HIGHER ORDER INVARIANTS IN SUPERGRAVITY

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STONY BROOK 2001

STONY BROOK 1979

3-LOOP
- DESER, KAY, STELLE '77
- FERRARA, VAN NIEUWENHUIZEN '78

$$\boxed{L^{(n)} \sim \kappa^{2(n-1)} R^{n+1}}$$

$$\int d^6x \left[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\lambda} \not{\partial} \lambda \right]$$

$$\int d^6x \left[J_{\mu\nu} J^{\mu\nu} + D_{\mu\nu} D^{\mu\nu} + \tfrac{i}{2} \bar{J}_\mu \not{\partial} J^\mu - \tfrac{3}{4} C_\mu \Box C^\mu \right]$$

$$\epsilon_{\mu\nu\rho\sigma\tau\lambda} F^{\rho\sigma} F^{\tau\lambda} + \ldots$$

DIMENSIONAL REDUCTION

W. PAUL HOWE
- PROCEEDINGS 413
- NPB 181 ('81) 487

CAMBRIDGE 1980

3-LOOP	$N \leq 3$
N-LOOP	$N \geq 4$
(N-1)	NONLINEAR

N=1
- FERRARA, ZUMINO (LIN) '78
- MOURA '01 (FULL OFF-SHELL)

SUPERSPACE !

N=8 ON-SHELL SUPERSPACE SUPERGRAVITY

- L. BRINK, P. HOWE PL 88B ('79)

TANGENT SPACE: $SL(2,\mathbb{C}) \times G$: $(G = SU(8))$
GLOBAL: G' $(= E_7)$

$$T_{\underline{\alpha}\,\underline{\beta}}{}^{\dot{\underline{\gamma}}} = 2\,\epsilon_{\alpha\beta}\,\bar{\chi}^{abc\dot{\gamma}}$$

$$\boxed{\mathcal{L}^{(3)} = \kappa^4 W^2 \bar{W}^2 E}$$

2 $N \leq 3$

$$\boxed{\int d^4x\, d^{4N}\theta\ \mathcal{L}^{(N)}(z)}$$

$$\Delta_1 \equiv C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$$

$$\boxed{\mathcal{L}^{(N)}(z) = \kappa^{2(N-1)} \epsilon^{\alpha\beta} \epsilon^{\dot{\alpha}\dot{\beta}} \chi_{\alpha abc} \chi_{\beta def} \bar{\chi}_{\dot{\alpha}}{}^{abc} \bar{\chi}_{\dot{\beta}}{}^{def} E}$$

LIN. $\kappa^{2(N-1)} \Delta_1 \Box^{(N-3)} \bar{\Delta}_1$

- R. KALLOSH LEB. PREP. '80 (3-LOOP LIN.)

STRING THEORY

- SUGR. EFFECTIVE THEORY
- D-BRANES, p-BRANES ; WV. THEORY
- M-THY $\rightarrow$ D=11 SUGR
-
-
-

$$\boxed{L_{DBI} + L_{WZ}}$$

- GREEN, HARVEY, MOORE '97
- CHEUNG, YIN '98

$$\boxed{T_p\, C \wedge \mathrm{tr}\, e^F} \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}$$

$$\sqrt{\hat{A}(R)} = 1 - \frac{1}{48} p_1(R) + \frac{1}{2560} p_1^2(R) + \cdots$$

p: PONTRYAGIN CLASS

$$(R_T)_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} + \delta_{\pi'\tau'} \left(\Omega^{\pi'}_{\mu[\rho} \Omega^{\tau'}_{\sigma]\nu} \right)$$

$$(R_N)_{\mu\nu}{}^{\pi'\tau'} = -R^{\pi'\tau'}{}_{\mu\nu} + g^{\rho\sigma} \left(\Omega^{\pi'}_{\rho[\mu} \Omega^{\tau'}_{\nu]\sigma} \right)$$

Ω 2ND FUNDAMENTAL FORM

$$\boxed{L_{DBI} = T_p \sqrt{-\det\left[(G_{\underline{\mu}\underline{\nu}} + B_{\underline{\mu}\underline{\nu}})\,\partial_\mu X^{\underline{\mu}} \partial_\nu X^{\underline{\nu}} + F_{\mu\nu}\right]}\; e^{-\phi}}$$

