

HIGHER ORDER INVARIANTS IN SUPERGRAVITY

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STONY BROOK 2001

STONY BROOK 1979

3-LOOP

- DESER, KATZ, STELLE 'FF
- FERARA, VAN NIEUWENHUIZEN 'TB

$$L^{(n)} \sim K^{2(n-1)} R^{n+1}$$

$$\int d^6x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\lambda} \not{\partial} \lambda \right]$$

$$\int d^6x \left[T_{\mu\nu} T^{\mu\nu} + D_{\mu\nu} D^{\mu\nu} + \frac{i}{2} \bar{\lambda} \not{\partial} \lambda \right]^n - \frac{3}{4} C_T \Omega C^n$$

↙
 $\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} F^{\mu\nu} + \dots$



DIMENSIONAL REDUCTION

- W. PAUL HONE
- PROCEEDINGS 413
- NPB 181 ('81) 487

CAMBRIDGE 1980

3-LOOP	$N \leq 3$
N-LOOP	$N \geq 4$
(N-1)	NONLINEAR

- FERRARA, GENTILE (LIV) '78
- MODENA '01 (FULL OFF-SHELL)

SUPERSPACE!

$N=8$ ON-SHELL SUPERSPACE SUPERGRAVITY

- L. BRINK, P. HONE PL 008 ('79)

TANGENT SPACE: $SL(2\mathbb{C}) \times G$; ($G = SU(8)$)
 GLOBAL: $G' (= E_7)$

$$T_{\alpha\beta}^{\dot{\alpha}\dot{\beta}} = 2 \epsilon_{\alpha\beta} \bar{\chi}^{abc\dot{\alpha}} \chi^{abc\dot{\beta}}$$

$$\mathcal{L}^{(3)} = \kappa^4 W^2 \bar{W}^2 \bar{E}$$

$2N \leq 3$

$$\int d^4x d^{4N}\theta \mathcal{L}^{(N)}(z)$$

$$\Delta_1 \equiv C_{\mu\nu\sigma} C^{\mu\nu\sigma}$$

$$\mathcal{L}^{(N)}(z) = \kappa^{2(N-1)} \epsilon^{\alpha\beta} \epsilon^{\dot{\alpha}\dot{\beta}} \chi_{\alpha abc} \chi_{\beta def} \bar{\chi}_{\dot{\alpha}}^{abc} \bar{\chi}_{\dot{\beta}}^{def} \bar{E}$$

Lin. $\kappa^{2(N-1)} \Delta_1 \square^{(N-3)} \bar{\Delta}_1$

- R. KALLOSH LEB. PREP. '80 (3-LOOP LIN.)

STRING THEORY

- SUPER. EFFECTIVE THEORY
- **D-BRANES**, P-BRANES; WV. THEORY
- M-THEORY → D=11 SUPER
-
-

$$L_{DBI} + L_{WZ}$$

- GREEN, HARTNELL, MOORE '97
- CHENNY, YIN '98

$$T_p \propto \int \text{tr} e^F \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}$$

$$\sqrt{\hat{A}(R)} = 1 - \frac{1}{48} p_1(R) + \frac{1}{2560} p_1^2(R) + \dots$$

p: Pontryagin class

$$(R_T)_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} + \delta_{\pi'\tau'} (\Omega^{\pi'}_{\mu\rho} \Omega^{\tau'}_{\nu\sigma})$$

$$(R_N)_{\mu\nu}{}^{\pi'\tau'} = -R^{\pi'\tau'}_{\mu\nu} + g^{\rho\sigma} (\Omega^{\pi'}_{\rho\mu} \Omega^{\tau'}_{\sigma\nu})$$

Ω 2ND FUNDAMENTAL FORM

$$L_{\text{DBI}} = T_p \sqrt{-\det[G_{\mu\nu} + B_{\mu\nu}] \partial_\mu X^\mu \partial_\nu X^\nu + F_{\mu\nu}} e^{-\phi}$$

