What is a SUGRA theory?

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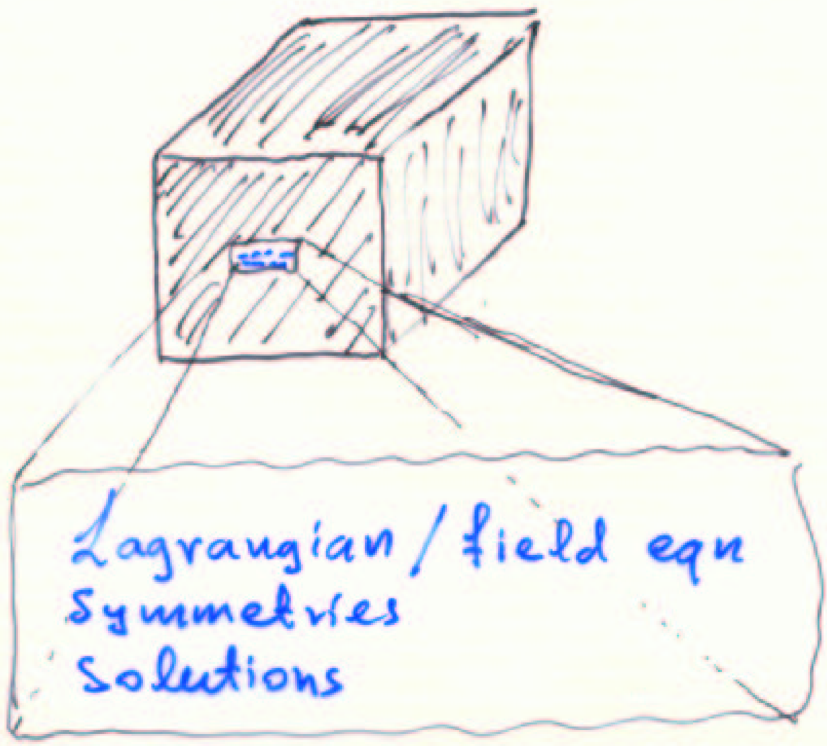
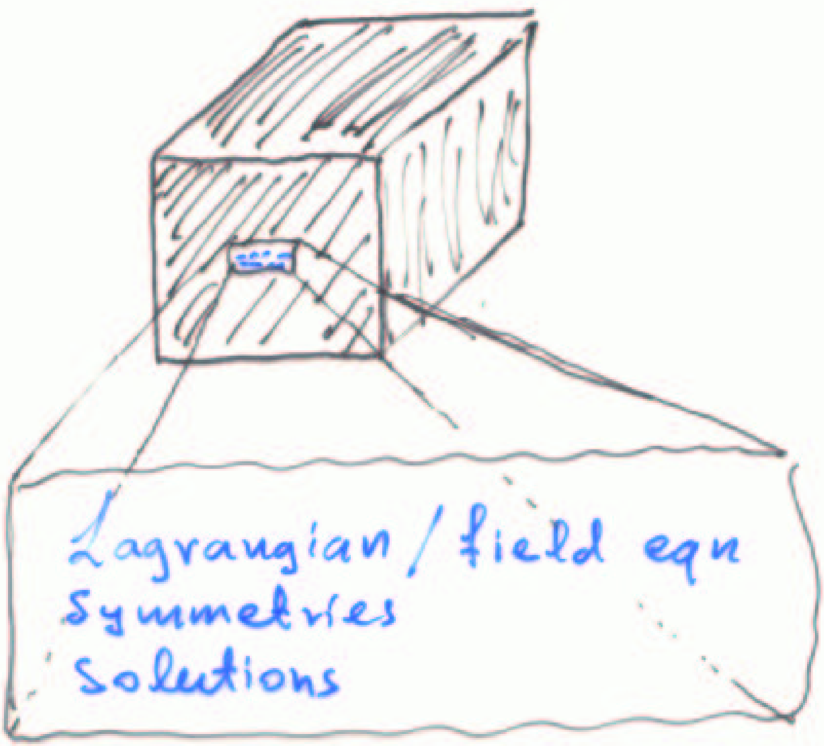


What is a SUGRA theory?