- ∂F - TERMS $\mathcal{O}(\alpha'^2)$. ANDREEV, TSEYTLIN '88
- $\partial^2 X$ - TERMS $\mathcal{O}(\alpha'^2)$. BACHAS, BAIN, GREEN '99 FOTOPOULOS '01

$\boxed{\text{BOSONIC.}}$

- WYLLARD '01

$$T_p\, e^{-\phi} \sqrt{g} \Big[1 - A(\alpha')^2 \Big\{ (R_T)_{\mu\nu\rho\sigma} (R_T)^{\mu\nu\rho\sigma} - 2 (R_T)_{\mu\nu} (R_T)^{\mu\nu} - (R_N)_{\mu\nu\rho'\sigma'} (R_N)^{\mu\nu\rho'\sigma'} + 2 \bar{R}_{\rho'\sigma'} \bar{R}^{\rho'\sigma'} \Big\} \Big]$$

AMBIGUITY $\sim \Omega_\mu^{\lambda' \mu}$.

Q: $\boxed{\text{κ-SYMMETRIC BRANE ACTION W. "RIGIDITY" TERM ?}}$

- "RIGID" STRING (BOS). • POLYAKOV '86
- (SPINNING) WEYL INV. • U.L., ROČEK, VAN NIEUWENHUIZEN '87; " " '88
- K-SYM. (PART. RES.) • CURTRIGHT, VAN NIEUWENHUIZEN '87
- K-SYM. SUPERPARTICLE • IVANOV, KAPUSTNIKOV '91 • GAUNTLETT '91
- p-BRANE ?

$$\boxed{S_{GS} = T\int d^{p+1}\xi \left[\sqrt{-\gamma} + \epsilon^{\mu\nu}\bar\theta \not\Pi_\mu \partial_\nu \theta\right]}$$

$$\Pi^{\underline{\mu}}_{\mu} \equiv \partial_\mu X^{\underline{\mu}} - i\bar\theta\Gamma^{\underline{\mu}}\partial_\mu\theta$$

$$\gamma_{\mu\nu} \equiv \Pi^{\underline{\mu}}_{\mu}\Pi^{\underline{\nu}}_{\nu}\eta_{\underline{\mu\nu}}$$

$$\delta\theta = \epsilon$$
$$\delta X^{\underline{\mu}} = i\bar\epsilon\Gamma^{\underline{\mu}}\theta$$

K-SYMMETRY

$$\boxed{\begin{aligned}&\delta\theta = \kappa\\ &\delta X^{\underline{\mu}} = -i\bar\kappa\Gamma^{\underline{\mu}}\theta\\ &\kappa = p\kappa\ ; \quad p^2 = p\end{aligned}}$$

POLYAKOV

$$\boxed{T\int d^2\xi\sqrt{-\gamma}\left[1+\mu\,\Omega^2\right]}$$

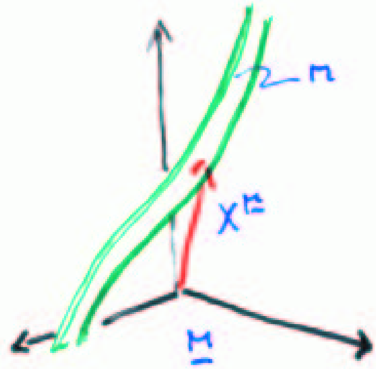
BOSONIC

$$\Omega^{\rho}{}_{\mu}{}^{\underline{\mu}}\cdot\Omega^{\sigma}{}_{\nu}{}^{\underline{\nu}}$$

$$\sim \eta_{\underline{\mu\nu}}\partial_\mu\delta^{\underline{\mu}} X^{\underline{\mu}}\partial_\nu\delta^{\underline{\nu}} X^{\underline{\nu}}$$

SUPEREMBEDDINGS

* SOROKIN, TKACH, VOLKOV
ZHELTUKHIN '89



$X^{\underline M} : M \longrightarrow \underline M$

(BOTH SUPERSPACES)

