$ gauge multiplet of supergravity in a locally supersymmetric way. The corresponding transformations of the fields which leave the action invariant are derived. The coupling of the scalar spin-0 – spin-$\frac{1}{2}$ multiplet to supergravity is also discussed. The same results are also obtained by the requirement of gauge invariance of the S-matrix.

1. Introduction

Supersymmetry was originally introduced as a global invariance of certain Lagrangian field theories in which bosons are transformed into fermions and *vice versa* [1]. Global supersymmetry transformations involve constant (anticommuting) spinor parameters ϵ.

Local supersymmetry invariance with arbitrary space-dependent parameters $\epsilon(x)$ can only be achieved by going to curved space and therefore the introduction of gravitation is necessary [2]. The gauge action for supergravity has recently been constructed [3]. It describes the interaction of the graviton with a massless spin-$\frac{3}{2}$ fermion, the gauge field related to local supersymmetry transformations. This theory provides the minimal extension of Einstein theory which incorporates a local fermionic symmetry and it has a particular simple form in the Cartan first order formulation with torsion of general relativity [4].

In a previous note [5] the first example of a consistent coupling of a multiplet of global supersymmetry to supergravity has been reported together with the transfor-

* On leave of absence from Frascati National Laboratories, Frascati, Italy.
** Laboratoire propre du CNRS associé à l'Ecole Normale Supérieure et à l'Université de Paris Sud. Postal address: ENS 24 rue Lhomond, 75230-Paris Cédex 05, France.
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Scalar multiplet coupled to supergravity

S. Ferrara,* D. Z. Freedman,† and P. van Nieuwenhuizen†
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794

P. Breitenlohner
Max-Planck Institut für Physik u. Astrophysik Munich, Germany

F. Gliozzi and J. Scherk
École Normale Supérieure Paris, France
(Received 30 September 1976)

We couple the massless spin-($0^{\pm}$,1/2) scalar multiplet of global supersymmetry to the gauge action of supergravity. The resulting locally supersymmetric Lagrangian and transformation laws are derived both in second- and first-order formalism. The results are valid for an arbitrary global internal-symmetry group. The commutator algebra is also obtained and has new features compared with previous models. The mass term of the scalar multiplet is also discussed.

I. INTRODUCTION

Supergravity is a newly proposed theory of gravitation which has an extra gauge invariance, namely local supersymmetry, in addition to the expected general coordinate invariance. The action of the gauge or gravitational multiplet describes a massless spin-2 boson field (the graviton) and a massless real spin-$\frac{3}{2}$ fermion field (the Rarita-Schwinger field[1]). The action and transformation rules were first constructed[2] using the more conventional second-order formalism for gravity and were then shown to be simplified[3] by the use of the first-order formalism. The commutator algebra of local supersymmetry[4] displays the connection between supersymmetry and general coordinate invariance.

A natural direction of development for supergravity is the extension of the actions and transformation rules of known[5] globally supersymmetric, four-dimensional field theories to achieve local symmetry. There has been recent progress in this matter-coupling problem. Namely, the locally supersymmetric Maxwell-Einstein theory has been obtained[6] by a constructive method (similar to that of Ref. 2), using the second-order description of gravity. These techniques[7] and similar methods[8] using first-order gravitation have been used to construct the locally supersymmetric extension of the Yang-Mills multiplet. Partial results have also been obtained for the scalar or chiral multiplet.[7] Locally supersymmetric systems in one and two spacetime dimensions (spinning particles[9] and strings[10]) have also been constructed.

In this article, the complete coupling of the massless scalar multiplet to supergravity is obtained, together with the transformation laws of the fields. The results are valid for an arbitrary global internal-symmetry group. The results are obtained both in second-order and in first-order (torsion) form. Unexpectedly, we found that the Lagrangian, presented in Ref. 7 as a partial result, is in fact the complete Lagrangian in second-order form. The commutator algebra of supersymmetry transformations is considered next and is found to have a more complicated structure than in previous models.[4,8] Finally, a mass term is added to the scalar multiplet. Results of the action up to order κ^3 and of the transformation laws up to order κ^2 seem to indicate that in this case the locally supersymmetric action is nonpolynomial in the scalar and pseudoscalar fields. No inconsistencies are present up to these orders.