Poincaré SUGRA



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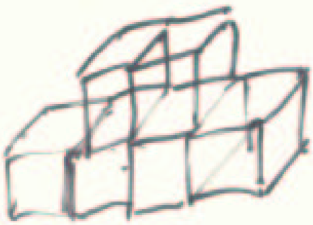
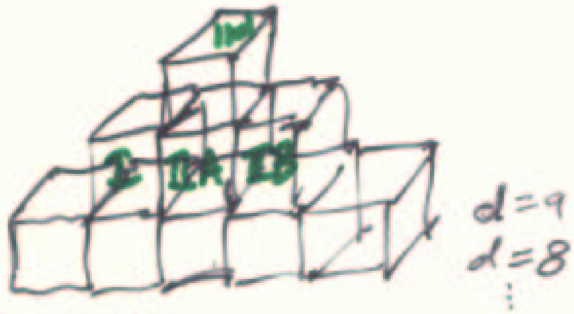
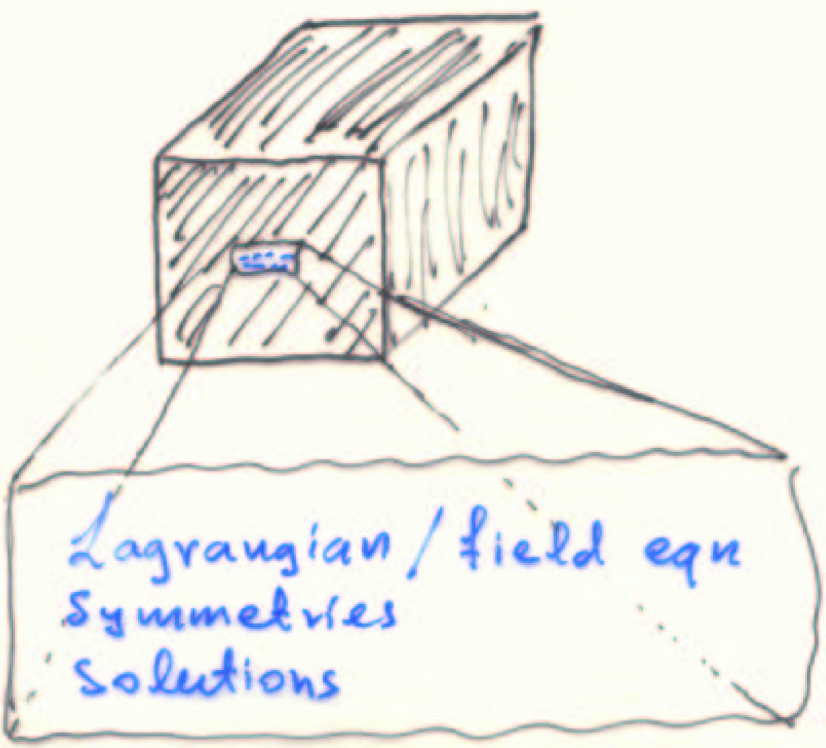


