SUPERGRAVITY DUALS OF $\mathcal{N}=1$ SUPERSYMMETRIC GAUGE THEORIES

Igor Klebanov

Talk at

SUPERGRAVITY AT 25

YITP, Stony Brook

Dec. 1, 2001

FROM ITS VERY INCEPTION,

SUPERGRAVITY

WAS CLEARLY A VERY DEEP AND RICH SUBJECT.

25 YEARS AGO IT WAS HARD TO ANTICIPATE, HOWEVER, THAT IT CONTAINS LARGE N GAUGE THEORY.

CONFORMAL GAUGE THEORIES IN 3+1 DIMENSIONS ARE DESCRIBED BY THE $AdS_5 \times X_5$ SOLUTIONS OF TYPE IIB SUGRA.

(I will emphasize a specific example

$$X_5 = T^{1,1} = \frac{SU(2)\times SU(2)}{U(1)} \quad)$$

Romans

Breaking of conformal invariance produces a variety of gauge theory phenomena:

logarithmic running of couplings, chiral symmetry breaking, confinement, etc.

ALL SEEN FROM SUGRA.

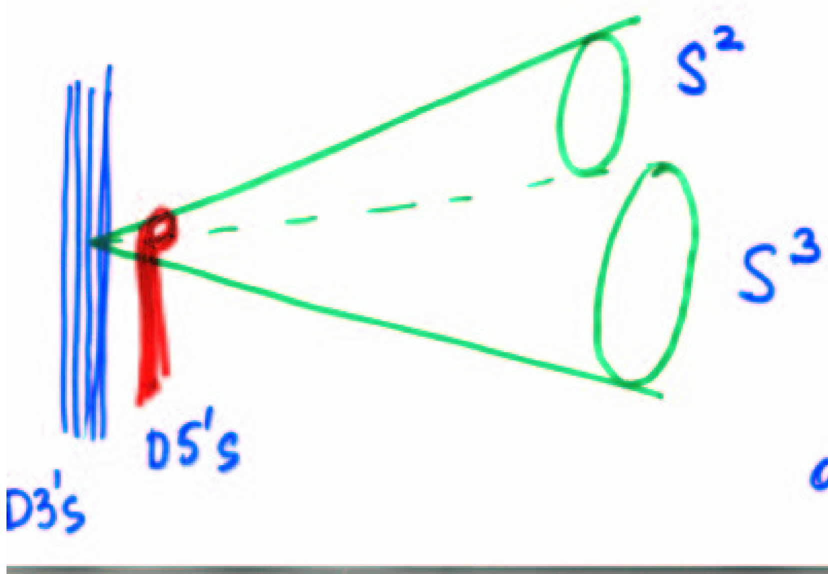
In $\mathcal{N}=1$ SUSY $SU(M)$ gauge theory XSB is through the appearance of gluino condensate

$$\langle \lambda\lambda \rangle = \Lambda^3 e^{2\pi i n/M} \; ; \quad n = 1, \ldots, M$$

It breaks the Z_{2M} chiral symmetry down to Z_2; n labels the M inequivalent vacua.

Recently, this pattern of chiral symmetry breaking was observed geometrically, in extensions of the AdS/CFT correspondence to non-conformal $\mathcal{N}=1$ SUSY gauge theories.

I.K., Strassler; Maldacena, Nunez; Vafa; ...



$SU(N+M) \times SU(N)$ gauge theory lives on N D3-branes and M wrapped D5-branes on the CONIFOLD

The Conifold is a 6-dimensional cone defined by the equation

$$\sum_{i=1}^{4} z_i^2 = 0$$

for 4 complex variables.

The metric created by the D-branes at the apex is a WARPED PRODUCT:

$$ds_{10}^2 = h^{-\frac{1}{2}}(r)(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + h^{\frac{1}{2}}(r)\, ds_6^2 ;$$

I.K., A. Tseytlin

$ds_6^2 \equiv$ explicitly known metric of the cone; $r \equiv$ the radius of the cone; $T^{1,1}$ is its base.

The CHIRAL SYMMETRY acts geometrically as $z_i \to z_i e^{i\alpha}$;

$$\alpha = \frac{\pi n}{M} ; \quad n = 1, 2, \ldots, 2M.$$

However, the warp factor has a zero: $h(r_*) = 0 \implies$ NAKED SINGULARITY!