The method by which the action and transformation laws are obtained is the same method as in Refs. 2, 4, 6, 7, and 8. One starts by adding the order-κ coupling of the spin-$\frac{3}{2}$ field ψ_μ to the Noether current of the globally supersymmetric matter system, and by covariantizing such that all results hold in curved space. Then one expands the variation of the action in powers of κ, and adds terms to action and transformation laws such that at each κ level invariance is maintained. It should be stressed that this method is unique, unambiguous, and straightforward. It is applicable to all cases where one wants to couple two systems in an invariant way. The method is based entirely on ordinary four-dimensional spacetime, but superspace methods[11] may eventually also illuminate the problem of matter couplings in supergravity.

In the next two sections the action and transformation laws for the massless scalar multiplet coupled to supergravity are obtained. In Sec. IV the algebra is discussed, while in Sec. V the mass-

Matter coupled (extended supergravity) in D dimensions

Theories of gravity coupled to non-linear σ models and $p+1$ gauge fields ($p \leq 3$)

A (non-compact) group G act linearly on the $p+1$ gauge form potential as well as (non-linearly) on the scalar field.

Bosonic part of the action

$$\int d^D x \left[\sqrt{-g}\, R - g_{AB}\, \partial_\mu \varphi^A \partial_\nu \varphi^B g^{\mu\nu} \right]$$

$$- \int G_{\Lambda\Sigma} H^\Lambda \wedge {}^* H^\Sigma - \int \theta_{\Lambda\Sigma} H^\Lambda \wedge H^\Sigma$$

(θ term exist if $p+2 = \frac{D}{2}$)

$p \neq \frac{D-4}{2} \quad G_{\Lambda\Sigma} \subset \frac{SL(n)}{SO(n)}$

$p = \frac{D-4}{2} \quad (G_{\Lambda\Sigma}, \theta_{\Lambda\Sigma}) \subset \frac{Sp(2n,R)}{U(N)}, \frac{SO(n,n)}{SO(n)\times SO(n)}$

p even $\qquad$ p odd

(Freedman, A. Gaumé; Bagger Witten; De Wit, Van Proeyen)

Magics of Quaternionic Geometry

(Alekseevsky ... Galiski)

$$\mathcal{M}(Q) = \{q^u\} \qquad u = 1 \ldots 4n_H$$

$$q^u \to q^u + K^u_\Lambda(q)\,\xi^\Lambda$$

$$\mathcal{M}_{SK}(z) = \{z^i\} \qquad i = 1 \ldots n_V$$

$$z^i \to z^i + K^i_\Lambda(z)\,\xi^\Lambda$$

$$L^\Lambda(z,\bar z) = e^{K/2} X^\Lambda(z)$$

(D'Auria, S.F. Frè)

$$V(q,z,\bar z) = \left(g_{i\bar j} K^i_\Lambda K^{\bar j}_\Sigma + h_{uv} K^u_\Lambda K^v_\Sigma\right) \bar L^\Lambda L^\Sigma$$

$$+ \left(-\tfrac{1}{2}(\mathrm{Im}\,\mathcal{N})^{-1\,\Lambda\Sigma} - \bar L^\Lambda L^\Sigma\right) P^x_\Lambda P^x_\Sigma - 3\, P^x_\Lambda P^x_\Sigma \bar L^\Lambda L^\Sigma$$

$$k^i_\Lambda = g^{i\bar j}\,\partial_{\bar j} P^0_\Lambda \quad , \quad K^u_\Lambda = \frac{1}{6}\,\Omega^{x\,vu}\,\nabla_v P^x_\Lambda$$