• ∂F - TERMS $\mathcal{O}(\alpha'^2)$

• ANDREEV, TSEYTLIN '88

• $\partial^2 X$ - TERMS $\mathcal{O}(\alpha'^2)$

• BACHAS, BAIN, GREEN '99
TOTOPOULOS '01

BOSONIC

• WYLLARD '01

$$\rightarrow T_p e^{-\phi} \sqrt{-g} \left[1 - A(\alpha')^2 \left\{ (R_T)_{\mu\nu\rho\sigma} (R_T)^{\mu\nu\rho\sigma} - 2 (R_T)_{\mu\nu} (R_T)^{\mu\nu} - (R_N)_{\mu\nu\rho\sigma} (R_N)^{\mu\nu\rho\sigma} + 2 \bar{R}_{\rho\sigma} \bar{R}^{\rho\sigma} \right\} \right]$$

ANOMALY $\sim \Omega_{2p}^{\alpha' p}$

Q:

K -SYMMETRIC BRANE ACTIONS
W. "RIGIDITY" TERM?

- RLWD STRINGS (bos).
- POLYAKOV '86
- (SPINNING) WEYL INV.
- U.L., ROČEK, VAN NIEUWENHUIZEN '87
- " " '88
- K-SYM. (PART. RES.)
- CURTIS, VAN NIEUWENHUIZEN '87
- K-SYM. SUPERPARTICLE
- IVANOV, KAPUSTNIKOV '91
- GAUNTLETT '91
- P-BRANE ?

$$S_{GS} = T \int d^{p+1} \xi \left[\sqrt{-\gamma} + \epsilon^{\mu\nu} \bar{\theta} \gamma_{\mu} \partial_{\nu} \theta \right]$$

$$\pi_{\mu}^{\kappa} \equiv \partial_{\mu} X^{\kappa} - i \bar{\theta} \Gamma^{\kappa} \partial_{\mu} \theta$$

$$\gamma_{\mu\nu} \equiv \pi_{\mu}^{\kappa} \pi_{\nu}^{\lambda} \eta_{\kappa\lambda}$$

K-SYMMETRY

$$\begin{aligned} \delta\theta &= \epsilon \\ \delta X^{\kappa} &= i \bar{\epsilon} \Gamma^{\kappa} \theta \end{aligned}$$

$$\begin{aligned} \delta\theta &= \kappa \\ \delta X^{\kappa} &= -i \bar{\kappa} \Gamma^{\kappa} \theta \\ \kappa &= p \kappa ; \quad p^2 = p \end{aligned}$$

POLYAKOV

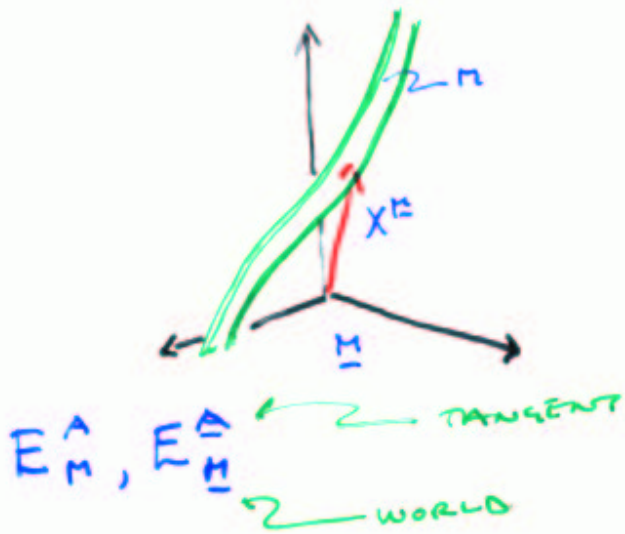
$$T \int d^2 \xi \sqrt{-\gamma} [1 + \mu \Omega^2]$$