$E_M{}^A \, , \, E_{\underline M}{}^{\underline A}$

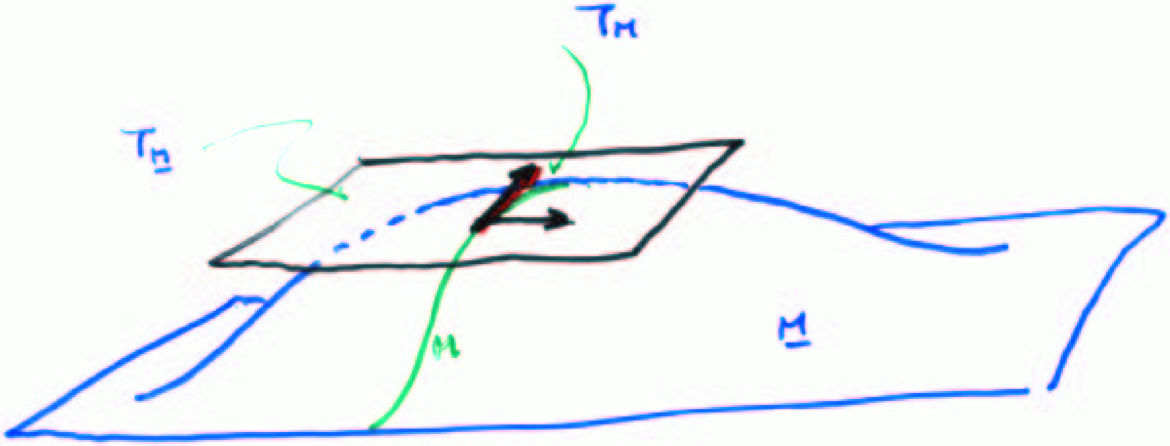
TANGENT

WORLD

$\underline A = (\underline a, \underline \alpha)$

$$E_A{}^{\underline A} \equiv E_A{}^M \partial_M X^{\underline M} E_{\underline M}{}^{\underline A}$$

THE EMBEDDING MATRIX



THE EMBEDDING CONDITION

$$\boxed{E_\alpha{}^{\underline a} = 0}$$

$$T^F_M \subset T^F_{\underline M}$$

$$T_{\underline{MN}}{}^{\underline{A}} = 2\nabla_{[\underline{M}} E_{\underline{N}]}{}^{\underline{A}}$$

$$\boxed{2\nabla_{[A} E_{B]}{}^{\underline{C}} + T_{AB}{}^{C} E_{C}{}^{\underline{C}} = (-)^{A(B+\underline{B})} E_{B}{}^{\underline{B}} E_{A}{}^{\underline{A}} T_{\underline{AB}}{}^{\underline{C}}}$$

- CHOOSE THE GEOMETRY OF $\underline{M}$
- PARAMETRIZE THE EMBEDDING CONDITION ETC.
- EXPLORE THE CONSEQUENCES OF THE TORSION REL'N.

THE SUPERMULTIPLET THAT DESCRIBES THE EMBEDDING WILL TYPICALLY BE ON-SHELL FOR LARGE NUMBER OF SUSY. FOR $N \leq 16$ IT CAN ALSO BE OFF-SHELL.

$\boxed{\text{K-SYMMETRY}}$

$$\delta X^{\underline{M}} = v^{M} \partial_{M} X^{\underline{M}}$$

$$\underline{\delta X^{\underline{A}}} \equiv \delta X^{\underline{M}} E_{\underline{M}}{}^{\underline{A}} = \underline{v^{A} E_{A}{}^{\underline{A}}}$$

THE EMB. MATRIX

$$v^{a} = 0 \quad \Rightarrow \quad \begin{cases} \delta X^{\underline{a}} = E_{\alpha}^{\underline{a}} v^{\alpha} = 0 \\ \delta X^{\underline{\alpha}} = v^{\alpha} E_{\alpha}{}^{\underline{\alpha}} \equiv \kappa^{\underline{\alpha}} \end{cases}$$

$$\kappa^{\underline{\alpha}} = \kappa^{\underline{\beta}} P_{\underline{\beta}}{}^{\underline{\alpha}}$$

PROJ. ONTO $T^F_M \subset T^F_{\underline{M}}$

$$(E^{-1})_{\underline{\alpha}}{}^{\gamma} E_{\gamma}{}^{\underline{\beta}}$$

MEMBRANE IN FLAT D=4 SUPERSPACE

$\exists$ OFF-SHELL MULTIPLET.

