SUPERGRAVITY
BLACK HOLES
COSMOLOGY

R. Kallosh

SUPEGRAVITY 25

STONY BROOK, December 1, 2001

* Nostalgia

* * * Supersymmetry and Cosmology

1976 and later... Moscow-Dubna-Kharkov
V. Ogievetsky, D. Volkov
SUPERGRAVITY IS SO BEAUTIFUL!

1979 Cambridge G. Gibbons Even if Supergravity has nothing to do with nature, it is enormously helpful for understanding Gravity

1977-1978 Modified Feynman Rules in Supergravity, BRST-story

2001
FIND T BRST: Result: 827 documents
FIND T BRS: Result: 276 documents

1980-1981 Sad story of Counterterms in Extended Supergravities: even $\mathcal{N} = 8$ was not finite (the counterterms respected $E_{7(7)}$ symmetry)

1984 Anomaly Cancellation in Superstring Theory Is Perturbative String Theory Quantum Gravity?

Mysterious Nilpotent Fermionic κ-symmetry of manifestly supersymmetric extended objects

1982, 1992 SUPERSYMMETRIC BLACK HOLES

Supersymmetry, Supergravity, General Relativity, String Theory

*** BPS, non-BPS Black Holes

***Branes as BPS solutions in Supergravity, as Hyperplanes in String Theory, on which strings end, new κ-symmetric brane actions

***BLACK HOLES NEAR HORIZON⇒ Doubling of Supersymmetries⇒ ADS/SCFT

U-duality of M-theory

One of the beautiful observations in non-perturbative supergravity in 90's

Black hole entropy is invariant under a discrete subgroup of

$E_{7(7)}$ in d=4

$$\sqrt{[\mathrm{Tr}(Z\bar{Z})^2 - \frac{1}{4}(\mathrm{Tr}\, Z\bar{Z})^2 + 4(\mathrm{Pf}\, Z + \mathrm{Pf}\, \bar{Z}\,)]}$$

$E_{6(6)}$ in d=5

$$\sqrt{\mathrm{Tr}(q\Omega q\Omega q\Omega)}$$

R. K., B. Kol ; S. Ferrara, R. K.

SUPERSYMMETRY AND COSMOLOGY

R.K., hep-th/0109168

R. K., Linde, Prokushkin, Shmakova, hep-th/0110089,

Herdeiro, Hirano, R. K., hep-th/0110271.

Dasgupta, Herdeiro, Hirano, R. K., in progress

Motivation

* Current experiments in cosmology indicate a possibility that the cosmological inflation took place in the early universe and that at present the universe is accelerating. The cosmological constant in such case is positive rather than negative. Future experiments in cosmology?

*Future experiments will hopefully detect supersymmetric particles

Meanwhile, can we study de Sitter space in supersymmetric theories?

More general issue in M/string theory and supergravity:

COSMOLOGICAL INSTABILITIES CONTROLLED BY SPONTANEOUS BREAKING OF SUPERSYMMETRY

New philosophy

Start with some supersymmetric theory, find de Sitter vacuum with spontaneously broken supersymmetry. Use the instability of such configuration in the cosmological context.

The best would be to find some local de Sitter minimum with slow-roll stage which eventually will bring the system down to the absolute minimum, Minkowski vacuum.

Many such possibilities are available in $\mathcal{N} = 1$ supergravity. In fact, too many...

N$\geq$ 2 Gauged Supergravity and de Sitter Space

Known de Sitter solutions of d=4 supergravity are unstable and always break supersymmetry spontaneously and may be obtained from d=11 with non-compact internal space, Gates and Zwiebach, 1983, Hull and Warner, 1988

We studied known de Sitter vacua of gauged supergravities with $N \geq 2$ and found that they are maxima/saddle points of the potentials with a peculiar relation between $m^2 = V''$ and V:

$$m^2_{\text{tachyon}} = -2V, \qquad V = \Lambda_{\text{deSitter}}$$

This relation can also be represented as

$$|m^2_{\text{tachyon}}| = 6H^2 = \frac{R}{2}$$

where H is the Hubble constant in de Sitter space and R is the curvature scalar.

Generally, this corresponds to a very unstable regime. However, much to our surprise, we have found that one can use this regime for the description of the present stage of acceleration of the universe.

R. K., Linde, Prokushkin, Shmakova, hep-th/0110089

? ? ? ?

PUZZLES OF GAUGED SUPERGRAVITIES

$$\mathcal{N} = 2, 4, 8$$

* Why in all cases of de Sitter critical points we never found a local minimum?

*** What is the explanation of

$$|m^2|_{\text{tach}} = 2\Lambda$$

Work in progress Fré, Trigiante, Van Proeyen, and Dall'Agata

A CHALLENGE

GAUGED $\mathcal{N} = 2$ SUPERGRAVITY

Rigid limit $M_P \to \infty$

Salam-Strathdee-Fayet model with FI terms

***A topological obstruction for FI terms in quaternionic spaces

$$P^r = \frac{1}{4n_H} D_u k_v \mathcal{R}^{ruv}$$

*** No obstruction for FI terms in rigid limit in hyperKäler spaces $\mathcal{R}^{ruv} = 0$

Ceresole, Dall'agata, R. K., Van Proeyen

Galicki and de Wit, Roček, Vandoren

D'Auria, Ferrara

CONCLUSION????? Make your bets

P-term inflation/acceleration in string theory

Carlos Herdeiro, Shinji Hirano, R. K., 2001

Hybrid inflation was invented by Linde in 1991. It has a long slow-roll de Sitter inflationary stage and a waterfall stage before the end of inflation and beginning of reheating. In 1994 it became clear that $\mathcal{N} = 1$ supersymmetric theories favor hybrid inflation. F-term inflation and D-term inflation have hybrid potentials.