Gauged SUGRA



Poincaré SUGRA



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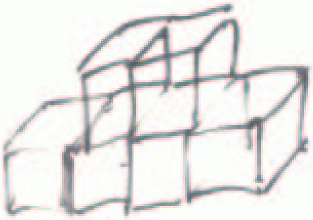
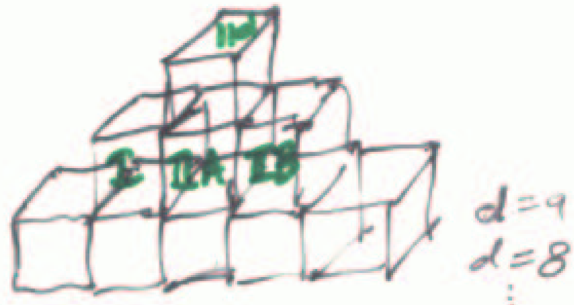
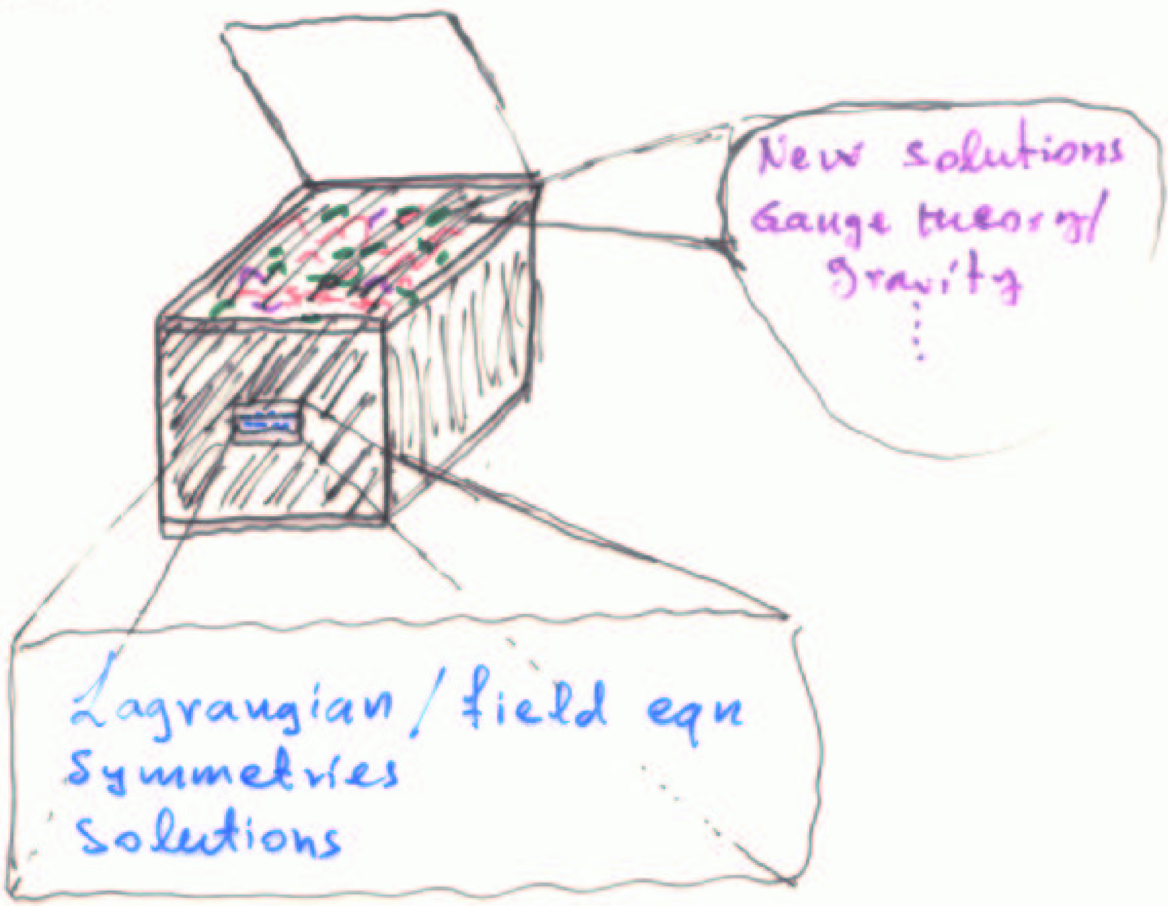


