SUPERGRAVITY

LIVES

AdS/CFT from a D3/D5 Brane System

0111135 with DeWolfe, Ooguri

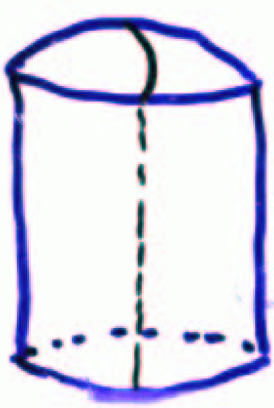
Develop gravity + field thy sides of duality proposed by Karch + Randall 0105132

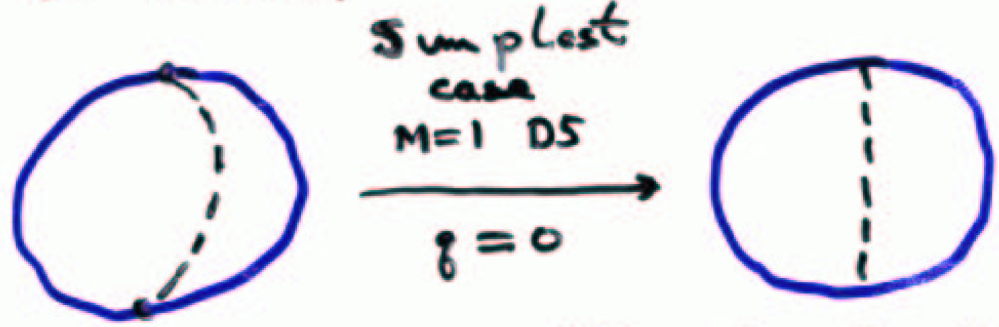
System of D3/D5's in IIB str Thy:

i) N D3's $\rightarrow$ usual $AdS_5 \times S_5$ geometry in near horizon limit

ii) stable position of D5 probe
D5 wraps max. radius $S_2 \subset S_5$
lives on $AdS_4 \subset AdS_5$

Basic setup in 5 dim:





AdS_4 bisects AdS_5

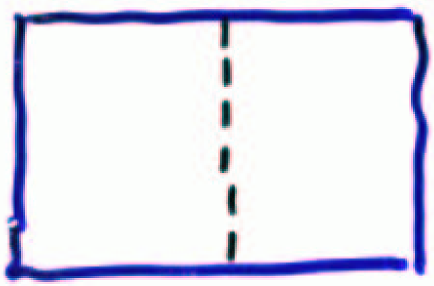
Two dualities in NH limit

1) Maldacena

IIB SG bulk fields in $AdS_5 \times S_5$ $\leftrightarrow$ $\mathcal{N}=4$ SYM on ∂AdS_5

2) D5 vector mult 55 open strs on $AdS_4 \times S_2$ $\leftrightarrow$ field thy on ∂AdS_4 elem fields are 3-5 strs $\to$ hypermultiplet

Field thy.



full $\mathbb{R}^4$ with usual $\mathcal{N}=4$ SYM

...

$\mathbb{R}^3$ defect with hyper.

Some $\mathcal{N}=4$ SYM fields interact on defect.

Bulk isom. gp. $SO(3,2) < SO(4,2)$ preserves AdS_4 brane

should match conf. gp of $Mink_4$ with $Mink_3$ defect, also $SO(3,2)$

Cardy

New type of dCFT = <u>defect</u> Conf Fld Thy

8 supercharges – $SU(2)_H \times SU(2)_V$ R-sym

$U(1)$ flavor sym.

Motivation:

1) no applics to particle thy in $d=4$ foreseen

2) One of the few things you can do to a CFT_2 is put in a bdy, so it is useful to see if AdS/CFT extends to these!

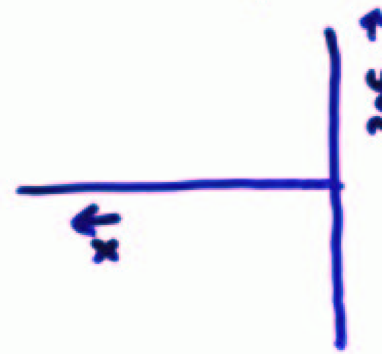
S. Shenker
2001

3) Novel

Implications of conf. sym. for CFT's with bdy.