$$E_\alpha{}^{\underline{a}} = 0$$

$$E_\alpha{}^{\underline{\alpha}} = u_\alpha{}^{\underline{\alpha}} + i\,\underline{h}\,\delta_\alpha{}^{\alpha'} u_{\alpha'}{}^{\underline{\alpha}}$$

$$u = \begin{pmatrix} u_\alpha{}^{\underline{\alpha}} \\ u_{\alpha'}{}^{\underline{\alpha}} \end{pmatrix} \Updownarrow \begin{pmatrix} u_a{}^{\underline{a}} \\ u_3{}^{\underline{a}} \end{pmatrix}$$

$$T_{\alpha\beta}{}^{c} = -i(\gamma^c)_{\alpha\beta}$$

$$E_a{}^{\underline{a}} = (1+h^2)\,u_a{}^{\underline{a}}$$

$$\boxed{\begin{array}{l} T_{\alpha\beta}{}^{\gamma} = 0, \quad T_{\alpha b}{}^{c} = 0 \\ T_{ab}{}^{c} = 0, \quad T_{a\beta}{}^{\gamma} = i(\gamma_a)_\beta{}^\gamma S \end{array}}$$

D=3 SUGRA

- BROWN, GATES '79

$$X = d u u^{-1}$$

$$E_a{}^{\underline{\alpha}} = \underline{\Lambda_a}{}^{\alpha} u_\alpha{}^{\underline{\alpha}} + \Psi_a{}^{\alpha'} u_{\alpha'}{}^{\underline{\alpha}}$$

$$X_{a,b3} = \tfrac{1}{2} X_{(a,b)3} = \underline{X_{ab3}}$$

$\boxed{\text{ACTIONS}}$

WANT

$$T_2 \int d^3\xi \left[\sqrt{-g}\,(1 + \mu\, \Omega^2) + \text{FERMIONS} \right]$$

• HOWE, RAETZEL, SEZGIN '98

$$\sqrt{-g}\left(\frac{1-h^2}{1+h^2}\right)\Big| - \frac{1}{6}\epsilon^{\mu\nu\rho} C_{\mu\nu\rho}\Big|$$

$?_{WZ}$

(κ-SYM.).

BY DIM. AN.

• HOWE, U.L. '01 hep-th/011036

$$\boxed{\begin{aligned} L_{(1)} &\equiv -\frac{i}{2}\Lambda^{a\alpha}\Lambda_{a\alpha} \\ L_{(2)} &\equiv -\frac{i}{2}(\gamma^{ab})^{\alpha\beta}\Lambda_{a\alpha}\Lambda_{b\beta} \end{aligned}}$$

MULT. BY ARB. $f(h)$...

$$\boxed{\begin{aligned} -i\nabla^\alpha\nabla_\alpha L_{(1)} &= \frac{1}{2}X_{abc}X^{abc} + \frac{h^2(2+9h^2)}{2(1+3h^2)^2} - \frac{2(1+2h^2+3h^4)}{(1+h^2)^2}(\nabla h)^2 \\ -i\nabla^\alpha\nabla_\alpha L_{(2)} &= \frac{1}{2}X_{abc}X^{abc} - \left(\frac{1}{2} + \frac{h^2(2+9h^2)}{(1+3h^2)^2}\right)X_3^2 \\ &\quad + \frac{4h^2(2+3h^2)}{(1+h^2)^2}(\nabla h)^2 \end{aligned}}$$

$\delta h \Rightarrow h(X_{abc})$ (DIFF. EQ.N) ....

REMARCABLY:

$$L^{0}\big|_{1}+\mu\left(L^{(1)}\big|_{1}+\frac{1}{2}L^{(2)}\big|_{1}\right)$$

$$\boxed{=\sqrt{-\det g}\left(\frac{1-h^2}{1+h^2}+\frac{\mu}{2}\hat{\Omega}^2-\frac{4\mu}{3(1+h^2)^2}(\nabla h)^2\right)}$$

THE TRACELESS PART
OF THE 2ND FUNDAMENTAL FORM

$$\phi=\tan^{-1}h$$

$$\delta\phi \Rightarrow \quad \underline{\nabla^2\phi-\frac{3}{4\mu}\sin 2\phi=0}$$

SINE-GORDON!

OUTLOOK

- D-BRANES, HIGHER ORDER IN α'
- MODIFY THE EMBEDDING CONDITION
- .