Gauged SUGRA



Poincare' SUGRA



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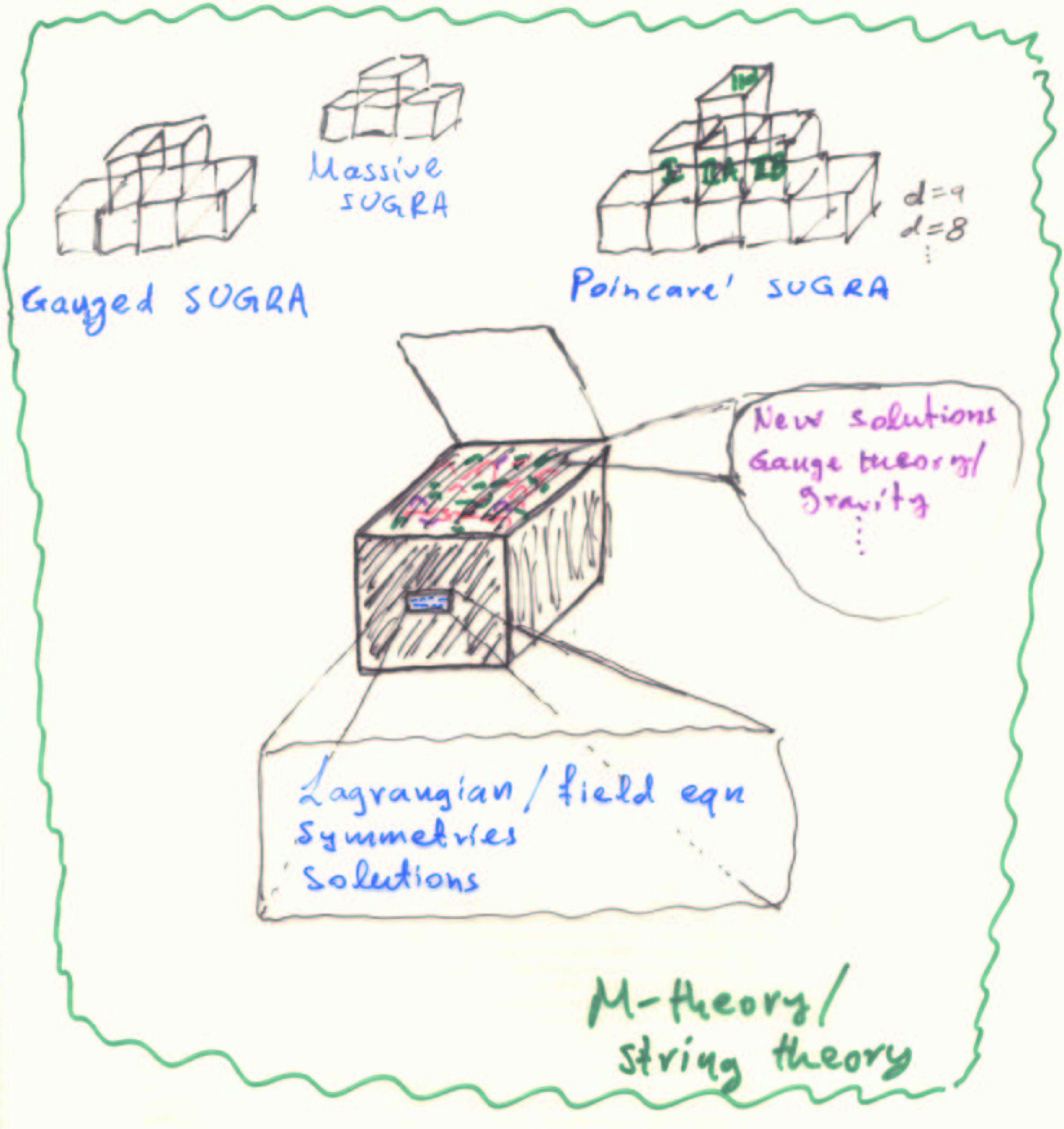

String theory corrections to p-brane solutions

K. Skenderis
Princeton University

based on work with

S. de Haro (UCLA)
A. Sinkovics (Swansea)

hepth/0112???

Outline

1. Introduction/Motivation
2. Higher-derivative terms in IIB strings
3. Corrections to D3 brane
4. First order eqns
5. Mass, charge / thermodynamics
6. Conclusions

Introduction / Motivation

Supergravity solutions have played instrumental role in most recent developments

- Existence of certain solutions is required by and motivates string dualities

- Black hole solutions can be constructed by "superimposing" elementary p-brane solutions
  → microscopic description of black holes

- Near-horizon limit of solutions leads to various
  gravity / gauge theory dualities

10d supergravity theories provide the leading terms in the low-energy expansion of the corresponding string theory effective action.

Worldsheet and string loops give rise to higher-derivative terms in the low energy effective action

Supergravity solutions that describe the long-range field of stringy configurations (such as collections of D-branes, fundamental strings, NS5 etc) should exist beyond the leading supergravity approximation.

Objective of this work:

Compute the leading corrections to p-brane solutions due to higher derivative terms

What will we learn?

- Check duality symmetries at subleading orders. Compute effects of quantum corrections to duality transformations

- Compute string corrections to black hole solutions

  - Both the solutions and the Bekenstein-Hawking entropy formula are expected to receive corrections.

  Providing a microscopic derivation of corrections will test our understanding of BH physics

- Compute corrections to near-horizon solutions

  The corrections carry information about subleading orders in N and $g_{YM}^2 N$.