F is an auxiliary field in the chiral multiplet, D is an auxiliary field in the gauge multiplet.

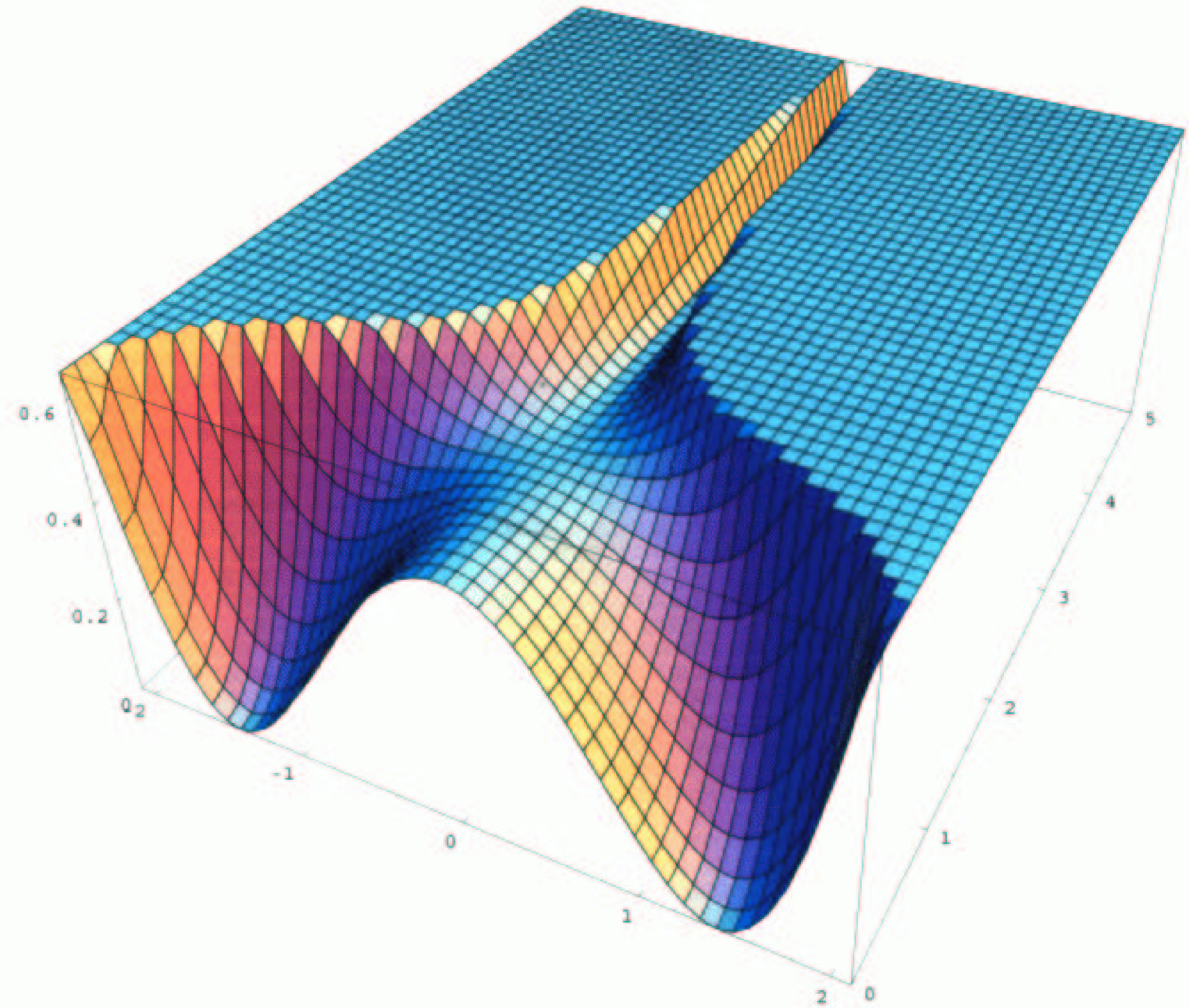
Our new observation: $\mathcal{N} = 2$ theory has a P-term inflation, P^r is the Killing prepotential triplet, an auxiliary field of $\mathcal{N} = 2$ gauge multiplet. When P^r has a constant part we have a FI term in $\mathcal{N} = 2$ theory.

