Gauged Supergravity and Holographic RG Flows

Stonybrook, December, 2001

Based upon:
- hep-th/9904017; hep-th/9906194; hep-th/004063
Freedman, S. Gubser, K. Pilch and NPW
- Recent work with R. Corrado, M. Gunaydin and M. Zagermann to appear

Overview

- Teaching an old dog new tricks: The AdS/CFT Correspondence
- Holomorphic Description of RG Flows
- Open problems in holographic field theory and supergravity
  - Half-maximal supersymmetric models
    $\mathcal{N}=2$ Quiver Gauge theories $\leftrightarrow \mathcal{N}=4$ supergravity + tensor multiplets
  - Half-maximal supersymmetric flows
    $\mathcal{N}=2$ Seiberg-Witten flows
  - Supersymmetric AdS geometries
- Other issues
D3-branes in IIB Supergravity

$\mathcal{N}=4$ Yang-Mills in strongly coupled, Large $N$ limit

Operating on the brane

Correlators and states

Sources in Supergravity

Classical supergravity solutions

Five-dimensional perspective:

- View $S^5$ as an internal space
- Decompose theory into generalized Fourier modes on $S^5$
- Space-time = D3-branes + radius, $r$
- Interpret as an RG scale for the theory on brane
Interpreting the metric

\[ ds_{10}^2 = \Omega^2(y) \left( e^{2A(r)} \eta_{\mu\nu} \, dx^\mu \, dx^\nu + dr^2 \right) + ds_5^2(y) \]

- **Warpfactor**
- **Physical scale on branes**
- **D3-brane**
- **Internal**

- **IR Limit**
- **Radius, r**
- **UV Limit**

- **Localize D3-branes at infinity**
  - Localize D3-branes finite

- **Conformal, UV limit**
  - Explicit cut-off

- **Redshifts**

- **Cosmological Entropy**

- **No hair theorems**

- **Gauged supergravity potentials**

- **Consistent truncation**

- **Phasediagrams/flows from relevant perturbations**

- **Universality in IR limit**

- **Large N structure of OPE in E.M. supermultiplets**

- **总局域D3-膜在无穷远处**
  - 在有限的D3-膜

- **Conformal, UV limit**
  - 显式切-off
I. Perturb the $\mathcal{N}=4$ theory:  

$$\left( A_\mu, \lambda^1, ..., \lambda^4, x^1, ..., x^6 \right)$$

Freedman, Gubser, Pilch and Warner, hep-th/9901017

Given as to an $\mathcal{N}=1$ chiral multiplet: 

$$W = W_0 + m \text{Tr}(\Phi^2)$$

$$\Delta \mathcal{L} = m_1 \text{Tr}(\lambda^1 \lambda^1) + m_2 \text{Tr}(x^1)^2 + (x^2)^2); \quad m_2 = m_1^2 = m^2$$

Non-trivial $\mathcal{N}=1$ fixed point 

(Leigh and Strassler) for flow was $m \to \infty$

\[ \frac{C_{\text{IR}}}{C_{\text{UV}}} = \frac{27}{32} \]

**Supergravity:** steepest descent on a supergravity superpotential, $W$:

II. Perturb an $\mathcal{N}=2$ superconformal quiver theory:

Klebanov and Witten, hep-th/9807080

Gubser, Nekrasov and Shatashvili, hep-th/9811230

\[ \frac{C_{\text{IR}}}{C_{\text{UV}}} = \frac{27}{32} \]
$\mathcal{N}=2,(SU(N))^p$ Yang-Mills
+ bi-fundamental matter

$Z_p$ Orbifold of pN D3-branes

Superconformal with $R$-symmetry = $SU(2) \times U(1)$

**Untwisted sector:** Vector multiplets

\[
\begin{pmatrix}
A^{(k)}_{\mu}; & \lambda_1^{(k)}, & \lambda_2^{(k)}, & \phi^{(k)} = X_1^{(k)} + iX_2^{(k)}
\end{pmatrix}
\]

$k=1,\ldots,p$

**Twisted sector:** Hypermultiplets

\[
\begin{pmatrix}
\psi_1^{(k)}, & \psi_2^{(k)}; & A^{(k)}, & B^{(k)}
\end{pmatrix}
\]

**Mass terms/flow:**

\[\Sigma_k m_{(k)} (\phi^{(k)})^2\]

**Klebanov and Witten:**

\[m \left( (\phi^{(1)})^2 - (\phi^{(2)})^2 \right) \quad p=2, \ Z_2 \text{ odd}\]