$\mathcal{N}_{\Lambda\Sigma}$ (coupling constant) $\qquad n_H P^x_\Lambda = -\frac{1}{2}\Omega^x_{uv} D^u K^v_\Lambda$

(D'Auria Frè)

(Cremmer, Kallosh, van Proeyen) dall'Agata

$$\nabla_u \nabla^u P^x_\Lambda = -4 n_H P^x_\Lambda$$

$$h_{uv} = \epsilon_{AB} C_{\alpha\beta}\, \mathcal{U}^{A\alpha}_u \mathcal{U}^{B\beta}_v$$

($\mathcal{U}^{\alpha A}_u$: quaternionic vielbein)

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SPECIAL AND QUATERNIONIC ISOMETRIES: GENERAL COUPLINGS IN $N = 2$ SUPERGRAVITY AND THE SCALAR POTENTIAL*

Riccardo D'AURIA

Dipartimento di Fisica, Universitá di Padova and Istituto Nazionale di Fisica Nucleare, Via Marzolo 8 I-35131 Padova, Italy

Sergio FERRARA

CERN, Theoretical Division, 1211 Geneva 23, Switzerland

Pietro FRÈ

SISSA-International School for Advanced Study, Via Beirut 2 I-34100 Trieste, Italy
and
INFN, Sezione di Trieste, Italy

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The general lagrangian for $N = 2$ matter-coupled supergravity is obtained, by gauging general isometries of quaternionic manifolds which can be coupled to supergravity. The resulting theories are purely geometrical and give an interplay between quaternionic and special Kähler geometry. The resulting scalar potential is expressed in terms of the two Killing prepotentials of the two geometries and it may be relevant to study transitions between different vacua in superstring theory. Furthermore from the geometrical point of view the prepotentials are hamiltonian functions yielding a poissonian realization of the gauge algebra on both the special Kähler and the quaternionic manifold. A possible cohomological obstruction to this realization is pointed out.

1. Introduction

One of the most striking results of supersymmetric quantum field theories is the deep relation between geometry, topology and supersymmetry.

In particular global and local supersymmetric theories exhibit deep geometrical structures inherent to the non-linear interactions of matter multiplets. Almost all of these couplings can be rephrased in a geometrical language as coupling of some non-linear σ-model to gravity and gauge fields [1–7].

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(Cremmer et al.)

Kahler-Hodge geometry and $N=1$ supergravity

$$V = (D_i L D_{\bar{j}} \bar{L} - 3 L \bar{L}) + \frac{1}{4} (\mathrm{Im} f^{-1})^{\Lambda\Sigma} P^0_\Lambda P^0_\Sigma$$

$$L = e^{K/2} W(z) \,, \quad (\partial_{\bar{i}} W = 0)$$

(M theory on manifolds with G_2 holonomy)
M. Duff.

unbroken supersymmetry:

$$D_i L = 0 \quad (\partial_i + K_i) W = 0 \,, \quad P^0_\Lambda = 0$$

(supersymmetric flow) $L = 0$ Superheavy
$L \neq 0$ anti de Sitter

In the horizon geometry of 4D-black-holes ($\frac{1}{2}$ BPS): (S.F., Kallosh, Strominger) (Gibbons, Kallosh SF)

$$D_i |Z| = 0 \qquad |Z| \text{ moduli dependent central charge}$$

$$(D_i \bar{Z} = 0)$$

The two situations get unified in $\mathrm{II}\,B$ supergravity on a Calabi-Yau manifold with black-hole charges or with H fluxes.

