INVENTING SYMMETRIES ?

B. JULIA ENS PARIS FRANCE

$D \geq 6$ 1975 KALUZA SCHERK + SCHWARZ

$D = 11$ $\begin{cases} 1977 \\ 1978\text{-}79 \end{cases}$ $U = E_n(n)(\mathbb{R})$ CJS / CJ

$N_4 \leq 6$ 1980 ADE EHLERS, G+H, J → MT_1, $N_4 = 0$

$D = 3 \to 2$ 1981 $U_2 = U_3^{(1)}$, E_9 … GEROCH J

$D = 1$ 1982 E_{10}, $D_8^{\wedge\wedge} = DE_{10}$ (BE_{10}?, CE_{10}? …) J

$U_1 \overset{\sim}{\longleftrightarrow} U_2$ 1997-10119 CREMMER-LU-POPE + J

$\langle U_1, U_2 \rangle$ GENERATE G BEYOND U-DUALITY

1998-06106 CJLP $_{II}$, $\begin{cases} \text{CJLP}_{III} \\ MT_2 \end{cases}$

WEYL ($U_3^{\wedge\wedge}$) CONTROLS ~1D CHAOS

DAMOUR HENNEAUX NICOLAI + J 2001-03094

SUPERGRAVITY AT 25

STONY BROOK Dec 1, 2001

I/ FINDING S-ACTIONS

WESS-ZUMINO 73-74 PARIS

SCHERK + SCHWARZ 75 PRINCETON

→ PROPAGANDA ...

PRINCETON : KALUZA-KLEIN + FERMIONS $d \geq 6 = 5+1?\ 4+2? \ldots$

DYONS ⇒ . BOSON ⟷ FERMION (DYON) 1975

1975 . LES HOUCHES
1976 . HARVARD LUNCH

1979 . DIRAC (TRIESTE)
. WITTEN (PARIS)

- SPONTANEOUS COMPACTIFICATION
- Q_{EL} / MASS
- SCALAR?
- MISSED = PSB = BPS + JZ !

LATER { DUALITY, CHIRALITY

(76) 1977 (78)

GSO PROJECTION

$d=2$ SUSY$_1$ $\rightleftarrows$ $D=10$ SUSY$_{\mathrm{I}, \mathrm{II}}$

* SCHERK + al.

SUPER YANGMILLS$_{10 \to 4}$

GELL-MANN $10-4 = 6 \to SO(6)$

NEED $SO(8) \cdot SU(8)$

TRIALITY.

NAHM 11 d SPECTRUM

↳ JOEL (PRIVATE)

RR $u^t \Gamma^i v$, $u^t \Gamma^{[ijk]} v$

II_A

A_μ $A_{\mu\nu\rho}$

SUPERGRAVITIES D = 4

N = 1 1976 FF vN (Z ...)

ENS SUMMER INSTITUTE

N = 2

N = 3

N = 4 SCALARS : NON POLYNOMIAL

SO(4) <=> SU(4)

DUALITIES

$$dF = 0$$

$$d{*}F \approx 0 \iff dG = 0 \qquad \boxed{G \approx {*}F}$$

(Z.S.F.C.) $\rightarrow \; SU(1,1)/U(1) \qquad K_{ij} = \begin{pmatrix} -1 & \\ & +1 \end{pmatrix}$

N = 8 LOWEST ORDER DW + F

$\downarrow$

D = 10 $8 \times 4 = 2 \times 16$

D = 11 $= 1 \times 32$

2 NEW TOOLS

AFTER B. ZUMINO'S PREPARATION

a) FIERZ (PAULI - F - de BROGLIE ...)

ψ^4

b) GUPTA PROGRAM (G - DESER - F^{2}vN - .... - CJS)

DEFORMATION OF

$\{$ GROUP, REPRESENTATION , INVARIANT $\}$

$\uparrow$

∞ DIMENSIONAL

TYPE I SUPERSTRINGS. ...

THE N=4 EXTENDED THEORY

(CHAMSEDDINE)

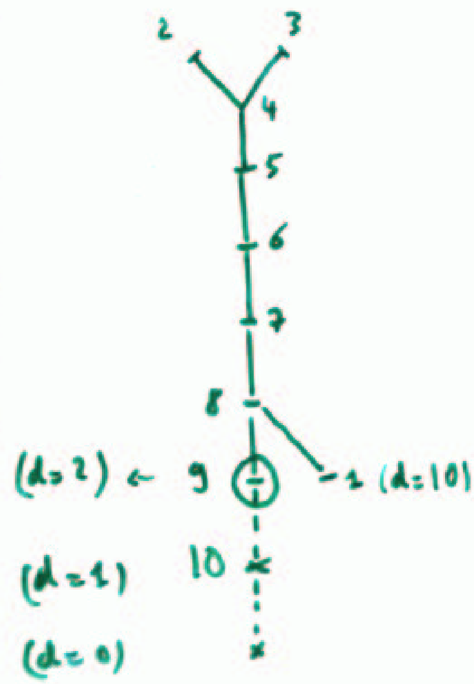
: 64 D.O.FREEDOM. (COMPLETED)