**Freedman, Gubser, Pilch and Warner:**

\[m \left( (\phi^{(1)})^2 + (\phi^{(2)})^2 \right) \quad Z_2 \text{ even}\]

Leigh-Strassler fixed points
**Five-dimensional, gauged supergravity and quiver flows**

Gauged $\mathcal{N}=4$ supergravity + 

**Vectoror Tensor multiplets:**

Dall'Agata, Herrmann and Zagermann, hep-th/0103106

**R-symmetry on the brane =**

**Gauge group:** $SU(2) \times U(1)$ (supergravity)

\[
\begin{align*}
\mathbf{m}^{ab}_{(k)} &\quad \lambda_a^{(k)} \lambda_b^{(k)} + \text{c.c.} &\quad 3(+2)+3(-2) \\
\mathbf{m}^{(k)} &\quad (\phi^{(k)})^2 + \text{c.c.} &\quad 1(+4)+1(-4) \\
\mathbf{T}_{(k)} &= \frac{4\pi i}{g_{(k)}^2} + \frac{\theta_{(k)}}{2\pi} &\quad 1(0)+1(0)
\end{align*}
\]

Scalar content of a pair of five-dimensional, charged tensor multiplets

**Scalar Manifold (Supergravity) \iff Couplings in Gauge theory**

\[
\begin{align*}
\text{SO}(1,1) \times \text{SO}(5,2p) &\supset \text{SO}(5) \times \text{SO}(2p) \\
\text{SO}(1,1) \times \text{SU}(p,1) &\Rightarrow \text{SU}(2)_R \times \text{U}(1)_R \times \text{SU}(p,1)
\end{align*}
\]

Generalizes:

$SU(1,1)/U(1)$ of IIB supergravity \iff $N=4$ Yang-Mills coupling, $T$

= Non-compact $CP_p$ parametrizing couplings: $T_{(k)}$

Invariance of supergravity potential

$\text{SU}(p,1)/\text{SU}(p) \times U(1)$
**Unexpected Consequences: SU(p) Symmetry**

Untwisted scalar sector in $\mathcal{N}=8$ supergravity

$+ \quad \mathcal{N}=4$ supergravity + two charged tensor multiplets

Twisted sector scalars from $2(p-1)$ additional tensor multiplets

$\xrightarrow{\text{Untwisted scalar sector}} \quad \begin{array}{c} \text{SO}(5,2) \\ \text{SO}(5) \times \text{SO}(2) \end{array}$

$\xrightarrow{\text{Twisted sector scalars}} \quad \begin{array}{c} (p-1) \times (5,2) \text{of} \\ \text{SO}(5) \times \text{SO}(2) \end{array}

**Flow of FGPW:**

$m \left( (\Phi^{(1)})^2 + (\Phi^{(2)})^2 \right)$

lies entirely in scalarsof untwisted sector

**Flow of KW:**

$m \left( (\Phi^{(1)})^2 - (\Phi^{(2)})^2 \right)$

lies entirely in scalarsof twisted sector

**SU(p) Symmetry**

Critical point of FGPW extends to a $\text{SU}(p)$ symmetric family of IR fixed points parametrized by $\Sigma_k m_k \left( \Phi^{(k)} \right)^2 \xrightarrow{\text{to smooth}} \text{SU}(p)$

considered as homogeneous coordinates on $\mathbb{C}P^{p-1}$.

**IIB Supergravity:**

- FGPW flow: topologically trivial flux on $S^5/\mathbb{Z}_2$
  $\xrightarrow{\text{U}(p)}$
- KW flow: topologically non-trivial flux on blow-up of $A_1$ singularity on $S^5/\mathbb{Z}_2$
Family of equivalent fixed points with same central charge: parametrized by initial “velocities”, \( m(k) \), off now.

This is a “large \( N \) result: ”the \( U(p) \) symmetry is broken by anomalies at finite \( N \).

Residual Symmetry: \( U(p) \supset G \supset (Z^{2N})^p \)

Discrete symmetry at finite \( N \) between FGPW and KW flow?

Singular behavior at finite \( N \) when one or more \( m(k) \) vanish?

Does consistent truncation work for \( \mathcal{N}=4 \) supergravity?

Unknown territory...

We have solutions in both five and ten dimensions.

- Solutions coincide at linearized level
- Same initial velocities
- Same supersymmetry
- Same \( R \)-symmetry

Try to find the \( SU(2) \) family of solutions in IIB supergravity that interpolates between topologically trivial flux on \( S^5/Z_2 \) and topologically non-trivial flux on the blow-up of an \( A_1 \) singularity on \( S^5/Z_2 \).