$$P^r = (0, 0, \xi)$$

$$\Lambda_{\rm deSitter} = 3H^2 = \frac{1}{2}\xi^2 > 0$$

$$H^2 = \frac{1}{6}\xi^2$$

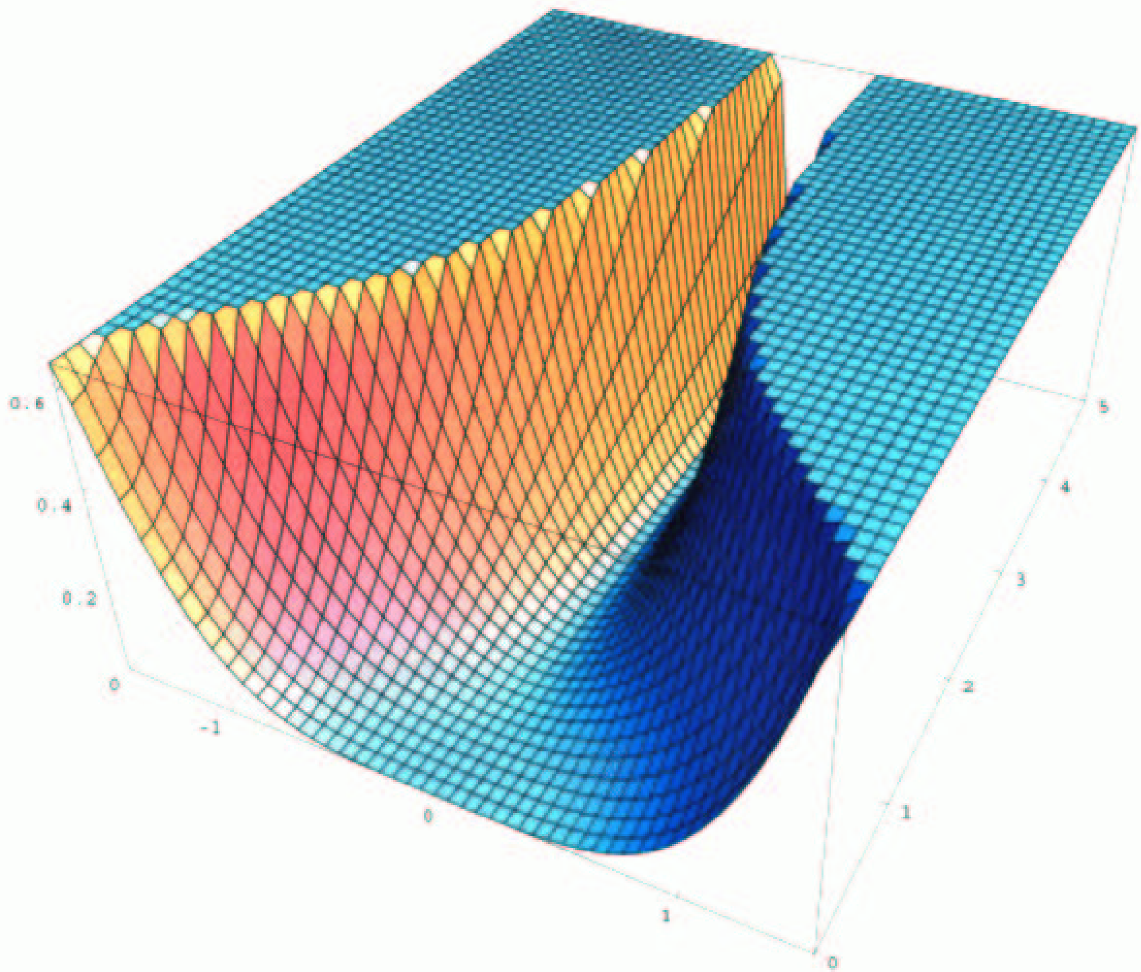
In this model only for non-vanishing Fayet-Iliopoulos term ξ there is a de Sitter vacuum, when the gauge model is coupled to gravity. The value of the Hubble constant is proportional to the FI term. When $\xi = 0$ we have a Minkowski space instead of de Sitter space.



Cosmological potential with Fayet-Iliopoulos term

De Sitter valley is classically flat; it is lifted by the one-loop correction corresponding to the one-loop potential between *D4-D6.* In this figure the valley is along the $|\Phi_3|$ axis; the orthogonal direction is a line passing through the origin of the complex Φ_2 plane and we have put $|\Phi_1| = 0$. The bifurcation point corresponds to $|\Phi_3| = \sqrt{\xi/g}$, $\Phi_2 = 0$. The absolute minimum is at $\Phi_3 = 0$, $\Phi_2 = \sqrt{2\xi/g}$.

D3-D7



Cosmological potential without Fayet-Iliopoulos term. The motion of the D4 corresponds to moving along the bottom of the valley, which has a zero potential.