d	$g_{\mu\nu}, A_{\mu\nu}, \varphi$	
10	$^{1}\ \mathbb{R}\big\|^{1}_{0}$	1
9	$^{2}\ \mathbb{R}\times\mathbb{R}\big\|^{2}_{0}$	2
8	$^{3}\ \frac{SO(2,2)}{SO(2)^2}\times\mathbb{R}\big\|^{7}_{2}$	3
→ 7	$^{4}\ \frac{SO(3,3)}{SO(3)^2}\times\mathbb{R}\big\|^{16}_{6}$	4
6	$^{5}\ \frac{SO(4,4)}{SO(4)^2}\times\mathbb{R}\big\|^{29}_{12}$	5
5	$^{6}\ \frac{SO(5,5)}{SO(5)^2}\times\mathbb{R}\big\|^{46}_{20}$	6
4	$^{7}\ \frac{SO(6,6)\times SU(1,1)}{SO(6)^2\times U(1)}\big\|^{69}_{31}$	7
3	$^{8}\ \frac{SO(8,8)}{SO(8)^2}\big\|^{120}_{56}$	8
2	$^{9}\ D_8^{(1)}$	9
1	HYPERBOLIC ?	
0		
−1		

$2+d-N=7$

$= SL(4)/\times\mathbb{R}$

BOURBAKI (456) p 134 ←



E_{10}, D_{10} : THE ONLY 2 SH.SL.KM ALG.

II / INVENTIONS ?

– $U_{\mathbb{R}}$ RIGID = GLOBAL SYMMETRY

① NON-COMPACT ($N=4$ ZCFS)

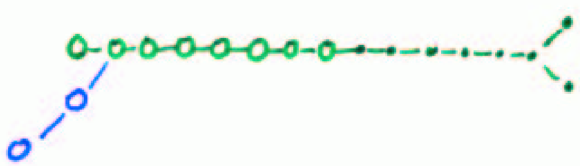
$N \geq 5$ ...

$\begin{cases} \text{NO GHOSTS} \\ \quad \text{KILLING}\big|_{Ku^\perp} > 0 \end{cases}$

· $(h, e, f) = (\sigma_3, \sigma_+, \sigma_-)$ $\mathbb{R}$-GENERATE $SO(2,1)$

$ih, (e-f), i(e+f)$ $\mathbb{R}$-GENERATE $SO(3)$

↑ COMPACT CARTAN GENERATOR



$SO(8,24)$ $\begin{cases} \text{PUZZLE} \\ 16+2 = 18 > 10 \end{cases}$

QUANTUM DUALITY IS "$U_{\mathbb{Z}}$" (W-S)

– KU = <u>LOCAL</u> SYMMETRY $\leftrightarrow$ LORENTZ

② $\cap$ $\qquad\qquad\qquad\qquad$ $\cap$

U $\qquad\qquad\qquad\qquad$ $GL(D, \mathbb{R})$

PERMITS FERMIONS

– AFFINE EXTENSION

③ $\nearrow D=2$ / $D=1 \searrow$ GENERATES MAGNETIC DUALS ...

– HYPERBOLIC CHAOS (NEGATIVE CURVATURE)

BE_{10}  $A_1^{\wedge\wedge}$ 

$10 = 10$

MODULAR STRINGS

GROUPS ← LIE

GROUPS ← KAC-MOODY AFFINE

GROUPS ← KAC-MOODY HYPERBOLIC

(BORCHERDS ?)

HETEROTIC, I. STRINGS

HAVE RANK 10 CHAOS

OBSERVED BY DAMOUR + HENNEAUX 2000-12172

RELATED TO TITS-SATAKE THEORY

IN HANANY-KEURENTJES-J. 2001-12xxx

INVENTIONS !

(BEYOND DISCOVERIES)

④ INSTANTONS EVERYWHERE

(QUANTUM HOPES)

FINITE EVEN DIM. SUPERALGEBRA

$\Theta = \{T, \tilde{T}\}$ $\quad A = e^{A^a \Theta_a}$ ALL p-FORMS

$A \to A\, e^{\Lambda^a \theta_a}$ $\quad (d\Lambda = 0)$

UNIVERSAL NON-ABELIAN STRUCTURE (ON T^4)

$$dA\,A^{-1} \equiv \mathcal{F} \simeq S(*\mathcal{F})$$

$S(T) = \tilde{T}$

$S(\tilde{T}) = (*)^2\, T \equiv \pm T$

Ex : $e^{\underset{\text{ODD}}{A_{(3)}\theta} + \underset{\text{EVEN}}{B_{(6)}\tilde{\theta}}}$ $\quad \{\theta\,\theta\}_+ = k\,\tilde{\theta}$

