Comments on Born-Infeld Theory

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CONTENTS

Selected References

A. Tseytlin, hep-th/9908105

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1 Abelian Born–Infeld (Single D-Brane)

D-branes are surfaces on which open strings can end. As such, their dynamics is described by *open string field theory* – e.g., of the type formulated by Witten. However, this can be difficult to work with, so it is sometimes useful to consider the low energy effective action obtained by integrating out all the massive modes, keeping only the massless super-Maxwell multiplet.

This becomes tractible if one only keeps terms in which the fields are slowly varying at the string scale – keeping fields strengths, but not their derivatives. The field strengths are allowed to be large, but one finds that they cannot exceed a certain critical value. At this value the stretching force on a string with charges on its ends matches the string tension.

In the case of Type II superstrings, the effective action is the sum of two terms: a Dirac–Born–Infeld term and a Chern–Simons term

$$S = S_{DBI} + S_{CS}.$$

1.1 Bosonic terms

Let us begin by considering a single D-brane ($N = 1$). In this case, ignoring fermi fields and taking a flat 10d background, the action for a D9-brane is

$$S_{DBI} = T_9 \int d^{10}\sigma \sqrt{-\det(g_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta})}.$$

Here T_9 is the D9-brane tension and $g_{\alpha\beta}$ is the pullback of the (flat) spacetime metric $\eta_{\mu\nu}$:

$$g_{\alpha\beta} = \eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$$

σ^α ($\alpha = 0, 1, \ldots, p$) are the world-volume coordinates

$X^\mu(\sigma)$ ($\mu = 0, 1, \ldots, 9$) are the embedding functions

The action S_{DBI} has world-volume diffeomorphism invariance. A natural gauge choice – called ***static gauge*** – is to identify the first $p+1$ components of X^μ with σ^α. In this gauge the D9-brane action becomes

$$S_{DBI} = T_9 \int d^{10}\sigma \sqrt{-\det(\eta_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta})}.$$

The actions for Dp-branes with $p < 9$ can be deduced from the D9-brane case using T duality. The result, which agrees with dimensional reduction, is

$$S_{DBI} = T_p \int d^{p+1}\sigma \sqrt{-\det(\eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i + 2\pi\alpha' F_{\alpha\beta})}.$$

The index $i = p+1, \ldots, 9$ labels the $9-p$ directions transverse to the Dp-brane. The world-volume scalars X^i can be regarded as Goldstone bosons associated to broken translational symmetries. In particular, as observed by Bachas, for a D0-brane this gives

$$T_0 \int d\sigma^0 \sqrt{1 - \partial_0 X^i \partial_0 X^i},$$

which is the standard action for a relativistic particle of mass T_0.

The formula for the Chern–Simons term (due to Douglas) in the presence of RR background fields is

$$S_{CS} = \int \left(C e^{2\pi\alpha' F} \right)_{p+1}$$

where $C = \Sigma C^{(n)}$ is a formal sum of RR n-form fields. (n is odd for IIA and even for IIB.)

1.2 Supersymmetrization

In the cases that are BPS, the inclusion of world volume fermions results in a supersymmetric D-brane action. The formulas were worked out by Aganagic, Popescu, and JHS in 1996 (and independently by Cederwall et al. and by

Bergshoeff and Townsend). The idea is to embed the D-brane in superspace $(X^\mu,\ \theta_1^a,\ \theta_2^a)$, where $(\theta_1,\ \theta_2)$ are MW spinors.