Cardy

McAvity-Osborn



$\mathcal{O}_4(x, \underset{\sim}{y})$ of scale dim Δ_4 on ambient R_4

$\mathcal{O}_3(\underset{\sim}{y}')$ of scale dim Δ_3 on R_3 defect

New Features:

i) non-vanishing $\langle \mathcal{O}_4(x, \underset{\sim}{y}) \rangle = c / x^{\Delta_4}$

ii) $$\langle \mathcal{O}_4(x, \underset{\sim}{y})\, \mathcal{O}_3(\underset{\sim}{y}') \rangle = \frac{c'}{x^{\Delta_4 - \Delta_3}\left[x^2 + (\underset{\sim}{y} - \underset{\sim}{y}')^2\right]^{\Delta_3}}$$

iii) $\langle \mathcal{O}_4(x, \underset{\sim}{y})\, \mathcal{O}_4'(x', \underset{\sim}{y}') \rangle$

$$= \frac{1}{x^{\Delta_4} x'^{\Delta_4'}} f(\xi) \qquad \xi = \frac{(x_\mu - x_\mu')^2}{4 x x'}$$

≠ scale dims. arb. fn.

iv) normal CFT_3 behavior of $\langle \mathcal{O}_3 \mathcal{O}_3 \ldots \mathcal{O}_3 \rangle$

Hope to calculate these for $N \to \infty$, $g_s N >> 1$ from duality.

III. Geometry

D3 - D5 conventions a là Hanany - Witten

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D3	×	×	×				×			
D5	∘	∘	∘	∘	•	•	$x^6=0$	·	·	·

$\underbrace{x^3\ x^4\ x^5}_{SU(2)_H}$ (wrapped $\qquad \underbrace{x^7\ x^8\ x^9}_{SU(2)_V}$

$AdS_5 \times S_5$ metric from N D3's:

$$d_{10}s^2 = ds^2_{AdS_5} + ds^2_{S_5}$$

$$ds^2_{AdS_5} = L^2\left[\frac{dv^2}{v^2} + v^2(d\underset{\sim}{y}^2 + dx^2)\right]$$

$\underset{\sim}{y} = x^{0,1,2}$

$x = x^6$

AdS_4 at $x=0$

Bi-spherical coords for round S_5

$$ds^2_{S_5} = L^2\left[d\psi^2 + \cos^2\psi\,\underbrace{(d\theta^2 + \sin^2\theta\, d\alpha^2)}_{S_{2H}} + \sin^2\psi\,\underbrace{(d\gamma^2 + \sin^2\gamma\, d\alpha^2)}_{S_{2V}}\right]$$

wrapped S_2 at $\psi = 0$

Its $\perp$ space in S_5 is

$$ds^2_\perp = L^2\left(d\psi^2 + \sin^2\psi\,(d\gamma^2 + \sin^2\gamma\, d\alpha^2)\right)$$

$$\approx dx_7^2 + dx_8^2 + dx_9^2$$

IV. Actions (very schematic)

$$S_{Tot} = S_{IIB} + S_{D5}$$

$$S_{D5} = S_{BI} + S_{WZ} \qquad T_{D5} \sim \frac{1}{g_s \alpha'^3}$$

$$S_{BI} = -T_{D5} \int d^6\xi \, e^{\Phi/2} \sqrt{-\det\left(g^{PB}_{ab} + e^{-\Phi} \mathcal{F}_{ab}\right)}$$

$$S_{WZ} = -T_{D5} \int C \wedge \mathcal{F}$$

KR result: the $AdS_4 \times S_2$ config of the D5 extremizes S_{D5} (with $q = 0$ flux of $\mathcal{F}_{ab}$)

2. Qual. form of S_{D5} for fluctuations

generic open str fields ψ (e.g. B_a, X^i, ...)

" closed " " φ (e.g. h_{mn}, Φ, B_{ab})

We scale φ to normalize $\langle O_4 O_4 \rangle \sim 1$

" ψ " " $\langle O_3 O_3 \rangle \sim 1$

$$S_{D5} \to \int d^6 z \left[(\partial\psi)^2 + \frac{1}{N^{m+\frac{n}{2}-1} \lambda^{n/4 - 1/2}} \phi^m \psi^n \right]$$

N, λ dependence of correlators can be read directly from this.