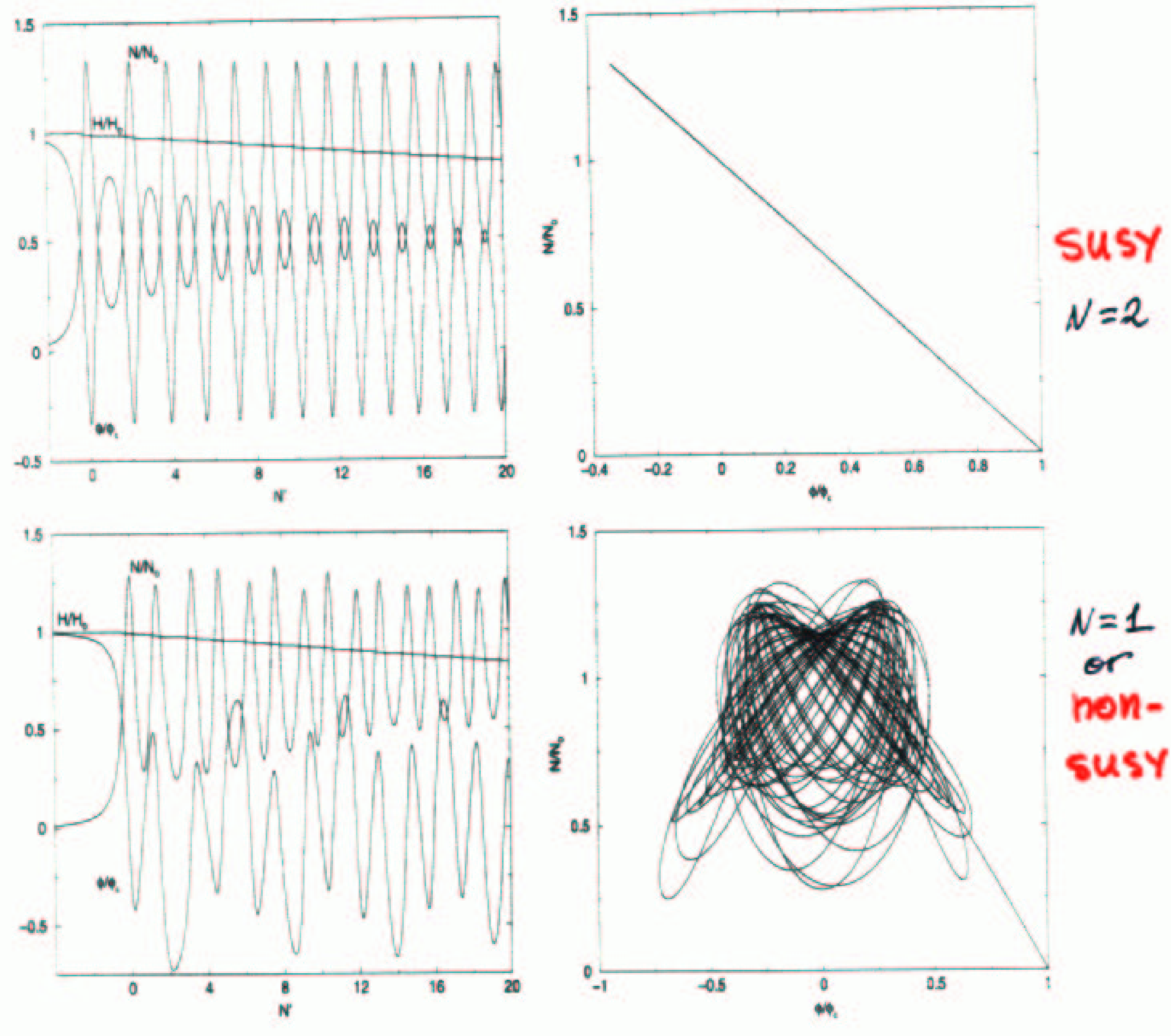


Figure 1: Oscillations of the classical fields (left) and the trajectory in the $N - \phi$ plane (right). The top panels correspond to the supersymmetric hybrid model with $\phi_c = 10^{16}\, GeV$, $\kappa = 10^{-3}$ and $m_\phi = 3.7 \times 10^9\, GeV$. The time scale is given in terms of the approximate number of oscillations of the fields, $N' = \bar{m}_\phi t/2\pi$, where $N' = 0$ has been defined for convenience as the point where the fields reach the first peak. The lower panels show for comparison the situation in a non-supersymmetric hybrid model with $g = \sqrt{\lambda} = 10^{-3}$, and $\phi_c (= \Lambda)$ and m_ϕ as before.

The potential of P-term inflation model

(Salam-Strathdee-Fayet model: $\mathcal{N}=2$ SQED with 1 vector multiplet, 1 charged hypermultiplet and FI term)

$$
\begin{aligned}
V &= \frac{g^2}{2}\left[(|\Phi_1|^2+|\Phi_2|^2)|\Phi_3|^2+|\Phi_1|^2|\Phi_2|^2\right] \\
&+ \frac{1}{2}\left(\frac{g}{2}(|\Phi_1|^2-|\Phi_2|^2)+\xi\right)^2
\end{aligned}
$$

R. K. 2001

Also F-term inflation Dvali, Shafi, Schaefer, 1994 and D-term inflation Binetruy, Dvali, 1996

Hypermultiplet mass splitting :

$$M_2^2 = g^2|\Phi_3|^2 - g\xi\ , \quad M_\psi = g|\Phi_3|\ , \quad M_1^2 = g^2|\Phi_3|^2 + g\xi\ .$$

Spontaneous susy breaking

$$\mathrm{STr}\, M^2 = 0$$

Low lying states in string spectrum ($g^2_{YM} = (2\pi)^2 g_s \frac{l_s}{L}$)

$$M_\pm^2 = \frac{(x^4)^2+(x^5)^2}{\pi^2 l_s^4} \pm \frac{\phi}{2\pi l_s^2}\ , \qquad M_\psi^2 = \frac{(x^4)^2+(x^5)^2}{\pi^2 l_s^4}$$

De Sitter valley; slow-roll with account of 1-loop corrections; Coulomb phase

Bifurcation point

The point where one of the scalars in the hypermultiplet becomes massless,

$$M_2^2 = g^2|\Phi_3|_c^2 - g\xi = 0 \quad \Leftrightarrow \quad |\Phi_3|_c = \sqrt{\frac{\xi}{g}}$$

is a bifurcation point. At $|\Phi_3|^2 \leq \xi/g$, de Sitter minimum becomes a de Sitter maximum; beyond it, such scalars become tachyonic. The system is unstable and the waterfall stage of the potential (mixed Coulomb-Higgs phase) leads it to a ground state (Fully Higgsed phase.)