IN $\mathbb{Z}$ (AND $\mathbb{Z}_2$) GRADED

IWASAWA $\quad \mathcal{A} = \underbrace{\sigma . d . t_2^+}_{SO(n) \backslash SL(n)}$

HIDDEN SYMMETRY $\Leftarrow$ FIXED GAUGE

RECALL MAXWELL'S GAUGE INVARIANCE

NEXT 3 INVENTIONS

WE HAVE FOUND $\underline{\text{TRIANG.}(G)}$

$\cup$

TRIANG $(\mathcal{U})$

BOREL $(E_{n(+n)})$

STILL MYSTERIOUS BECAUSE $\neq$ RIGID

SUPERALGEBRA = VERY SOLVABLE

ie

VERY NON-SIMPLE

BUT STILL NON-ABELIAN

$T_1 T_2 = T_3 \dots$

1'

NEED $\underline{\text{FULL} \quad G}$: = RIGID?

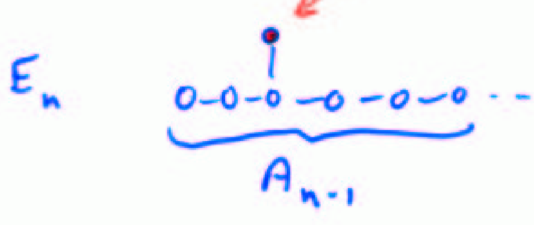
$G \supset KG \not\subset$ TRIANG (G)

$\downarrow$

PERMITS FERMIONS

BUT RECALL ALSO $A_{ijk} = \bar{u}\, \Gamma_{ijk}\, v$

SO(8) TRIALITY



(2') - GRAVITY DOUBLING $d=4$ $S_* R = R$

ARBITRARY Λ_{cosm} !

- NUT CHARGE

D

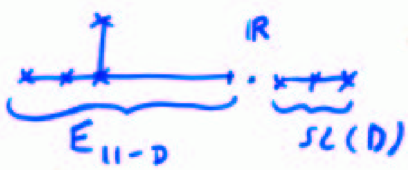
- CONTRAST | YANG-MILLS - DUAL?

$G_{(7)} = dB_{(6)} + \kappa A_{(3)} F_{(4)}$ | - DOUBLING

- BEYOND LIE GROUPS ...

- CONSERVATIVE ATTEMPTS

J. 1998-05083 CARGESE LECTURES



$\subset E_{11}$ FOR ANY D.

$A_7^{++} \equiv EA_9$ LAST CHAOS FOR PURE GRAVITY $D \leq 10$

$\mathcal{F}_{12}$?

$SL(11-D)$ FROM TORUS = 1980-81-82 (J)

YET EVEN E_{10} REMAINS CONJECTURE

(3') FERMIONIZATION

MAJOR SPIN-OFF CLASSICAL LIMITS ⟲ DUALITIES

FERMIONS, DYONS AND D-BRANES ARE $\frac{1}{2}$ WAY.

FERMION - BOSON DEMOCRACY.

MAGIC TRIANGLES

1) FROM PURE 4D. SUGRAS

$m = 0$ (NO MATTER)

$D - 3 \leftrightarrow 7 - N_4$

$$\boxed{4D - 12 \leftrightarrow 28 - \# Q_{SUGRA}}$$

2) FROM OXYDISINTEGRATION OF $E_{n(n)}/KE_n$

$\mathbb{C}\mathbb{P}_{\mathrm{III}}$

del Pezzo's SPECIAL ALGEBRAIC SURFACES

+

MORE ...

$D=11$	-								
$D=10$	$\mathbb{R}$ / A.								
$D=9$	$\mathbb{R}\times A_1$	$\mathbb{R}$							
$D=8$	$A_1\times A_2$	$\mathbb{R}\times A_1$	A_1						
$D=7$	E_4	$\mathbb{R}\times A_2$	$\mathbb{R}\times A_1$	$\mathbb{R}$	-				
$D=6$	E_5	$A_1\times A_3$	$\mathbb{R}\times A_1^2$	$\mathbb{R}^2$	$\mathbb{R}$				
$D=5$	E_6	A_5	A_2^2	$\mathbb{R}\times A_1^2$	$\mathbb{R}\times A_1$	A_1			
$D=4$	E_7	D_6	A_5	$A_1\times A_3$	$\mathbb{R}\times A_2$	$\mathbb{R}\times A_1$	$\mathbb{R}$		
$D=3$	E_8	E_7	E_6	E_5	E_4	$A_1\times A_2$	$\mathbb{R}\times A_1$	$\mathbb{R}$ / t A.	-

Table 3: Disintegration (*i.e.* Oxidation) for E_n Cosets

"SPLIT" FORMS

$E_n(n)$