BOSONIC \rightarrow

$$\begin{aligned} &\Omega_{\mu}^{\sigma} \cdot \Omega_{\nu}^{\rho} \\ &\sim \eta_{\mu\nu} \partial_{\mu} X^{\sigma} \partial_{\nu} X^{\rho} \end{aligned}$$

SUPEREMBEDDINGS

• SOROKIN, TRACH, VOLKOV
ZHELTUKHIN '89



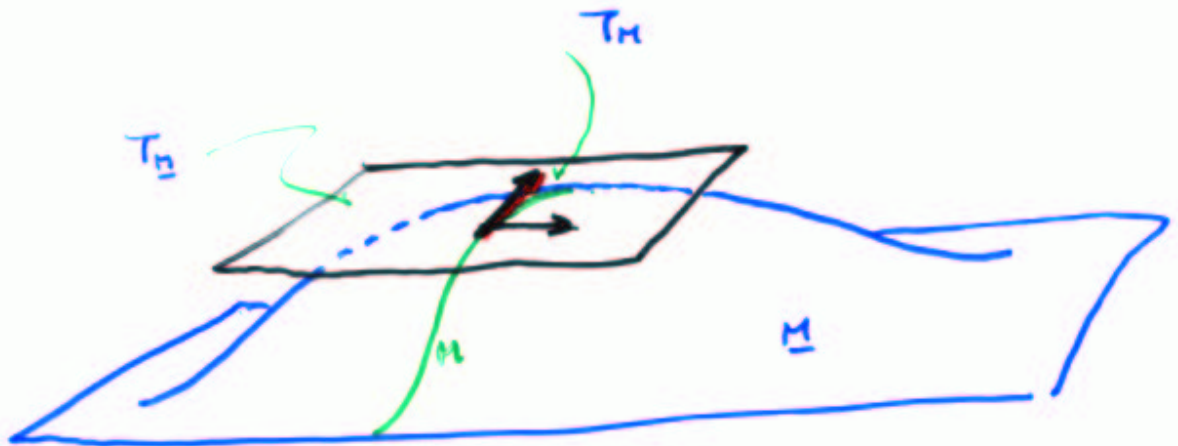
$$X^M : M \rightarrow \underline{M}$$

(BOTH SUPERSPACES)

$$A = (a, \alpha)$$

$$E_{\hat{A}}^{\hat{\alpha}} = E_A^M \partial_M X^{\hat{\alpha}} E_M^{\hat{\alpha}}$$

THE EMBEDDING MATRIX



THE EMBEDDING CONDITION

$$E_{\hat{\alpha}}^{\hat{\alpha}} = 0$$

$$T_{\hat{M}}^F \subset T_M^F$$

$$T_{MN}^A = 2 \nabla_{[M} E_{N]}^A$$

$$2 \nabla_{[A} E_{B]}^C + T_{AB}^C E_C^D = (-)^{A(B+B)} E_B^D E_A^E T_{DE}^C$$

- CHOOSE THE GEOMETRY OF M
- PARAMETRIZE THE EMBEDDING CONDITION ETC.
- EXPLORE THE CONSEQUENCES OF THE TORSION RELN.

THE SUPERMULTIPLY THAT DESCRIBES THE EMBEDDING WILL TYPICALLY BE ON-SHELL FOR LARGE NUMBER OF SUSY. FOR $N \leq 16$ IT CAN ALSO BE OFF-SHELL.

K-SYMMETRY

$$\delta X^M = U^M \partial_M X^M$$

$$\delta X^A \equiv \delta X^M E_M^A = \underline{U^A E_A^A}$$

THE
EMB.
MATRIX

$$U^a = 0 \Rightarrow \begin{cases} \delta X^a = E_a^\alpha U^\alpha = 0 \\ \delta X^\alpha = U^\alpha E_\alpha^a \equiv K^a \end{cases}$$

$$K^a = K^b P_b^a \leftarrow \text{PROJ. ONTO } T_M^F \text{ CTF}_M^F$$

$$(E^{-1})_\alpha^\gamma E_\gamma^a$$

MEMBRANE IN FLAT D=4 SUPERSPACE

∃ OFF-SHELL MULTIPLICET.