The Einstein metric on T^{11} is

$$ds^2_{T^{11}} = \frac{1}{9}\left(d\psi + \cos\theta_1 d\varphi_1 + \cos\theta_2 d\varphi_2\right)^2 + $$
$$+ \frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2 + \sin^2\theta_i\, d\varphi_i^2\right)$$

$\psi \in [0, 4\pi)$; $\varphi_1, \varphi_2 \in [0, 2\pi)$; $\theta_1, \theta_2 \in [0, \pi)$

The harmonic 2- and 3-forms are

$$\omega_2 = \frac{1}{2}\left(\sin\theta_1 d\theta_1 \wedge d\varphi_1 - \sin\theta_2 d\theta_2 \wedge d\varphi_2\right),$$

$$\omega_3 = e^\psi \wedge \omega_2,$$

$$e^\psi = d\psi + \cos\theta_1 d\varphi_1 + \cos\theta_2 d\varphi_2$$

$$\int \omega_2 = 4\pi \; ; \qquad \int \omega_3 = 8\pi^2 \; ;$$

To have M units of RR 3-form flux, we need $\frac{1}{4\pi^2\alpha'}\int F_3 = M$;

The solution with this extra flux through T^{11} is

I.K., Tseytlin

$$F_3 = \frac{\alpha'}{2} M \,\omega_3 \,; \qquad B_2 = \frac{\alpha'}{2} 3 g_s M \ln(r/r_0)\, \omega_2$$

The dilaton $\Phi = 0$.

$$ds^2 = h^{-\frac{1}{2}}(r)\, dx_\mu dx^\mu + h^{\frac{1}{2}}(r)\left(dr^2 + r^2 ds^2_{T^{11}}\right)$$

$$h(r) = \frac{4\pi g_s}{r^4}\left(N_0 + \frac{3}{2\pi} g_s M^2 \left[\ln\left(\frac{r}{r_0}\right) + \frac{1}{4}\right]\right)\frac{27}{16}$$

The 5-form field strength $\tilde{F}_5 = \mathcal{F}_5 + * \mathcal{F}_5$,

$\mathcal{F}_5 \sim N(r)\, \mathrm{vol}\,(T^{11})$.

$$N(r) = N_0 + \frac{3}{2\pi} g_s M^2 \ln(r/r_0).$$

The number of colors N in $SU(N) \times SU(N+M)$ has become scale dependent.

As $\ln(r/r_0)$ decreases by $\frac{2\pi}{3 g_s M}$,

$N(r) \to N(r) - M$.

This is the cascade of Seiberg dualities.

Connection with the NSVZ β-functions.

$$\frac{d}{d\ln(\Lambda/\mu)}\frac{8\pi^2}{g_1^2} \sim 3(N+M) - 2N(1-\gamma)$$

$$\frac{d}{d\ln(\Lambda/\mu)}\cdot\frac{8\pi^2}{g_2^2} \sim 3N - 2(N+M)(1-\gamma)$$

γ is the anomalous dimension of $\mathrm{Tr}(A_iB_j)$

$$\gamma = -\frac{1}{2} + \mathcal{O}\left(\frac{M^2}{N^2}\right).$$

$$\frac{8\pi^2}{g_1^2} - \frac{8\pi^2}{g_2^2} \sim 6M\ln(\Lambda/\mu)$$

In SUGRA we have

$$\int B_2 = (2\pi\alpha')\,3g_s M\ln(r/r_0)$$

Using the relation

$$\frac{1}{2\pi\alpha' g_s}\int B_2 = \frac{4\pi^2}{g_1^2} - \frac{4\pi^2}{g_2^2}$$

we find exact agreement with FT.
The factor 3 in the SUGRA calculation is geometrical; related to self-duality of G_3.

Correct β-function also follows from $W \sim \int G_3\wedge\Omega$
Vafa.

THIS SINGULARITY IS A SIGNAL OF THE CHIRAL SYMMETRY BREAKING

To achieve a non-singular solution of the supergravity equations of motion, the conifold must be deformed to