Higher-derivative terms in IIB

The low-energy effective action can be computed by

- σ-model β-function computations
- string scattering amplitudes

In IIB strings, the leading corrections appear at order α'^3

$$S = S_{SUGRA} + \alpha'^3 S_8$$

- 8-derivative term
- 4-loop effect in σ-model computations
- 4 and higher-point function scattering amplitudes

S_8 is constrained by symmetries

- supersymmetry
- $SL(2,\mathbb{Z})$
- world-sheet parity

Neither the 4-loop nor the scattering amplitude computations have been done in full generality. One needs to use symmetry considerations to obtain the full set of terms

4-pt graviton scattering and 4-loops beta function computations lead to

$$\mathcal{L}_8 \sim \zeta(3)\, \alpha'^3 \left(t_8 t_8 + \frac{1}{8} \epsilon_{10} \epsilon_{10}\right) C^4$$

Gross, Witten
Gross, Sloan
⋮
Grisaru, Zanon
Freeman, Pope, Sohnius, Stelle
Grisaru, Van de Ven, Zanon
⋮

C_{ijkl} Weyl tensor

t_8 specific combination of δ_i^j

4pt - NS scattering amplitudes

$$C \longrightarrow C + \alpha_1 \nabla H_3 + \alpha_2 \nabla^2 \phi$$

One can incorporate **linearized** susy by utilizing IIB on-shell superspace

[Howe, West]

$$D^* \Phi = 0$$

$$D^4 \Phi = D^{*4} \Phi$$ → linearized IIB field equs

$$\Phi = \tau + i\,\bar\theta^* \lambda + \dots + C_{ijkl}\,\bar\theta^* \gamma^{ikp}\theta\,\bar\theta^* \gamma^{jl}{}_p \theta$$
$$+ \partial_\mu F_{5\,\nu\rho\sigma\tau\omega}\,\bar\theta^* \gamma^{\mu\nu\rho}\theta\,\bar\theta^* \gamma^{\sigma\tau\omega}\theta$$
$$+ \dots + \theta^8 \partial^4 \bar\tau$$

$$S_8 \sim \zeta(3)\,\alpha'^3 \int d^{10}x\, d^{16}\theta \left(\sum_n c_n \Phi^n\right)$$

$c_n(\tau_0)$ fixed by non-linear susy to be specific non-holomorphic modular forms

[Green, Gutperle

⋮

Green, Sethi

⋮

To compute corrections to solutions we need to know the full set of bosonic terms that appear to this order.

To obtain them one may evaluate the fermionic part of the superspace integral. This computation is in progress ...

In this talk I will only discuss corrections to the D3 brane solution.

Since the lowest order solution involves only g_{ij}, $F_{(5)}$ and has constant dilaton we only need to obtain terms that contain R and $F_{(5)}$.

Terms allowed by symmetries are

$$C^4 \qquad C^2 (DF_5)^2 \qquad (DF_5)^4$$

Terms with (DF_5)-dependence have not been computed previously.

To obtain these terms we need to compute

$$S_8 \sim \int d^{10}x \, d^{16}\theta \;\; \Phi^4$$

In the light-cone gauge the result is

$$S_8 \sim \left(t_8 t_8 - \tfrac{1}{4} \epsilon_8 \epsilon_8 \right) \bar{C}^4$$

$$\begin{aligned} \bar{C}_{ijkl} = \; & C_{ijkl} \\ & + \alpha_3 \left[\nabla_{[i} \epsilon_{j]klmnpqr} F_{(5)}{}^{mnpqr} \right. \\ & \qquad \left. - (ij) \leftrightarrow (kl) \right] \end{aligned}$$

Equations of motion

$$E_{ij} \equiv R_{ij} - \frac{1}{2} g_{ij} R - \frac{1}{2}\left[\partial_i \phi \partial_j \phi - \frac{1}{2} g_{ij} (\partial\phi)^2\right]$$
$$- \frac{1}{96} F_{i l_1 \ldots l_4} F_j{}^{l_1 \ldots l_4} + \alpha'^3 \left[\frac{\delta \mathcal{L}_8}{\delta g_{ij}} - \frac{1}{2} g_{ij} \mathcal{L}_8\right] = 0$$