The global $\mathcal{N} = 2,\ D = 10$ susy is realized on superspace in the usual way ($\delta\theta = \epsilon$, $\delta X^\mu = \bar{\epsilon}\,\Gamma^\mu\theta$). The D-brane action is constructed out of the susy invariants

$$\Pi^\mu_\alpha = \partial_\alpha X^\mu - \bar{\theta}\Gamma^\mu\partial_\alpha\theta$$

and $\partial_\alpha\theta$. In order that the θ's give the desired number of fermions, half must be compensated by a local fermionic symmetry called *kappa symmetry.*

The requirements of global susy and local kappa symmetry determine the action. One finds

$$S_{DBI} = T_p \int d^{p+1}\sigma\sqrt{-\det(G_{\alpha\beta} + 2\pi\alpha'\mathcal{F}_{\alpha\beta})},$$

where

$$G_{\alpha\beta} = \eta_{\mu\nu}\Pi^\mu_\alpha\Pi^\nu_\beta$$

$$\mathcal{F}_{\alpha\beta} = F_{\alpha\beta} - B_{\alpha\beta} - b_{\alpha\beta}.$$

Here B is the pullback of the NS-NS 2-form background field and b is a two-form involving the fermi fields, which in the IIA case is

$$b = -\bar{\theta}\Gamma_{11}\Gamma_\mu d\theta \wedge (dX^\mu + \frac{1}{2}\bar{\theta}\Gamma^\mu d\theta).$$

There is also a complicated CS term.

1.3 Static Gauge

Let us focus on the $p = 9$ case, since the formulas for $p < 9$ can be inferred by dimensional reduction. As before, the local diffeomorphism symmetry is used to identify the embedding functions X^μ with the world volume coordinates σ^α. In addition, the local kappa symmetry is used to eliminate half of the θ coordinates. A simple choice that preserves the manifest 10d covariance is to simply set $\theta_2 = 0$. This has the remarkable consequence of completely eliminating the Chern–Simons term.

Renaming $\theta_1 = \lambda$ and setting $2\pi\alpha' = 1$ leaves

$$\int d^{10}\sigma\sqrt{-\det(\eta_{\alpha\beta} + F_{\alpha\beta} - 2\bar{\lambda}\Gamma_\alpha\partial_\beta\lambda + \bar{\lambda}\Gamma^\rho\partial_\alpha\lambda\bar{\lambda}\Gamma_\rho\partial_\beta\lambda)}.$$

This is the $\mathcal{N} = 1$, $D = 10$ super-Maxwell theory supplemented by higher-dimension interaction terms.

In addition to the 16 linearly realized supersymmetries of the free theory there are 16 additional nonlinearly realized supersymmetries. That is why this formula is reminiscent of the Volkov-Akulov action – λ can be interpreted as the Goldstone field for the broken supersymmetries.

When this D9-brane action is dimensionally reduced to give the Dp-brane action:

- 16 supersymmetries and $p + 1$ translation symmetries are linearly realized

• 16 supersymmetries and $9-p$ translation symmetries are nonlinearly realized and correspond to Goldstone modes on the world volume

In the standard realization of nonlinear supersymmetry, the Goldstino transformation rule is

$$\delta\lambda = \eta + \xi^\alpha \partial_\alpha \lambda,$$

where

$$\xi^\alpha = \bar{\eta}\Gamma^\alpha \lambda,$$

and every other field undergoes an induced general coordinate transformation with infinitesimal parameter ξ^α. This is not the rule for the action given here, however. This rule is achieved by a different gauge choice, namely $\theta_1 = \theta_2$. The action in this gauge is much more complicated, because the Chern–Simons term does not vanish. The actions corresponding to the two different gauge choices are related by a complicated field redefinition.

1.4 $\mathcal{N}=1$, $D=4$ Analog

We have seen that the super D9-brane action can be interpreted as a nonlinear extension of supersymmetric Maxwell theory in ten dimensions that incorporates a second nonlinearly realized supersymmetry. It is natural to ask whether there is an analogous construction in four dimensions (Bagger and Galperin 1996, Rocek and Tseytlin 1999). In this case one can use superfields to make the linear supersymmetry manifest. The result is remarkably simple, when properly formulated.

$$S = \int d^4x\, d^2\theta\, \Phi(W, \bar{W}) + \text{h.c.}$$

Here Φ is a chiral superfield given in terms of the field strength superfield W_α by

$$\Phi = \frac{1}{2}\Phi \bar{D}^2 \bar{\Phi} + \frac{1}{2} W^\alpha W_\alpha.$$

This equation can be solved explicitly. The nonlinear supersymmetry is given by

$$\delta\Phi = -\eta^\alpha W_\alpha,$$

which implies the invariance of the action. One can work out the corresponding transformation of the vector superfield. I think it would be interesting to describe the ten-dimensional theory in a similar manner.