$$\sum_{i=1}^{4} z_i^2 = \epsilon^2 ;$$

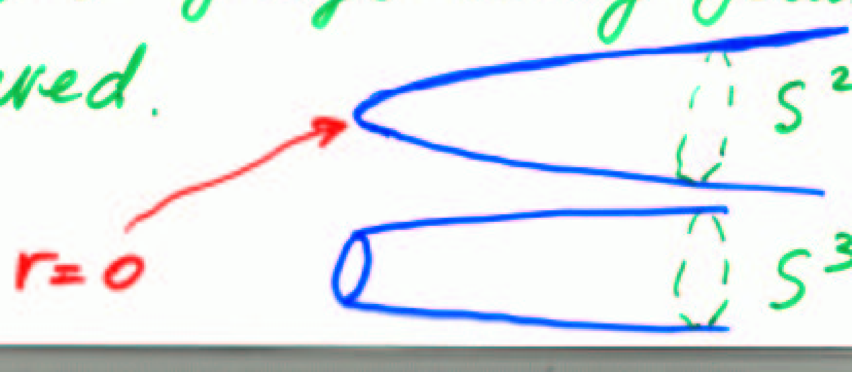
I.K., Strassler

This breaks the Z_{2M} symmetry down to Z_2: $z_i \to -z_i$.

$$ds_{10}^2 = h^{-\frac{1}{2}}(r)(-dt^2 + d\vec{x}^2) + h^{\frac{1}{2}}(r)\, d\tilde{s}_6^2 .$$

Now the warp factor $h(r)$ has NO unwanted zeroes.

$\sqrt{h(0)} \sim g_{YM}^2 M \equiv$ the 't Hooft coupling of the $SU(M)$ gauge theory found in the infrared.



A fundamental string at the apex ($r=0$) along the world volume directions $\mathbb{R}^{3,1}$ has tension $\sim \frac{1}{\sqrt{h(0)}}$.

It is finite $\Rightarrow$ CONFINEMENT

In contrast, for $AdS_5 \times S^5$, $h = \frac{L^4}{r^4}$ blows up at $r=0 \Rightarrow$

NO CONFINEMENT FOR $\mathcal{N}=4$ SUPER-YM.

<u>IN THE $\mathcal{N}=1$ $SU(M)$ GAUGE THEORY THERE ARE CONFINING q-STRINGS</u>

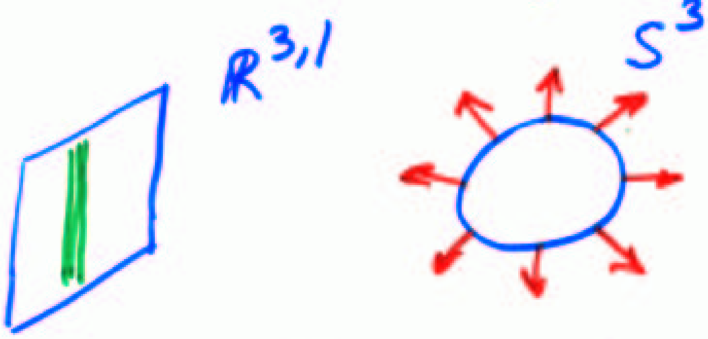
They connect q probe quarks with q corresponding anti-quarks.

Wilson loops in the fully antisymmetric representation with q indices.

The tensions T_q, $q = 1, 2, \ldots, M-1$.

$T_q = T_{M-q}$ ($q \to M-q$ exchanges quarks with antiquarks).

$T_M = 0$ (M quarks form a colorless object, the baryon).



In our SUGRA dual the q-string is described by q coincident fundamental strings at the apex.

The blown-up 3-sphere has metric ($b \approx .933$)

$$ds_3^2 = b\, g_s M \alpha' [d\psi^2 + \sin^2\psi (d\theta^2 + \sin^2\theta\, d\varphi^2)]$$

and M units of R-R 3-form flux

$$F_3 = 2M\alpha' \sin^2\psi \sin\theta\, d\psi \wedge d\theta \wedge d\varphi.$$

$\psi \in [0, \pi]$ is the azimuthal angle.

The R-R flux blows up the q F-strings into a D3-brane wrapped over an S^2 at fixed azimuthal angle ψ C. Herzog, I.K.

$$T \sim \sqrt{b^2 \sin^4\psi + \left(\psi - \frac{\sin 2\psi}{2} - \pi\frac{q}{M}\right)^2}$$