In this state $|\Phi_2| = \sqrt{\frac{2\xi}{g}}$, gauge symmetry is broken, but $\mathcal{N} = 2$ supersymmetry is restored.

What drives hybrid inflation?

The tree level splitting of the masses in supermultiplets in de Sitter vacuum leads to the effective 1-loop potential for large inflaton field Φ in gauge theory:

$$V_{1-\text{loop}} = \frac{\xi^2}{2}\left[1 + \frac{g^2}{8\pi^2}\ln\frac{|\Phi^2|}{|\Phi^2|_c}\right]$$

This leads to inflation. Friedmann equation is

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{V}{3} \approx \frac{\xi^2}{6}$$

$$a(t) = a(0)\ \exp\frac{\xi\ t}{\sqrt{6}}\ .$$

Slow-roll near de Sitter regime

$$3H\dot\Phi = -V'(\Phi)$$

$$\Phi^2(t) = \Phi^2(0) - \frac{g^2\xi\ t}{2\sqrt{6}\pi^2}\ .$$

To have 60 e-foldings we take $\frac{\xi\ t}{\sqrt{6}} = 60$. Linde, Riotto, Lyth, 1997 Under certain conditions using COBE normalization for inflationary perturbations of metric on the horizon scale one finds, for P-term inflation

$$\frac{\xi}{g} \approx 10^{-5} M_P^2\ , \qquad |\Phi_2| = 10^{16}\text{GeV}.$$

NS5-D4/D6-NS5 model

Witten, 1997; Hanany-Witten, 1996, Giveon, Kutasov, 1998, Brodie,2001

Supersymmetric vacua

	0	1	2	3	4	5	6	7	8	9
$D4$	×	×	×	×			×			
$D6$	×	×	×	×				×	×	×
$NS5$	×	×	×	×	×	×				

The system of $D4$ and $D6$-brane at an angle $\phi + \pi/2$ and separated by some distance. The boundary conditions of several types of open strings are summarized in the table.

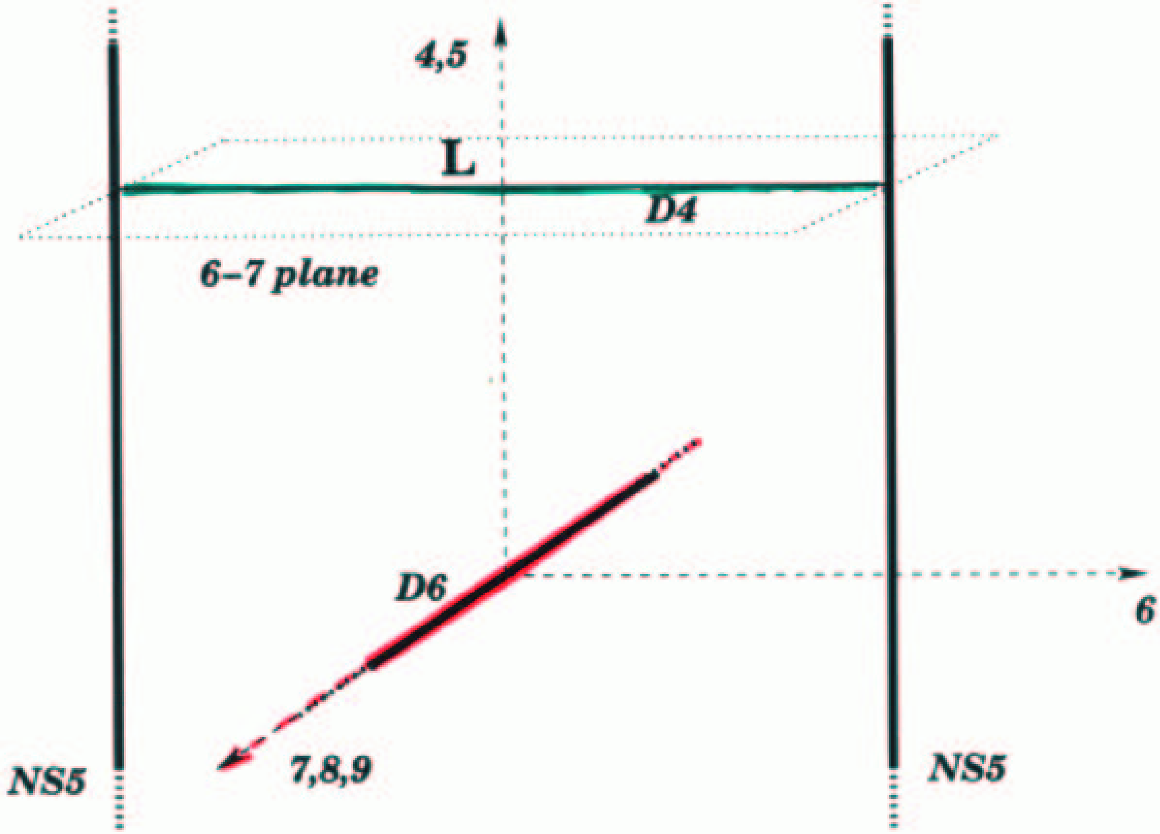
	$X^{0,1,2,3}$	$X^{4,5}$	ReZ	ImZ	Re$(e^{i\phi}Z)$	Im$(e^{i\phi}Z)$	$X^{8,9}$
4-4	NN	DD	- -	- -	NN	DD	DD
6-6	NN	DD	DD	NN	- -	- -	NN
4-6	NN	DD	-D	-N	N-	D-	DN
6-4	NN	DD	D-	N-	-N	-D	ND