$$E \equiv D\phi + \alpha'^3 \frac{\delta \mathcal{L}_8}{\delta \phi} = 0$$

$$E^{l_1 \ldots l_4} \equiv D_l F_{(5)}{}^{l l_1 \ldots l_4} + \alpha'^3 \frac{\delta \mathcal{L}_8}{\delta C_{(4) l_1 \ldots l_4}} = 0$$

$$\mathcal{L}_8 = C^4 + C^2 (D F_5)^2 + (D F_5)^4$$

- In most previous works

$$C^4 (\text{lowest order solution}) = 0$$

- When $\mathcal{L}_8 \neq 0$ (more precisely $\frac{\partial \mathcal{L}_8}{\delta C} \neq D_l F^{l l_1 \ldots l_4} f(R,F)$)

$$F_5 (C_4 + \alpha'^3 C_4^{(1)}) \neq * F_5 (C_4 + \alpha'^3 C_4^{(1)})$$

We look for perturbative solutions

$$g_{ij} = g^{(0)}_{ij} + \alpha'^3 g^{(1)}_{ij}$$

$$\phi = \phi_{(0)} + \alpha'^3 \phi_{(1)}$$

$$C_{(4)ijkl} = C^{(0)}_{4\,ijkl} + \alpha'^3 C^{(1)}_{4\,ijkl}$$

We consider spherically symmetric solutions of the form

$$ds^2 = e^a\left[-f\,dt^2 + d\vec{x}^2 + e^h\left(\frac{1}{f}dr^2 + r^2 d\Omega_5^2\right)\right]$$

The lowest order D3-brane solution is

$$e^{-2a_0} = e^{h_0} = 1 + \frac{\ell^4}{r^4} \qquad \ell^4 = 4\pi g_s N \alpha'^2$$

$$e^{\phi_0} = g_s \qquad f = 1$$

$$F_{tabcr} = \epsilon_{abc}\, e^{-h_0} r^{-5}\, 16\pi N \alpha'^2$$

$$F_{m_1 \dots m_5} = \epsilon_{m_1 \dots m_5}\, 16\pi N \alpha'^2$$

Effective 1d action

For spherically symmetric solutions one can derive an effective 1d action whose variation gives the field equs evaluated on the ansatz

$$S_1 = \int dr \sqrt{g_5}\Big[R_5 - \frac{1}{2} g_5^{rr} (\partial_r \phi)^2 - \frac{40}{3} g_{5s}^{rr} (\partial_r v)^2 - V(r) + \alpha'^3 e^{-\frac{10}{3} v} \mathcal{L}_8 \Big]$$

$$ds_5^2 = e^{\frac{1}{3}(8a+5h)} \left(\frac{r}{\ell}\right)^{\frac{10}{3}} \left(-f dt^2 + d\vec{x}^2 + \frac{\ell^4}{f} dr^2\right)$$

$$v(r) = \frac{1}{2}(a+h) + \log \frac{r}{\ell}$$

$$V(r) = \frac{1}{\ell^2}\left[8 e^{-\frac{40}{3} v} - 20 e^{-\frac{16}{3} v} \right]$$

[Gubser, Klebanov, Tseytlin]

This potential admits a stable AdS_5 solution

$\Downarrow$

[Townsend
Skenderis, Townsend]
DeWolfe et.al.

$$V(v) = \frac{3}{10}\left[\left(\frac{\partial W_\pm}{\partial v}\right)^2 - \frac{16}{3} W_\pm^2 \right]$$

$$W = \frac{1}{\ell}\left[e^{-\frac{20}{3} v} \pm \frac{5}{2} e^{-\frac{8}{3} v} \right] \Longrightarrow \text{first order equs}$$

$$a' + h' + \frac{2\ell^4}{r^5} e^{-2(a+h)} = 0$$

$$4a' + \frac{5}{2}h' + \frac{2\ell^4}{r^5} e^{-2(a+h)} = 0$$

$$\phi' = 0$$

Equivalent to Killing spinor eqns

[Duff, Liu]

Include α'^3-terms

$$a' + h' + \frac{2\ell^4}{r^5} e^{-2(a+h)} + \alpha'^3 j_1 = 0$$

$$4a' + \frac{5}{2} h' + \frac{2\ell^4}{r^5} e^{-2(a+h)} + \alpha'^3 j_2 = 0$$

$$\phi' + \alpha'^3 j_3 = 0$$

Equivalent to Killing spinor equns (?)
[Duff, Liu]