Minimizing wrt ψ, we find

$$\psi - \frac{\pi q}{M} = \frac{1-b^2}{2}\sin(2\psi)$$

If $b=1$, then $\psi = \frac{\pi q}{M}$

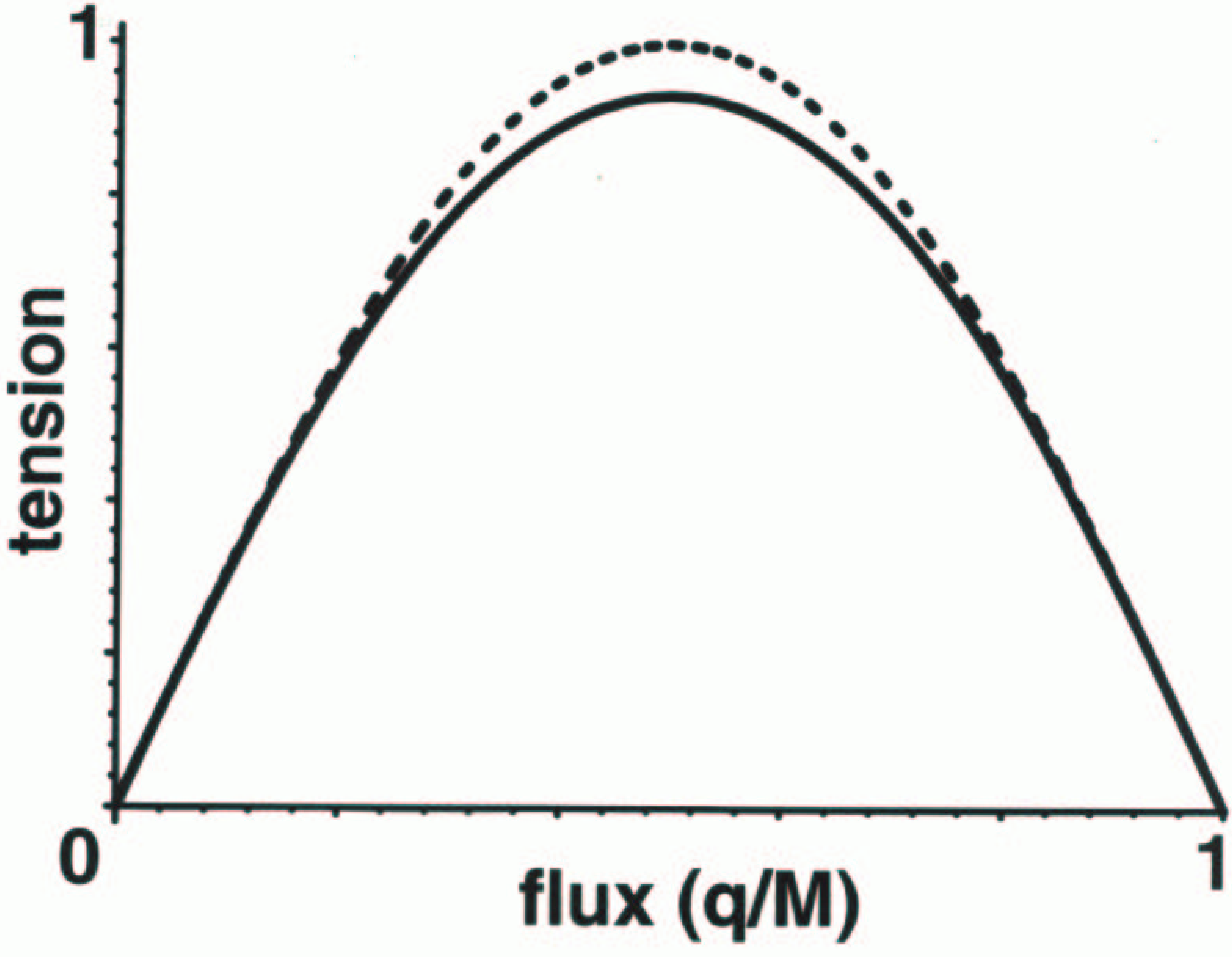
$$T_q \sim \sin\left(\pi\frac{q}{M}\right)$$

This formula is valid in softly broken $\mathcal{N}=2$ and in MQCD.

For the KS soltn, $b \approx .933 \Rightarrow T_q$ is almost the same.

For the Maldacena-Nuñez SUGRA dual, which also has a blown-up S^3, $b=1$ exactly.

q-string tensions



The D-brane calculation has the exact $q \to M-q$ symmetry crucial for the gauge theory interpretation.

Our calculation is reliable for large $g_s M$. To describe pure $\mathcal{N}=1$ $SU(M)$ gauge theory we need small $g_s M$.

The KS background describes a different gauge theory: $SU(N+M) \times SU(M)$ which "cascades" to pure $SU(M)$ only far in the IR.

A lot remains to be understood...

Nevertheless,

$$\frac{T_q}{T_{q'}} \approx \frac{\sin(\pi q/M)}{\sin(\pi q'/M)}$$

may be a good model for what to expect from the LATTICE (Lucini + Teper).
Del Debbio et al.