$$E_a^{\dot{a}} = 0$$

$$E_a^{\alpha} = u_a^{\alpha} + i h \delta_{\alpha}^{\alpha'} u_{\alpha'}$$

$$u = \begin{pmatrix} u_a^{\alpha} \\ u_{\alpha'}^{\beta} \end{pmatrix} \begin{matrix} \downarrow \\ \downarrow \end{matrix} \begin{pmatrix} u_a^{\dot{a}} \\ u_a^{\dot{b}} \end{pmatrix}$$

$$T_{\alpha\beta}^c = -i (\gamma^c)_{\alpha\beta}$$

$$E_a^{\dot{a}} = (1 + h^2) u_a^{\dot{a}}$$

$$T_{\alpha\beta}^{\dot{a}} = 0, T_{ab}^c = 0$$

$$T_{ab}^c = 0, T_{\alpha\beta}^{\dot{a}} = i (\gamma_a)_{\beta}^{\dot{a}} S$$

D=3 SU(2)

* BROWN, GATES '79

$$X = du^i$$

$$E_a^{\alpha} = \Lambda_a^{\alpha} u_a^{\alpha} + \gamma_a^{\alpha'} u_{\alpha'}$$

$$X_{2,1b3} = \frac{1}{2} X_{(ab)3} = \underline{X_{ab3}}$$

WANT

$$T_2 \int d^3x \left[\sqrt{-g} (1 + \mu \Omega^2) + \text{FERMIONS} \right]$$

= HOWE, RAETZEL, SERGIN '98

$$\sqrt{-g} \left(\frac{1-h^2}{1+h^2} \right) - \frac{1}{6} \epsilon^{\mu\nu\sigma} C_{\mu\nu\sigma}$$

? WZ

(K-SYM. !)

BY DIM. AN.

= HOWE, U.L. / 01 hep-th / 011036

$$L_{(1)} \equiv -\frac{i}{2} \Lambda^{\alpha\beta} \Lambda_{\alpha\beta}$$

$$L_{(2)} \equiv -\frac{i}{2} (\gamma^{ab})^{\alpha\beta} \Lambda_{\alpha\beta} \Lambda_{\gamma\delta}$$

MULT. BY
ARB.
 $f(h) \dots$

$$-i \nabla^\alpha \nabla_\alpha L_{(1)} = \frac{1}{2} X_{ab3} X^{ab3} + \frac{h^2 (2+9h^2)}{2(1+3h^2)^2} - \frac{2(1+2h^2+3h^4)}{(1+h^2)^2} (\nabla h)^2$$

$$-i \nabla^\alpha \nabla_\alpha L_{(2)} = \frac{1}{2} X_{ab3} X^{ab3} - \left(\frac{1}{2} + \frac{h^2 (2+9h^2)}{(1+3h^2)^2} \right) X_3^2$$

$$+ \frac{4h^2 (2+3h^2)}{(1+h^2)^2} (\nabla h)^2$$

$$\delta h \Rightarrow h(X_{ab3}) \quad (\text{DIFF. EQN}) \quad \dots$$

REMARKABLY:

$$L_0 + \mu \left(L^{(1)} + \frac{1}{2} L^{(2)} \right)$$

$$= \sqrt{-\det g} \left(\frac{1-h^2}{1+h^2} + \frac{\mu}{2} \hat{\Omega}^2 - \frac{4\mu}{3(1+h^2)^2} (\nabla h)^2 \right)$$

THE TRACELESS PART
OF THE 2ND FUNDAMENTAL FORM

$$\phi = \tan^{-1} h$$

$$\delta\phi \Rightarrow \nabla^2 \phi - \frac{3}{4\mu} \sin 2\phi = 0$$

SINE-GORDON !

OUTLOOK

- D-BRANES, HIGHER ORDER IN α'
- MODIFY THE EMBEDDING CONDITION
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