Somewhat unusual:

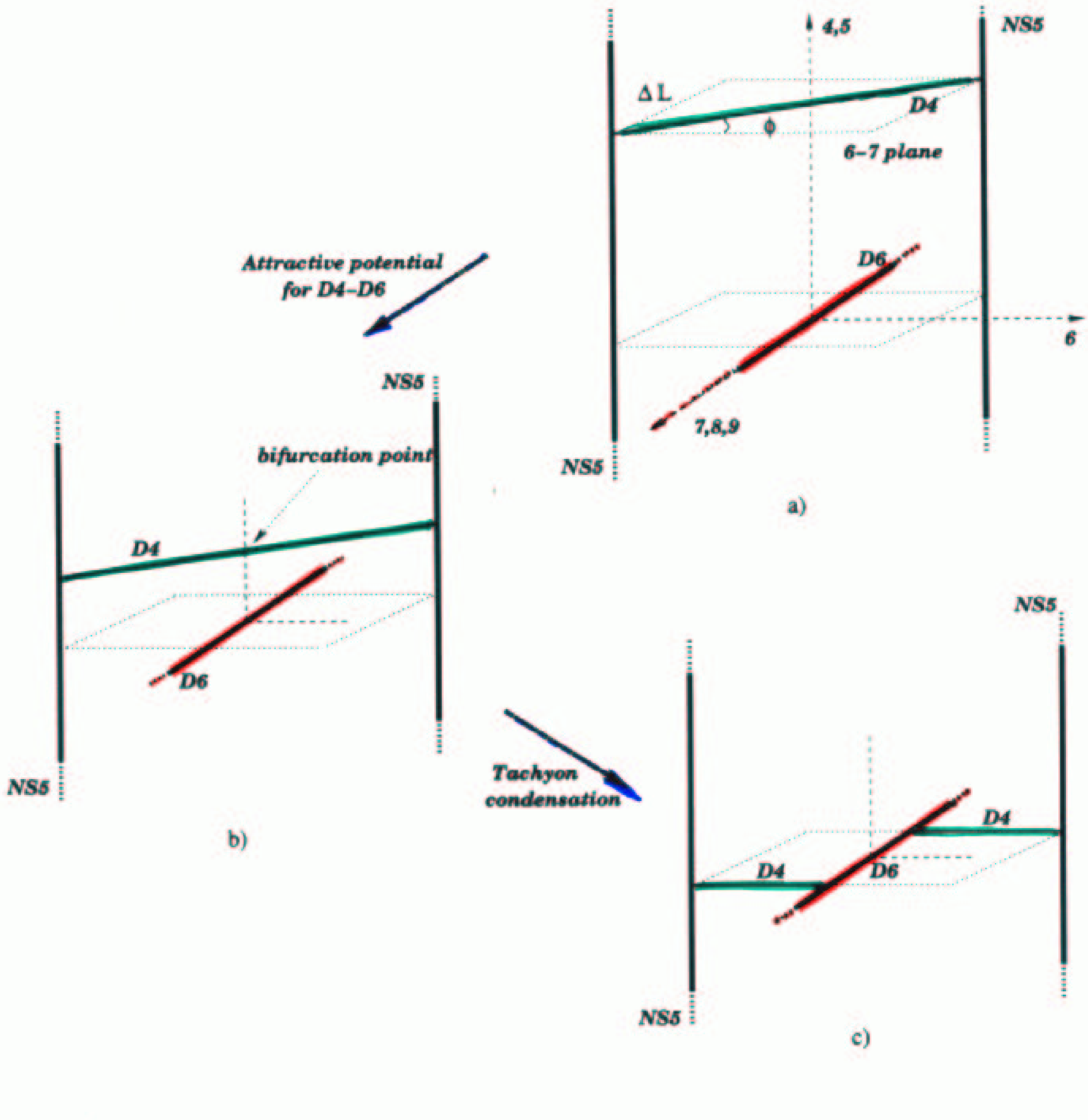
$$\sigma_1 = 0 \quad (D4) \qquad \partial_{\sigma_1} \mathrm{Re}\,(\mathrm{e}^{i\phi} Z) = \mathrm{Im}\,(\mathrm{e}^{i\phi} Z) = 0,$$
$$\sigma_1 = \pi \quad (D6) \qquad \partial_{\sigma_1} \mathrm{Im}\ Z = \mathrm{Re}\ Z = 0,$$

Computation of i)string spectrum, ii)one-loop open string potential, ii)Supertrace of M^2 for all states

Almost standard, following Joe's Big Book of String



Brane configuration without Fayet-Iliopoulos term. We are free to move D4 in 4,5 directions with no energy cost.



Brane configuration evolution with Fayet-Iliopoulos term.

a) For $\phi \neq 0$, supersymmetry is broken and D4-D6 experience an attractive force.
b) At the bifurcation point, a complex scalar in the hypermultiplet becomes massless.
When we overshoot, tachyonic instability develops, taking the system to the zero energy ground state shown in c).