$N=2$ extremal black holes

Sergio Ferrara*
Theory Division, CERN, 1211 Geneva 23, Switzerland

Renata Kallosh†
Physics Department, Stanford University, Stanford California 94305-4060

Andrew Strominger‡
Department of Physics, University of California, Santa Barbara, California 93206-9530
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It is shown that extremal magnetic black hole solutions of $N=2$ supergravity coupled to vector multiplets X^Λ with a generic holomorphic prepotential $F(X^\Lambda)$ can be described as supersymmetric solitons which interpolate between maximally symmetric limiting solutions at spatial infinity and the horizon. A simple exact solution is found for the special case that the ratios of the X^Λ are real, and it is seen that the logarithm of the conformal factor of the spatial metric equals the Kähler potential on the vector multiplet moduli space. Several examples are discussed in detail.

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I. INTRODUCTION

Black holes seem to be a never-ending source of surprises. While much has been learned about their behavior, much remains to be understood, even at the classical level. In this paper we study the classical supersymmetric solutions in a general theory with $N=2$ supersymmetry. Previous work on this subject can be found in [1,2]. The solutions appear to have a richer structure than the more thoroughly studied $N=4$ case. In Sec. II we recall some aspects of $N=2$ supergravity. In Sec. III the magnetic solutions are described in terms of trajectories in the special geometry of the $N=2$ moduli space which terminate at a supersymmetric fixed point at the horizon. In Sec. IV we find that the equations can be integrated for a restricted but large class of cases. An intriguing relation between the Kähler potential on the moduli space and the metric conformal factor emerges. Some simple examples are worked out in detail in Sec. V. We do not attain a complete characterization of the classical geometry of $N=2$ black holes in this paper, but we hope that our results prove useful for future efforts in this direction.

II. SPECIAL GEOMETRY AND $N=2$ SUPERSYMMETRY

We study $N=2$ supergravity coupled to n $N=2$ vector multiplets in the framework of special geometry [3–6]. In this section some formulas that will be needed in the following are recalled. Further details can be found in [4] whose notation we adopt. The supergravity theory is defined in terms of a projective holomorphic section $(X^\Lambda(\phi^i), -i/2F_\Lambda(\phi^i))$, $\Lambda=0,1,\ldots,n$, $i=1,\ldots,n$, of an $Sp(2n+2)$ vector bundle over the moduli space parametrized by ϕ^i. (We note that alternate conventions are often employed in which the definition of F_Λ differs by a factor of $2i$.) In some cases the theory can be described in terms of a holomorphic function $F(X)$ of degree two:

$$F_\Lambda(\phi^i)=F_\Lambda(X(\phi^i))=\frac{\partial}{\partial X^\Lambda}F(X). \tag{1}$$

Given a prepotential $F(X)$, or a holomorphic section $(X^\Lambda, -i/2F_\Lambda)$, one can construct the entire scalar and vector parts of the action.

It is convenient to introduce the inhomogeneous coordinates

$$Z^\Lambda=\frac{X^\Lambda(\phi_i)}{X^0(\phi_i)}, \quad Z^0=1. \tag{2}$$

We assume $Z^\Lambda(\phi_i)$ to be invertible, so that, in special coordinates, $\partial Z^\Lambda/\partial\phi^i=\delta_i^\Lambda$. In this case the complex scalars $Z^i=\phi^i$ $(i=1,\ldots,n)$ represent the lowest component of the n vector multiplets of $N=2$ supersymmetry. The Kähler potential determining the metric of these fields is

$$K(Z,\bar Z)=2\ln|X^0|=-\ln[N_{\Lambda\Sigma}(Z,\bar Z)\,Z^\Lambda\bar Z^\Sigma]$$
$$=-\ln\tfrac{1}{2}[f(Z)+\bar f(\bar Z)+\tfrac{1}{2}(Z^i-\bar Z^i)(\bar f_i-f_i)], \tag{3}$$

where $N_{\Lambda\Sigma}=1/4(F_{\Lambda\Sigma}+\bar F_{\Lambda\Sigma})$ and $f(Z)=(X^0)^{-2}F(X)$. In the conformal gauge [4],

$$N_{\Lambda\Sigma}X^\Lambda\bar X^\Sigma=1. \tag{4}$$

The graviphoton field strength, as well as the field strengths of the n Abelian vector multiplets, are constructed out of $n+1$ field strengths $\hat F^\Lambda_{\mu\nu}=\partial_\mu W^\Lambda_\nu-\partial_\nu W^\Lambda_\mu$. The graviphoton field strength is