Compare $N \to \infty$ $\lambda \gg 1$ gravity with planar graphs in QFT.

$$\langle \ldots\ldots \rangle = N^a f(\lambda) \xrightarrow[\lambda \ll 1]{} N^\alpha \lambda^\beta$$

Generically λ dependence disagrees,

$\Rightarrow$ no generic non-renorm thms

But N-dependence generically agrees.

weak test that AdS/CFT can work

#3? $S_{BI} + S_{WZ}$ contain (after dim red. on S^5)

$S_1 = \int d^4z \sqrt{g_{AdS_4}}\, \mathcal{O}$ and $S_2 = \int d^4z \sqrt{g_{AdS_4}} \left(\mathcal{O}\psi + \mathcal{O}^2\right)$

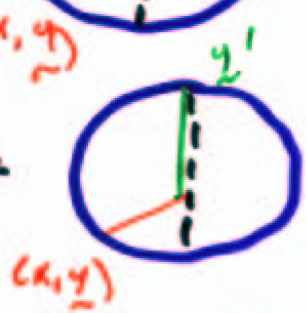
example $\mathcal{O}$ = dilaton or h^a_α from expansion of $\sqrt{g_{10}}$

These lead to Witten diags. for "special correlators"

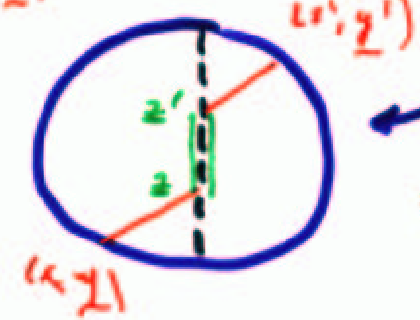
$\langle O_4 \rangle =$ 

bulk to bdy props.

$K_{\Delta_4} = c \left(\frac{z_0}{z_0^2 + x^2 + \vec{y}^2} \right)^{\Delta_4}$

$\langle O_4 O_3 \rangle =$ 

$K_{\Delta_3} = c' \left(\frac{z_0}{z_0^2 + \vec{y}'^2} \right)^{\Delta_3}$

$\langle O_4 O_4 \rangle =$ 

bulk to bulk

$= G_3(z, z')$

AdS$_4$ integrals can be done by methods of MIT 9804058

$\langle O_4 \rangle = c_4 \lambda^{1/2} \frac{1}{x^{\Delta_4}} \Gamma\left(\frac{\Delta_4 - 3}{2}\right) \cdot$ (other Γ's with no poles)

$\langle O_4 O_3 \rangle = c_{43} \frac{\lambda^{1/2}}{x^{\Delta_4 - \Delta_3} \left(x^2 + (\vec{y} - \vec{y}')^2\right)^{\Delta_3}} \Gamma\left(\frac{\Delta_4 - \Delta_3}{2}\right)$ (other Γ's)

$\langle O_4 O_4 \rangle$ can be done by method of "without really trying" 9905049

Features :

1. Spacetime forms req'd by conf sym of dCFT do emerge from gravity calcs.

Reason: usual AdS/CFT match of bulk isom gp + bdy conf gp.

2 Poles eg. at $\Delta_3 - \Delta_4 = 2n$ from generic couplings are not compatible with

$SO(6) \longrightarrow SU(2)_H \times SU(2)_V$ R sym

in actual 8 superchg. model. So couplings $c_{43} = 0$ in all but $\Delta_3 = \Delta_4$ case.

Similar to poles of $\langle \mathrm{Tr}\, X^k\, \mathrm{Tr}\, X^l\, \mathrm{Tr}\, X^m \rangle$

all but one sing case vanishes by $SO(6)$

one extremal case $k = l + m$ where

0. ∞ gives non-sing $\langle \quad \rangle$.

MIT '98

Princeton '98

V. KK reduction of D5 fields on $AdS_4 \times S_2$

$$X^i(v, \underline{y}, \theta, \varphi) \qquad i = 6,7,8,9$$

$$B_a(v, \underline{y}, \theta, \varphi) \qquad a = 0,1,2,3,4,5$$

A kindergarden KK problem:

i) no gravity

ii) high school harmonics in

$$X^i = \sum X^i_{\ell m}(v, \underline{y})\, Y_\ell^m(\theta, \varphi)$$

$$B_\alpha = \sum B_{\ell m}(v, \underline{y})\, \epsilon_{\alpha\beta} D^\beta Y_\ell^m(\theta, \varphi)$$

$$\alpha, \beta \in S^2$$

iii) non-trivial mixing of X^6 and B_α from S_{WZ}

Translate $m_\ell^2 \to \Delta_\ell$ for operators in CFT_3 by

$$\Delta_\ell = \tfrac{1}{2}\left(3 + \sqrt{9+4m_\ell^2}\right)$$

Full Results on KK decomposition:

Mode	Δ	$SU(2)_H$	$SU(2)_V$
$(B_a + Z^i)^{(-)}$	ℓ	$\ell \geq 1$	0
Z^i	$\ell+2$	$\ell \geq 0$	1
$(B_a + Z^i)^{(+)}$	$\ell+4$	$\ell \geq 0$	0
B_μ	$\ell+2$	$\ell \geq 0$	0

← Primaries of short reps of $OSp(4,4)$

← lowest vector with $\Delta = 2$ is dual to conserved J_c on defect.

Would like to organize these into reps of $OSp(4,4)$. Detailed decomp. of short $s_{max}=1$ reps of $OSp(3,4)$ given by F. + Nicolai NPB 1984

Perfect match ~~match~~ if assume $SO(3)$ = diag. subgp. of $SU(2)_H \times SU(2)_V$

Nontrivial because bosonic content of reps is 1 vector + 5 scalars of specific Δ and ℓ.

This is one of ~~that~~ D3-wrapped D5 brane system is SUSY (with ≥ 6 superchs.)

Spectrum contains 3 (stable) tachyons

$\Rightarrow$ 3 relevant deformations of dCFT.

VI The Field Theory

i) ambient $\mathbb{R}^4$ $\mathcal{N}=4$ SYM usual action

ii) $\mathbb{R}^3$ defect: hyperm. q^i, ψ^i $\in N$ of $SU(N)$

$i = 1,2$ of $SU(2) =$ diag of $SU(2)_H \times SU(2)_V$

A subset of 4d fields interacts on defect.

Strategy to find classical action: Hori 0012179

1) extend "superspace bdy" method

$\rightarrow$ general action with 2 supchgs.

(i.e. $\mathcal{N}=1$ in $d=3$) and $SU(2)_{diag}$ sym.

2) impose $SU(2)_H \times SU(2)_V$ on component action

to achieve 8 supchgs ($\mathcal{N}=4$ in $d=3$).

A. Susy generators in $\mathcal{N}=1$ $d=4$ Majorana

superspace: $S_\alpha = \frac{\partial}{\partial \bar\theta_\alpha} - i(\gamma^\mu \theta)_\alpha \frac{\partial}{\partial x^\mu}$

those containing $\frac{\partial}{\partial x^3}$ are broken by defect,

the rest are preserved.

Hori: an efficient way to separate good + bad SUSY's and write actions inv. under the former.

i) Choose Majorana basis of γ^μ so that

any spinor $\theta_\alpha \longrightarrow \theta_{\alpha(\tau)}$ $\quad \alpha = 1,2$, $\tau = 1,2$

α is index on which $SO(2,1)$ acts.

ii) restrict superspace by $\theta_{\alpha(2)} = 0$

Then $S_{\alpha(1)}$ is indep of $\frac{\partial}{\partial x^3}$ and generates std $\mathcal{N}=1$ $d=3$ SUSY.

iii) take any 4d sf: $\Xi(x^\mu, \theta_\alpha)$

Define its restriction to "supspace bdy" by

$$\Xi(\underset{\sim}{y}, \Theta_\alpha)\Big|_\partial = \Xi(\underset{\sim}{y}, x^3 = 0, \theta_{\alpha(1)} = \Theta_\alpha, \theta_{\alpha(2)} = 0)$$

? This acts as a std 3-d sf. and can be multiplied by any intrinsic 3d sf, such as

$$Q^i = q^i + \bar{\Theta}\psi^i + \tfrac{1}{2}\bar{\Theta}\Theta f^i \quad \leftarrow \text{hyperm.}$$

For any 4d chiral sf: $\Phi(x, \theta)$

both $\mathrm{Re}\,\Phi\big|_\partial$ and $\mathrm{Im}\,\Phi\big|_\partial$ are 3d sfs.