Corrado, Pilch and Warner: work in progress

Another duality that attracts 3-form flux for Kähler moduli....
Open Problems: Supersymmetric $RR$-Geometries

Geometry of supersymmetric solutions with background $RR$-fluxes?

- Ricci-flat manifolds $+ \mathcal{N}=2$ supersymmetry: Kähler
- Ricci-flat manifolds $+ \mathcal{N}=4$ supersymmetry: Hyperkähler

What is the analogue of this for holographic RG flows?

The issue for flow solutions in IIB supergravity:

- The flows generally start from AdS$_5$, with a background 4-form $RR$-tensor field
- Softly broken $\mathcal{N}=4$ Yang-Mills: fermion masses are holographically dual to 2-form tensor field components
- The dilaton and axion are dual to the Yang-Mills gauge couplings, and thus generically run.

Interesting supersymmetric flow solutions involve fluxes for all the background tensor fields.

Classification theorems for such solutions: very few, and none of them relevant to the important physical holographic flow solutions.

Important Example: (unsolved)

Find the Holographic Dual of the $\mathcal{N}=2$ Seiberg-Witten effective action
Simplest version:

\[ N=4 \text{Yang-Mills} \quad \rightarrow \quad N=2 \text{Yang-Mills} + \text{massive hypermultiplet} \]

Coulomb branch:

\[ u_n = \text{Tr} \ (\phi^n) \]

D3-brane distribution

Source:

\[ \rho(y) = \sum u_n y^n \]

Complex scalar, \( \phi \)

Complex coordinate, \( y \)

Problem: Find the general \( N=4 \) supersymmetric flow solution determined by an arbitrary source function, \( \rho(y) \), of two variables.

What is known: The solution for one point in the moduli space

\[ \rho(y) = (a^2 - |y|^2)^{1/2} \]

A uniform disk-like distribution of D3-branes

Pilch and Warner; hep-th/0006066
Buchel, Peet and Polchinski; hep-th/0008076
Evans, Johnson and Petrini; hep-th/0008081

The general \( N=8 \) supersymmetric flow solution (the Coulomb branch of \( N=4 \) Yang-Mills) is well-known, and is determined by an arbitrary harmonic source function, \( H(y) \), of six variables. The half-maximal supersymmetric solutions shouldn’t be much more difficult.

Harder problem: Find the general \( N=4 \) supersymmetric flow solution for the quiver gauge theories

Holography and the \( N=2 \) Seiberg-Witten effective action

What is known: the solution for one point in the moduli space

\[ \rho(y) = (a^2 - |y|^2)^{1/2} \]

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Other Issues

- $\mathcal{N}=1$ supersymmetric flows: generalized Kähler structure in the presence of RR fluxes?
  - Brane probe results find the Kähler structure on the moduli space of the probe.
    - Johnson, Lovis, C. Page, hep-th/0107261

- Non-compact gaugings: Holographic interpretation???
  - Vacua: De-Sitter space $\rightarrow$ All supersymmetry broken
  - Domain wall solutions can be (half-maximally) supersymmetric
    - Hull, hep-th/0110048; Gibbons and Hull, hep-th/0111072

- Potentials without critical points:
  - Domain wall solutions...

- Most of the exactly known flows solutions have been obtained by lifting solutions of gauged supergravity
  - Systematic of lifting
  - General method of construction in 10 or 11 dimensions
Conclusions

- Gauged supergravity is a very valuable tool in the study of holographic RG flows.

- $\mathcal{N}=4$ gauged supergravity coupled to tensor multiplets can be used to study a class of $\mathcal{N}=1$ supersymmetric flows in large N, $\mathcal{N}=2$ quiver gauge theories.

- The large N, $\mathbb{Z}_p$ quiver models have a $(p-1)$-dimensional surface, $\mathbb{C}P^{p-1}$, of $\mathcal{N}=1$ supersymmetric fixed points, and all these fixed point theories are equivalent under the action of an SU(p).

- In ten dimensions, this SU(p) must act as a "generalized duality symmetry", mapping compactifications with fluxes on $S^5/\mathbb{Z}_p$ to compactifications with blow-ups of the $A_{p-1}$ singularity:

  Flux $\leftrightarrow$ Kähler Moduli

- Important open question in holographic RG flows:

  Geometry of supersymmetric compactifications with RR fluxes???