*Electronic address: ferraras@cernvm.cern.ch
†Electronic address: kallosh@physics.stanford.edu
‡Electronic address: andy@denali.physics.ucsb.edu

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Supersymmetry and attractors

Sergio Ferrara*
Theory Division, CERN, 1211 Geneva 23, Switzerland
and Physics Department, University of California at Los Angeles, Los Angeles, California 90024-1547

Renata Kallosh†
Physics Department, Stanford University, Stanford, California 94305-4060

(Received 1 March 1996)

We find a general principle which allows one to compute the area of the horizon of $N=2$ extremal black holes as an extremum of the central charge. One considers the ADM mass equal to the central charge as a function of electric and magnetic charges and moduli and extremizes this function in the moduli space (a minimum corresponds to a fixed point of attraction). The extremal value of the square of the central charge provides the area of the horizon, which depends only on electric and magnetic charges. The doubling of unbroken supersymmetry at the fixed point of attraction for $N=2$ black holes near the horizon is derived via conformal flatness of the Bertotti-Robinson-type geometry. These results provide an explicit model-independent expression for the macroscopic Bekenstein-Hawking entropy of $N=2$ black holes which is manifestly duality invariant. The presence of hypermultiplets in the solution does not affect the area formula. Various examples of the general formula are displayed. We outline the attractor mechanism in $N=4,8$ supersymmetries and the relation to the $N=2$ case. The entropy-area formula in five dimensions, recently discussed in the literature, is also seen to be obtained by extremizing the $5d$ central charge. [S0556-2821(96)03714-9]

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I. INTRODUCTION

Supersymmetry seems to be related to dynamical systems with fixed points describing the equilibrium and stability.[1] The particular property of the long-term behavior of dynamical flows in dissipative systems is the following: In approaching the attractors the orbits lose practically all memory of their initial conditions, even though the dynamics is strictly deterministic.

The first example known to us of such attractor behavior in the supersymmetric system was discovered in the context of $N=2$ extremal black holes [1,2]. The corresponding motion describes the behavior of the moduli fields as they approach the core of the black hole. They evolve according to a damped geodesic equation [see Eq. (20) in [1]] until they run into the fixed point near the black hole horizon. The moduli at fixed points were shown to be given as ratios of charges in the pure magnetic case [1]. Recently Strominger has further shown that this phenomenon extends to the generic case when both electric and magnetic charges are present [2]. The inverse distance to the horizon plays the role of the evolution parameter in the corresponding attractor. By the time moduli reach the horizon they lose completely the information about the initial conditions, i.e., about their values far away from the black hole, which correspond to the values of various coupling constants; see Fig. 1.

The main result of this paper is the derivation of the universal property of the stable fixed point of the supersymmetric attractors: the fixed point is defined by the new *principle of a minimal central charge*[2] and the area of the horizon is proportional to the square of the central charge, computed at the point where it is extremized in moduli space. In $N=2$, $d=4$ theories, which is the main object of our study in this paper, the extremization has to be performed in the moduli space of the special geometry and is illustrated in Fig. 1. This results in the following formula for the Bekenstein-Hawking entropy S, which is proportional to the quarter of the area of the horizon:

$$S=\frac{A}{4}=\pi|Z_{\rm fix}|^2, \quad d=4. \tag{1}$$

This result allows generalization for higher dimensions; for example, in five-dimensional space-time one has

$$S=\frac{A}{4}\sim|Z_{\rm fix}|^{3/2}, \quad d=5. \tag{2}$$

There exists a beautiful phenomenon in black hole physics: According to the no-hair theorem, there is a limited number of parameters[3] which describe space and physical fields far away from the black hole. In application to the recently studied black holes in string theory, these parameters include