Let $\Phi^A = (X_V^A + i X_H^A) + \bar\theta \chi^A + \frac{1}{2}\bar\theta\theta (F_V^A + i F_H^A)$

$X_V^A = X^{7,8,9}$ $\quad X_H^{3,4,5}$ $\quad A = 1,2,3$

a chiral sf in $\mathcal{N}=4$ SYM thy.

Defect action involves:

$$\mathrm{Re}\,\Phi^A\big|_\partial = X_V^A + \bar\Theta \chi^A_{(1)} + \frac{1}{2}\bar\Theta\Theta (F_V^A - i D_3 X_H^A)$$

X_V^A, $\chi^A_{(1)}$ are main 4d fields which interact on defect (as do $A_{0,1,2}$, λ_1 from $V(x,\theta)$)

but $D_3 X_H^A = \partial_3 X_H^A + g[A_3, X_H^A]$ also interact.

(ii) Defect action in $\mathcal{N}=1$ $d=3$ supspace:

$$S = \int d^3y\, d^2\Theta \left[\overline{\nabla_\alpha Q^i}\, \nabla_\alpha Q^i + g' \bar Q^i \sigma^A_{ij} Q^j\, \mathrm{Re}\,\Phi^A \big|_\partial \right]$$

$$\nabla_\alpha = D_\alpha - i g \Gamma_\alpha = D_\alpha - i g (D_{\alpha(0)} V)\big|_\partial$$

Invariant under: $\mathcal{N}=1$ $d=3$ SUSY $\quad SU(N)$ gauge

$SU(2)_{diag} \times U(1)$ flavor sym

on Q^i

B. Implement $SU(2)_H \times SU(2)_V$ in components:

eg $\mathcal{L}_{Yuk} = \bar{\Psi}^i [i g \lambda \delta_{\lambda j} + g' \sigma^A_{\lambda j} \chi^A] q^j$

gaugino

chiral matter

To get full R-symmetry

i) set $g = g'$

as in linear σ-mo

ii) define bispinor $\lambda_{\lambda j} = \lambda \delta_{\lambda j} - i \sigma^A_{\lambda j} \chi^A$

$\mathcal{L}_{Yuk} = i g \bar{\Psi}^i \lambda_{\lambda j} q^j$

(0, 1/2) (1/2, 1/2) (1/2, 0) of $SU(2)_H \times SU(2)_V$

Potential terms have manifest R sym after elimination of f^i, F^A:

Final defect action in components

$$S = \int d^2 y \, [D_k q^+ D^k q - i \bar{\Psi} \not{D} \psi$$ kinetic

$$- g \bar{\Psi} \sigma^A X^A_V \psi + i g \bar{\Psi} \lambda q + \text{h.c.}$$ Yukawa

$$- g \bar{q} \sigma^A D_3 X^A_H q + g^2 \varepsilon^{ABC} \bar{q} \sigma^A [X^B_H, X^C_H] q$$

$$+ \tfrac{1}{2} g^2 \bar{q} \{ X^A_V, X^A_V \} q - g^2 \delta(0) (\bar{q} \sigma^A q)^2]$$ potential

$\delta(0)$ from elim of $F_V^A \sim X X^2 - \delta(x_3) \bar{q} \sigma^A q$

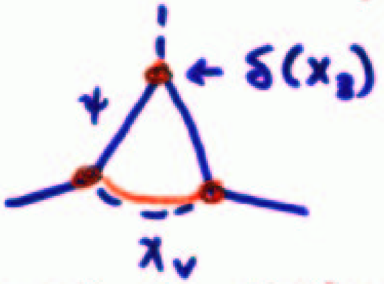
Artefact 

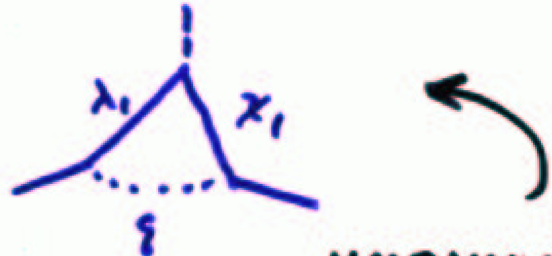
$\doteq \dfrac{1}{(\underset{\sim}{y} - \underset{\sim}{y}')^4}$

Mirabelli Peskin 97

VII Perturbative Field Thy

A. Basics: 1-loop graph for $\langle X_v(x,\vec{y})\, \Psi(\underset{\sim}{y}_1)\, \bar{\Psi}(\underset{\sim}{y}_2)\rangle$