The open string one-loop potential is expected to provide an attractive potential between the $D4$ and $D6$-branes which drives the inflaton towards the bifurcation point and the D4 brane towards the D6 brane.

The one-loop vacuum amplitude corresponding to the effective interaction between $D4$ and $D6$-branes we calculated and the result is

$$V = \left(\frac{1}{8\pi^2\alpha'}\right)^2 \frac{\sin^2\phi}{\cos\phi} \int_{\alpha'/\Lambda^2}^{\infty} \frac{dt}{t} \exp\left[-2\pi t \frac{(\Delta s)^2}{\alpha'}\right] + \mathcal{O}(e^{-\pi/t}) \sim \frac{1}{16\pi^2}\left(\frac{\phi}{2\pi\alpha'}\right)^2 \log\frac{x_4^2+x_5^2}{(x_4^2+x_5^2)_c}.$$

This exactly reproduces the one-loop correction in the field theory, including the numerical coefficient, in the small angle and large separation approximation.

At the critical separation between branes

$$\frac{(x_4^2+x_5^2)_c}{\pi^2 l_s^4} = \frac{\phi}{2\pi l_s^2}$$

corresponding to the vanishing of the mass of the lowest lying state there is a bifurcation point: beyond it, such state becomes tachyonic.

Spontaneous supersymmetry breaking in string theory

In field theory, spontaneous breaking of supersymmetry manifests itself through the vanishing of the supertrace. It is natural to ask if the supertrace vanishes for the whole tower of string states in the $D4$-$D6$ system with an angle.

$$\mathrm{STr} M^2 = \frac{2}{\alpha'} q \frac{\partial}{\partial q} \mathcal{Z} \bigg|_{q=1} .$$

The partition function is

$$q^{d^2/\alpha'} \prod_{m=1}^{\infty} (1-q^m)^{-4} (1-q^{m-1/2-\phi/\pi})^{-1} (1-q^{m-1/2+\phi/\pi})^{-1}$$
$$(1-q^{m-1/2})^{-2} \times \left[q^{-\phi/(2\pi)} \prod_{m=1}^{\infty} (1+q^{m-1/2})^4 (1+q^{m-\phi/\pi}) \right.$$
$$(1+q^{m-1+\phi/\pi})(1+q^m)^2 - 2 \prod_{m=1}^{\infty} (1+q^m)^4 (1+q^{m-1/2-\phi/\pi})$$
$$\left. (1+q^{m-1/2+\phi/\pi})(1+q^{m-1/2})^2 \right] .$$

An explicit calculation then shows that the supertrace is indeed vanishing. The first 3 levels show a

Beautiful 'stringy Zeeman effect'

SPONTANEOUS SYPERSYMMETRY BREAKING LEADS TO STRINGY ZEEMAN EFFECT

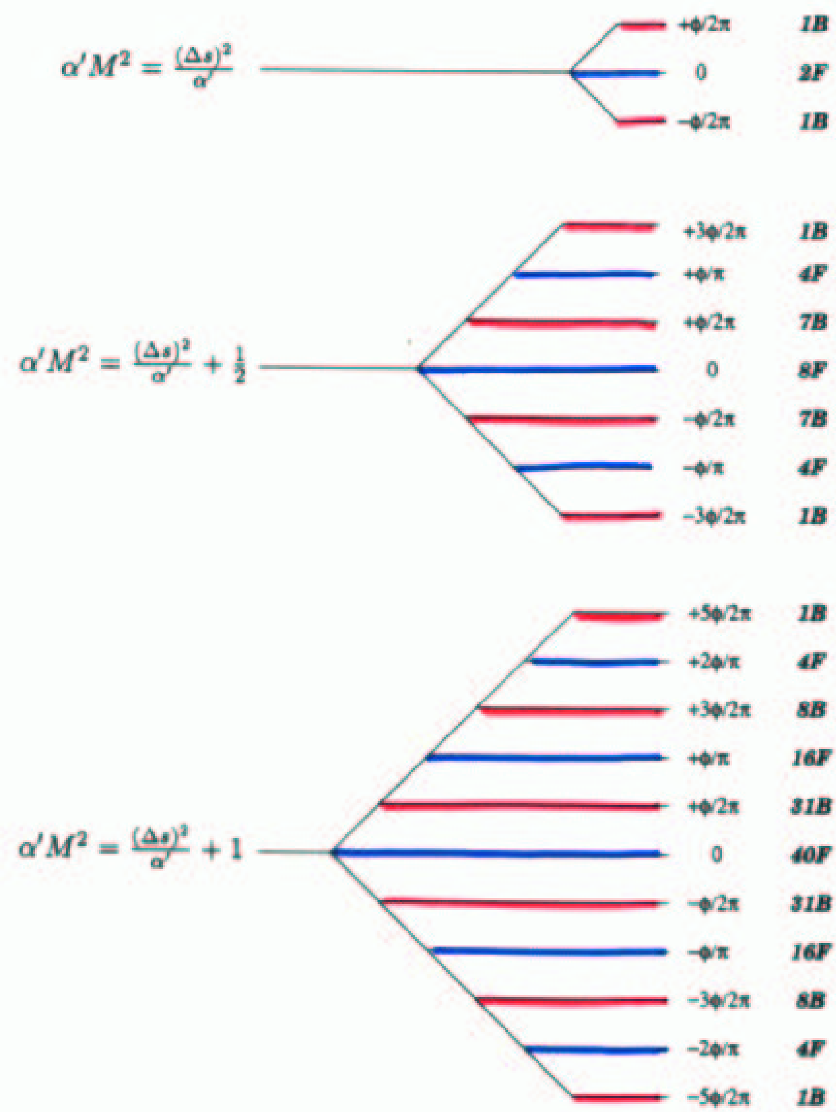


Figure 5: Splitting of mass for the first three levels due to the presence of an angle ϕ. Notice that at each level the supertrace vanishes. On the right we show the number of bosonic (B) or fermionic (F) states with such mass.