*Electronic address: ferraras@cernvm.cern.ch
†Electronic address: kallosh@physics.stanford.edu

[1]A point $x_{\rm fix}$ where the phase velocity $v(x_{\rm fix})$ is vanishing is named a *fixed point* and represents the system in equilibrium, $v(x_{\rm fix})\sim 0$. The fixed point is said to be an *attractor* of some motion $x(t)$ if $\lim_{t\to\infty}x(t)\sim x_{\rm fix}(t)$.

[2]We are assuming that the extremum is a minimum, as it can be explicitly verified in some models. However, for the time being we cannot exclude situations with different extrema or even where the equation $D_iZ=0$ has no solutions.

[3]This number can be quite large; e.g., for $N=8$ supersymmetry one can have 56 charges and 70 moduli.

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Calabi-Yau compactification of IIB

Five and three forms F_5, H_3^{NS}, H_3^{RR}

$(F_5 = dA_4 + \ldots,\ H_3 = dB_2 + \ldots)$

On a C-Y manifold one can turn on "fluxes" which are related to holomorphic quantities which depend on the deformation of the "complex structure of the manifold".

Vector multiplets (scalars): h_{21} moduli

hypermultiplet (scalar): h_{11} moduli

(Mirror symmetry $h_{11} \Leftrightarrow h_{21}$)

3-form cohomology complex basis:

$$\Omega,\ D_i\Omega\ (\bar{\Omega},\ D_i\bar{\Omega})$$
$$H^{(3,0)}\quad H^{(2,1)}\quad H^{(0,3)}\quad H^{(1,2)}$$

$$\Omega = X^\Lambda \alpha_\Lambda - F_\Lambda \beta^\Lambda \quad (\alpha, \beta \text{ dual coh. basis})$$

$$\int_B \Omega = X^\Lambda \qquad \int_A \Omega = F_\Lambda$$

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GEOMETRY OF TYPE II SUPERSTRINGS AND THE MODULI OF SUPERCONFORMAL FIELD THEORIES*

S. CECOTTI

CERN, CH-1211 Geneva 23, Switzerland, and International School for Advanced Studies, Trieste, Italy

S. FERRARA

CERN, CH-1211 Geneva 23, Switzerland, and University of California, Los Angeles, CA 90024, USA

and

L. GIRARDELLO

Dipartimento di Fisica dell'Universita' di Milano, Italy and INFN, Sezione di Milano, Milano, Italy

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We study general properties of the low-energy effective theory for 4D type II superstrings obtained by the compactification on an *abstract* (2, 2) superconformal system. This is the basic step towards the construction of their moduli space. We give an explicit and general algorithm to convert the effective Lagrangian for the type IIA into that of type IIB superstring defined by the same (2, 2) superconformal system (and vice versa). This map converts Kahler manifolds into quaternionic ones (and quaternionic into Kahlerian ones) and has a deep geometrical meaning. The relationship with the theory of normal quaternionic manifolds (and algebras), as well as with Jordan algebras, is outlined. It turns out that only a restricted class of quarternionic geometries is allowed in the string case. We derive a general and explicit formula for the (fully nonlinear) couplings of the vector-multiplets (IIA case) in terms of the basic three-point functions of the underlying superconformal theory. A number of illustrative examples is also presented.

1. Introduction

A very interesting aspect of two-dimensional (super-)conformal field theories is the possibility of describing the abstract space of all such theories in standard geometrical terms. In particular, Zamolodchikov[1] has shown that the space of the conformal field theories is equipped with a *natural* Riemannian structure. For the applications, one is particularly interested in the connected components of this space, which are the moduli spaces of the various (super-)conformal theories.

A very powerful tool[2] to construct the moduli spaces and to compute their geometrical properties, is the study of the low-energy field theory for a (super-)string compactified on the given (super-)conformal theory.