2 Non-Abelian Generalizations

The world-volume theory of N coincident Dp-branes is a $U(N)$ gauge theory. As such, it must be a non-Abelian generalization of the formulas of the preceding section. The explicit construction of such an action is a difficult problem that has been studied extensively, but is not yet completely settled.

Tseytlin (1997) proposed a specific recipe for generalizing Abelian formulas to non-Abelian ones. His proposal was to resolve ordering ambiguities by a *symmetrized trace prescription.* Studies by Hashimoto and Taylor (1997) and others suggest that this is a correct rule through terms of order F^4, but that it fails at higher orders.

Part of the rationale for Tseytlin's symmetrized trace prescription is that a field strength commutator

$$[F_{ij}, F_{kl}] \sim [D_i, D_j]F_{kl}$$

can be regarded as being higher-order in derivatives. This reflects an inherent ambiguity in the meaning of "slowly varying fields" in the non-Abelian case. It is an important question whether or not there is a well-defined counterpart of the slowly-varying field expansion in the non-abelian case. If not, the next meaningful approximation beyond super Yang–Mills theory would be the complete effective action for superstring amplitudes given by disk world-sheets.

Myers (1999) discovered an interesting part of the answer by exploring consistency with T duality. He worked in the static gauge and focused on the dependence on the bosonic fields A_α and X^i; each of which are now $N \times N$ matrices. He included the dependence on B and C background fields. For the Chern–Simons term he obtained the result

$$S_{CS} = T_p \int \mathrm{Str}\left(P[e^{iI_X I_X} C e^B] e^F\right).$$

This is a subtle formula that requires some explanation. First of all $C = \Sigma C^{(n)}$, as before. $P[\ldots]$ means the pullback to the world volume, since B and C are bulk fields. X refers to the $9-p$ scalars X^i, which are now $N \times N$ matrices. The operation $I_X I_X$ acting on an n-form gives an $(n-2)$-form. For example,

$$I_X I_X C^{(2)} = X^j X^i C^{(2)}_{ij} = \frac{1}{2} C^{(2)}_{ij} [X^j, X^i]$$

Moreover, in the pullback of a function $f(x^\alpha, x^i)$, the matrices X^i need to substituted for the bulk coordinates x^i. This requires an ordering prescription, since $[X^i, X^j] \neq 0$. The proposed formula is

$$P[f] = \exp(X^i \frac{\partial}{\partial x^i}) f(\sigma^\alpha, x^i)|_{x^i=0}.$$

A crucial feature of this formula is that multi D-brane systems can be sources of higher D-brane charge as well as lower D-brane charge, since all the RR fields appear. This is to be contrasted with the Abelian case where $(Ce^F)_{p+1}$ only depends on $C^{(p+1)}, C^{(p-1)}, \ldots$.

Myers discovered a *dielectric effect* in which a background RR field strength can cause the brane to expand into new dimensions. For example, a system of N D0-branes in the presence of an electric $F^{(4)} = dC^{(3)}$ becomes a fuzzy two-sphere with $[X^i, X^j] \sim N\epsilon^{ijk}X^k$. For large N this describes an ordinary S^2 with radius proportional to N. This can be interpreted as a spherical D2-brane with N D0-branes bound to it. The Myers effect is relevant to the appearance of "giant gravitons" on the AdS side of the AdS/CFT correspondence (McGreevy, Susskind, and Toumbas).

3 Gravitational Analog of Born–Infeld?

The Born-Infeld theory arose as the effective field theory description of open-string amplitudes in the tree-level (disk world-sheet) approximation. The result turned out be informative for the strong field behavior of the theory. It is natural to wonder whether analogous formulas can be constructed for closed strings. They could tell us about the implications of classical string theory for strong gravitational fields. It would be very interesting to know, for example, how the Schwarzschild metric is modified.