$$\mathrm{STr}\, M^2 \equiv \sum_j (-1)^{2j}(1+2j)M_j^2 = 0,$$

Ferrara, Girardello and Palumbo, 1979

This motion of branes at angle in string theory shows a precise correspondence with the motion on the ridge of the potential after the bifurcation point, since the de Sitter valley becomes a hill top there, hence an unstable maximum.

The actual behaviour of perturbations in such potentials has been investigated numerically by Felder, Garcia-Bellido, Greene, Kofman, Linde, Tkachev where the tachyonic instability was studied in the context of preheating of the universe after inflation.

- tachyonic instabilities in models of hybrid cosmology
- phenomenon of tachyon condensation in open string theory discovered by Sen

The attention in the latter studies was towards brane/antibrane systems with the consequent brane/antibrane annihilation.

Interesting brane inflation models have been suggested in the framework of brane/antibrane configurations recently. In this framework tachyon instability develops at brane/antibrane separations of the order of l_s, with the consequent tachyon condensation. It was conjectured that such inflation may be of the hybrid inflation type, which is exactly coming out in our study.

In our case, we know from the cosmological part of our construction that after the bifurcation point the waterfall stage (tachyon condensation) takes place.

Instead of continuing at the ridge (in Coulomb phase) the brane system undergoes a phase transition. The system reconfigures itself as to reach the supersymmetric state.

This is the only supersymmetric configuration possible if we allow only the $D4$-branes to move. It is an $\mathcal{N}=2$ supersymmetric, fully Higgsed phase of the field theory. All fields are massive. This absolute ground state is in precise agreement with the Minkowski ground state of the hybrid potential.

In the final configuration

the branes reconnect.

Dasgupta, Herdeiro, Hirano, R. K.,
in progress

A COSMOLOGICAL D3/D7 MODEL

	0	1	2	3	4	5	6	7	8	9
$D3$	×	×	×	×						
$D7$	×	×	×	×			×	×	×	×

De Sitter stage of a hybrid inflation/acceleration

on D7 worldvolume : $\mathcal{F}_{67} = \tan\phi$

D7-D3 STRINGS

$\sigma = 0(D7)$	$\sigma = \pi(D3)$
$\partial_\sigma X^6 - \tan\phi\,\partial_\tau X^7 = 0$	$\partial_\tau X^6 = 0$
$\partial_\sigma X^7 + \tan\phi\,\partial_\tau X^6 = 0$	$\partial_\tau X^7 = 0$
$\partial_\sigma X^8 = 0$	$\partial_\tau X^8 = 0$
$\partial_\sigma X^9 = 0$	$\partial_\tau X^9 = 0$

For large r D3-D7 supergravity solution with harmonic function

$$H(r) \sim \log r \sim V$$

gives a good description of the attractive potential between D3 and D7
Near the bifurcation point

$$r = \arctan \mathcal{F}_{67}$$

supergravity approximation is not valid anymore

Tachyon instability and tachyon condensation sets on

Phase transition, EXCISION OF SINGULARITY

Impossible to come close to naked singularity

$$r = 0$$

Fully Higgsed Phase D3 is melted in D7

The worldvolume field $\mathcal{F}_{89}$ gets excited

$$\mathcal{F} = \mathcal{F}^* = \tan\phi$$

Highly Non-linear in $\mathcal{F}$ Killing Spinor Equation on D7 with the imprint of D3 is defined by κ-symmetry Bergshoeff, R.K., Ortin, Papadopoulos

$$(1 - \Gamma)\epsilon = 0 \qquad \Gamma = e^{-\frac{a}{2}}\Gamma_{(0)}e^{\frac{a}{2}}$$

$$\epsilon = i\sigma_2 \otimes e^{-\frac{\phi\sigma_3 \otimes \Gamma_{67}(1-\Gamma_{6789})}{2}}\Gamma_{01236789}\, e^{\frac{\phi\sigma_3 \otimes \Gamma_{67}(1-\Gamma_{6789})}{2}}\epsilon$$

Solution with 1/4 of unbroken non-linear susy on the worldvolume

$$\epsilon = i\sigma_2 \otimes \Gamma_{01236789}\epsilon$$
$$\epsilon = \Gamma_{6789}\epsilon$$

For small $\mathcal{F} \sim \tan\phi \sim \phi$ reduces to $\mathcal{N} = 2$ supersymmetric gauge theory vacuum with FI term $\xi \sim \phi$ and D-flatness condition

Combinations of Supersymmetric Gauge Theories, Supergravity, String Theory, κ-symmetry on the Worldvolume of Branes may lead to new surprises in future.

HAPPY BIRTHDAY

SUPERGRAVITY 25!!!