Roughly speaking, the moduli space is just the manifold of classical vacua for the low-energy theory. The method is especially profitable when the resulting low-energy

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$$F_5 = \mathcal{F}^\Lambda \alpha_\Lambda - \mathcal{G}_\Lambda \beta^\Lambda \qquad \Lambda = 1 \ldots h_{21}+1$$

($\mathcal{F}^\Lambda$, $\mathcal{G}_\Lambda$ electric and magnetic dual field strengths)

F_5 fluxes $\longrightarrow$ 4D ($\frac{1}{2}$BPS) black-holes

$$\int_{S_2 \times C} F_5 \wedge \Omega = Z = e^{-K/2}(X^\Lambda q_\Lambda - F_\Lambda g^\Lambda)$$

$$\int_{A \times S_2} F_5 = q_\Lambda \qquad \int_{B \times S_2} F_5 = g^\Lambda$$

$$\int_{S_2 \times C} F_5 \wedge \mathcal{D}_i \Omega \rightarrow \mathcal{D}_i Z$$

$\mathcal{D}_i Z = 0$ Attractor equation for h_{21} moduli:

horizon geometry of 4D black holes

$$S \sim |Z|^2_{extr}$$

$$H_3 = s\,H_3^{NS} + H_3^{RR} \qquad (H_3^i)$$

H_3 fluxes : gauging of $N=2$ supergravity

$N=1$ effective theory emerges from C-Y compactification

(Polchinski, Strominger; Vafa, Taylor ...)

$$\int_C H_3 \wedge \Omega = W(s,z) = (s e_\Lambda^N + e_\Lambda^R) X^\Lambda - (s m^{N\Lambda} + m^{R\Lambda}) F_\Lambda$$

$$\int_B H_3 = s e_\Lambda^N + e_\Lambda^R \qquad \int_A H_3 = s\, m^{N\Lambda} + m^{R\Lambda}$$

$$\int_C H_3 \wedge \mathcal{D}_i \Omega = \mathcal{D}_i W \qquad (L = e^{K/2} W)$$

$$(W \rightarrow (P_\Lambda^1 - i P_\Lambda^2) L^\Lambda = I_1)$$

Attractor equation $\mathcal{D}_i W = 0$

(unbroken $N=1$ supersymmetry for the reduced theory) where

$\mathcal{Y}^\Lambda$, λ^{i2} are truncated away

$b_{\mu\nu}^{RR}$, $b_{\mu\nu}^{NS}$ are truncated away

(S.F. D'Auria, Andrianopoli)

General study of supersymmetry reduction in N-extended supergravity

(One can study the general case with non-abelian gauge symmetries and non trivial reduction of the gauge sector)

$$\xi^\alpha \to \quad C_{\alpha\beta}\, \xi^\alpha \sigma_{\mu\nu} \xi^\beta = 0$$

$$\lambda^A \to \quad g_{i\bar{j}}\, \bar{\lambda}^{\bar{j}} \gamma_\mu \lambda^i = 0 \; (\to \mathcal{N}_{\Lambda\Sigma} \to \bar{f}_{\Lambda\Sigma},$$

(by supersymmetry also imply a reduction of quaternionic and special Kähler geometry)

In presence of gauging

$$\delta\psi_\mu = \delta\zeta = \delta\lambda = 0 \quad \text{imply}$$

$$g_{(\Lambda)}\, \mathcal{F}^\Lambda (\mathcal{P}^1_\Lambda - i\mathcal{P}^2_\Lambda) = 0 \qquad \left(\begin{matrix}\text{on C.K.}\\ \mathcal{F}^\Lambda = 0\end{matrix} \right)$$

$$g_{(\Lambda)}\, L^\Lambda \mathcal{P}^3_\Lambda = 0 \qquad (\mathcal{P}^3_\Lambda = 0)$$

But in other reductions has many more solutions.