X_v pinned to defect unpinned

std 4d scalar prop: $\dfrac{1}{(x_1-x_1')^2+(\underset{\sim}{y}-\underset{\sim}{y}')^2} = FT_4\, \dfrac{1}{h^2+\underset{\sim}{k}^2}$

when pinned on both ends $\dfrac{1}{(\underset{\sim}{y}-\underset{\sim}{y}')^2} = FT_3\, \dfrac{1}{|\underset{\sim}{k}|}$

more singular than

std 3d prop $\dfrac{1}{|\underset{\sim}{y}-\underset{\sim}{y}'|} = FT_3\, \dfrac{1}{\underset{\sim}{k}^2}$

For this simple reason, a 3d defect thy. with gauge + Yuk. couplings — normally superrenorm. — is promoted to critically renorm. Generically couplings g, g' run logarithmically but with 8 supercharges, it might be conformal!

B. $\delta(x_3)$ gives clean separation of pinned + unpinned contribs. to correlators.

Gauge gp also gives separation:

i) unpinned graphs always occur with f^{abc} of non-abelian $SU(N)$ thy, so

ii) pinned graphs are whole story for $U(1)$ thy. (in which $\mathcal{N}=4$ $d=4$ thy is free)

C. Detailed pwr ctg + sym. arg. for pinned graphs

i) only divergences in wave fn renormal of q^c, ψ^c. ← gauge dependent non-observable don't spoil conf. sym.

ii) g is not renormalized by defect interacts
g' would be " if $g \neq g'$
but not if $g = g'$ by R-sym.

iii) no new divergent couplings by various symmetries.

A rigorous pf. that U(1) thy is a dCFT!

D. Possible problems in SU(N) thy from unpinned graphs.
more singular than pinned
non-trans inv in x.

Unsettled question...

$\delta(0)(\bar{q}\sigma q)^2$ is a harmless artefact. It cancels a $\delta(0)$ from $\partial_3 X_H$ exchange

$$= g^2 \frac{1}{(\underset{\sim}{y}-\underset{\sim}{y}')^4}$$

VI Operator Matching: lowest KK mult. on gravity side contains

Spin	Δ	(ℓ_H, ℓ_V)	Operators
0	1	$(1, 0)$	$\bar{q}^i \sigma^A_{ij} q^j$
1/2	3/2	$(1/2, 1/2)$	$\bar{q}^i \psi^j$
0	2	$(0, 1)$	$\psi \sigma^A \psi + 2\bar{q} X_V^A q$
1	2	$(0, 0)$	$J_k = \bar{\psi}\rho_k \psi + i\bar{q} \overleftrightarrow{D}_k q$

Primary Op $\bar{q}\sigma^A q$ is unique op with $\Delta = 1 = \ell_H$ others found by action of bispinor supchg. Q^{ij} This mult. contains conserved J_k on defect.

Comments:

1. $\Delta_0 = 1$ R-sym singlet $\bar{q}^i q^i$ is <u>not</u> in op. map

Expect $\Delta = 1 + a_1(g^2 N) + \dots$ ← not protected dual to str. state

A weak coupling calc. proves $a_1 \neq 0$

2. What ops. are dual to higher KK primaries with $\Delta = l_H > 1$?

Candidates: i) highest $l_H = l$ components of

$$(\bar{q}\,\sigma^{A_1} q) \ldots\ldots\ldots (\bar{q}\,\sigma^{A_l} q) \qquad *$$

ii) $l_H = l$ components of

$$(\bar{q}\,\sigma^{A}\, X_H^{B_1} \ldots\, X_H^{B_{l-1}}\, q) \qquad \checkmark$$

Which is right?

i) intuitive arg that ops. should be formed from defect fields favors $*$

ii) a brane T-duality arg. favors $X_H^A = X^{3,4,5}$ ✓

Weak coupling calcs.

i) 1-loop calc shows that highest l projection of $*$ has no anom dim to order $g^2 N$

ii) a more precise calc is req'd to decide the question for ✓. Not yet done.

Thus:

i) Field thy has natural candidate operators for duals to gravity fields

ii) Weak coupling calcs can illuminate situation.

Conclusion:

i) evidence for new type of dCFT, and

ii) extended AdS/CFT of KR

but $\exists$ open ?*s.