Requiring that the 2^{nd} order equations follow from the first order ones yields

$$j_1 = -2\left(1 + \frac{10}{r h_0'}\right) b_1 + \frac{1}{2h_0'} (w_h + w_f - w_a)$$

$$j_2 = -5\left(1 + \frac{10}{r h_0'}\right) b_1 + \frac{1}{2h_0'} (w_h + w_f - w_a)$$

$$j_3 = -\frac{1}{r^5} \int^r dr' r'^5 w_\phi + \frac{c_1}{r^5}$$

where $\quad b_1' + \frac{9}{r} b_1 = \frac{1}{10} (w_f - w_a)$

Integration constants are to be fixed by requiring regularity at horizon.

Mass, charge and thermodynamics

- $Q \sim \int_{S^5_\infty} * F_{(5)} \sim$ integer $\Rightarrow$ cannot renormalize

- In supersymmetric solutions the mass (tension) is directly related to the charge.

$$M \sim Q$$

$\Rightarrow$ mass (tension) cannot renormalize

- Smarr formula

$$TS = M - \phi Q$$

For extremal solutions: $T=0$

$$\Rightarrow \quad M = \underbrace{\phi}_{\text{const.}} Q$$

Consider the spacetime described by the D3-brane solution. There is a global timelike Killing vector

$$\xi = \xi^i \frac{\partial}{\partial x^i} \qquad \xi^t = 1 \quad \xi^i = 0 \quad i \neq t$$

in this spacetime.

The mass of the solution is given by the Komar mass formula

$$M = \frac{1}{8\pi G} \int_{S^5_\infty \times T^3} * d\xi$$

Using Stokes' thm we can rewrite this as

$$M = -\frac{1}{4\pi G} \int_{\Sigma} * R(\xi) + \underbrace{\frac{1}{8\pi G} \int_{\mathcal{H} \times T^3} * d\xi}_{\sim A \text{ of horizon} \; = 0 \text{ for D3}}$$

$(R(\xi) = R_{ij}\xi^j)$

$\partial\Sigma = S^5_\infty \times T^3 \cup \mathcal{H} \times T^3$

$\Downarrow$ Einstein's eqns

$$M = -\frac{1}{32\pi G}\int_{\Sigma}(E\wedge * F + B\wedge F)$$

$$= -\frac{1}{32\pi G}\int_{\partial\Sigma}\left[(\phi\wedge * F) + (\psi\wedge F)\right]$$

$$E_{l_1\cdots l_4} = \xi^{i_1}F_{i_1 l_1\cdots l_4} \quad ; \quad i_\xi F = E = d\phi \Rightarrow \boxed{\phi = \xi^i A_{i i_2 i_3 i_4}}$$

$$i_\xi * F = B = d\psi$$

Since F is SD

$$M = -\frac{1}{16\pi G}\int_{S^5_\infty\times T^3}\phi(*F)$$

$$\boxed{M \sim \underbrace{(\xi^i A_{i i_2 i_2 i_3})}_{\phi}\Big|_{\infty} Q}$$

In the lowest order D3-solution : $\phi = 1$

$Q = \int * F_5$ does not renormalize

metric , $*$ renormalize

$\Downarrow$

$F_5 \sim d A_{i_1 \ldots i_4}$ renormalize

$\Downarrow$

$\phi (= \xi \cdot A)$ renormalize

$\Downarrow$

$M \; (= \phi Q)$ renormalize !

Inconsistent with SUSY!

$Q = \int * F_5$ does not renormalize

metric , $*$ renormalize

$\Downarrow$

$F_5 \sim d A_{i_1 \cdots i_4}$ renormalize

$\Downarrow$

$\phi (= \xi \cdot A)$ renormalize

$\Downarrow$

$M \; (= \phi Q)$ renormalize !

Inconsistent with SUSY!

Higher derivative terms modify both the mass formula and the formula for the potential.

The formula for the potential receives a contribution from the corrections to SD condition.

Still in progress ...

Summary / Future Directions

We have started a systematic investigation of string corrections to p-branes.

- Results for D1-D3-D5 under way
  F1-NS5
- Work out corrections for IIA and M-branes
- Non-extremal branes
- Intersecting branes
- Black holes

⋮

HAPPY BIRTHDAY

SG!