①

TACHYON DYNAMICS IN STRING THEORY

① STATIC PROPERTIES

② DYNAMICAL PROPERTIES

PLAN

LECTURE 1: GENERAL INTRODUCTION AND REVIEW OF RESULTS

LECTURES 2-5: DETAILS OF THE ANALYSIS LEADING TO THESE RESULTS

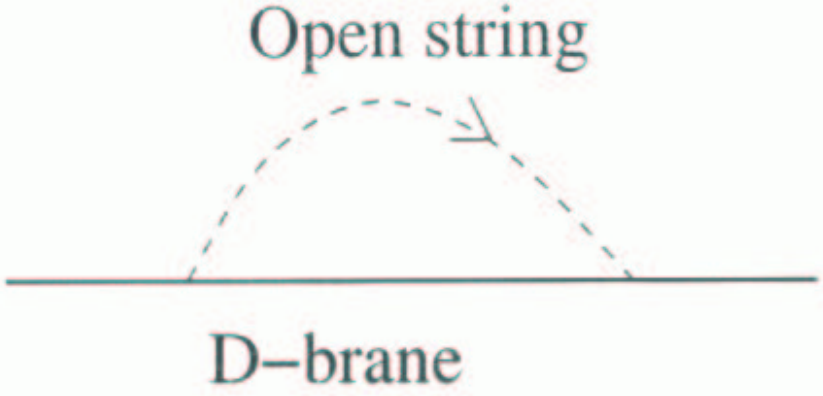
D-p-brane: p-dimensional object

D0-brane $\equiv$ particle, D1-brane $\equiv$ string,

D2-brane $\equiv$ membrane etc.

Consider static flat D-p-brane in flat space-time, lying along a p-dimensional subspace

Definition of a D-p-brane: Fundamental strings can end on a D-brane



Quantum states of a fundamental string with ends on a D-brane represent quantum excitation modes of the D-brane.

Spectrum of D-branes in type IIA/IIB string theory in (9+1) dimensions:

1. Oriented (BPS) D-p-brane for even/odd p

   Mass per unit p-volume (tension $\mathcal{T}_p$) $= \frac{1}{(2\pi)^p g_s}$

   (in $\alpha' = 1$ *i.e.* string tension $= \frac{1}{2\pi}$ unit)

   g_s: string coupling constant

   Anti-D-brane ($\bar{\text{D}}$-brane) $\equiv$ a D-brane with opposite orientation

   All open string states on a BPS D-brane or $\bar{\text{D}}$-brane have mass$^2 \geq 0$

2. Unoriented (non-BPS) D-p-brane for odd/even p

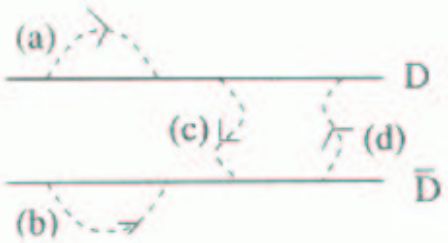
   Tension $\tilde{\mathcal{T}}_p = \frac{\sqrt{2}}{(2\pi)^p g_s}$

   Each such D-brane has one open string mode with mass$^2 = -\frac{1}{2}$

   $\rightarrow$ tachyonic mode

NOTE : IN THE ABSENCE OF A D-BRANE IIA/IIB HAS ONLY CLOSED STRING STATES

A coincident BPS D-p-brane $\bar{\text{D}}$-p-brane pair also has two tachyonic modes, each of mass^2 = $-\frac{1}{2}$ from the open strings with one end on the brane and one end on the antibrane (sectors (c) and (d)).



For a non-BPS D-brane, the dynamics of the tachyonic mode can be described by a real scalar field T with negative mass^2, coupled to infinite number of other massless and massive fields associated with other states of the string, in $(p+1)$-dimension.

$T \equiv$ the tachyon field

For the D-$\bar{\text{D}}$ system, the dynamics of these tachyonic modes is described by a complex scalar field T with negative mass^2, coupled to infinite number of other massless and massive fields.

In either case we can formally integrate out all fields other than the tachyon field, and define a tachyon effective action $S_{eff}(T)$. (AT TREE LEVEL)

For space-time independent field configurations,

$$S_{eff}(T) = -\int d^{p+1}x V(T)$$

$V(T)$: tachyon effective potential

In actual practice we shall never try to construct $S_{eff}(T)$ but

1) either work with the infinite set of fields describing the dynamics of open strings on the D-brane (string field theory)

2) or use indirect method *e.g.* of two dimensional CFT to study the dynamics of open strings on the D-brane.

Nevertheless it is more convenient to state various results in terms of $S_{eff}(T)$ as if the dynamics can be described in terms of a single (complex) scalar field T.

Properties of $V(T)$ and $S_{eff}(T)$:

- For a non-BPS D-brane $S_{eff}(-T) = S_{eff}(T)$.

  For a D-$\bar{\text{D}}$ system, $S_{eff}(e^{i\phi}T) = S_{eff}(T)$.

- mass^2 of the field T is $V''(T)|_{T=0} < 0$.

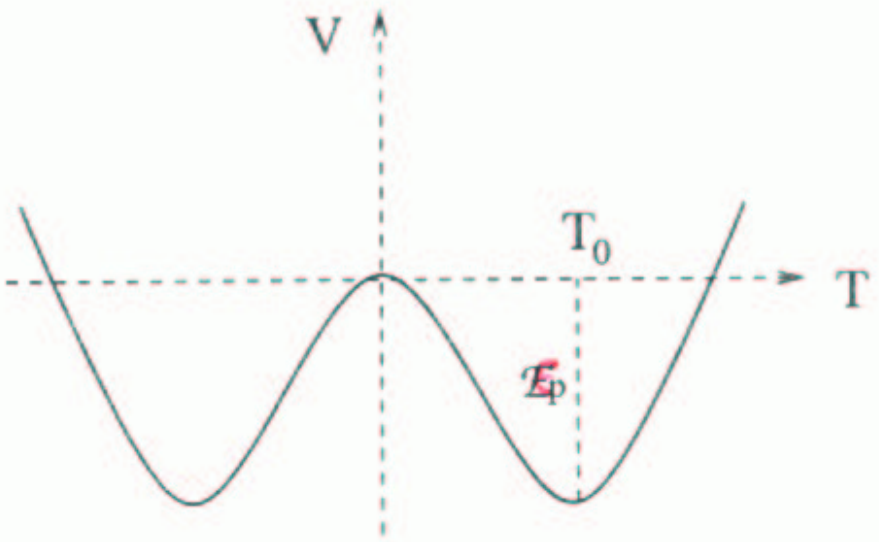
  Thus $V(T)$ has a maximum at $T = 0$.

  Question: Does $V(T)$ have a minimum?

Conjectures: A.S.

1. $V(T)$ does have a minimum at $|T| = T_0$.

$$V(T_0) + \mathcal{E}_p = 0$$



$\mathcal{E}_p$: Total energy density of the original system.

$\mathcal{E}_p = \widetilde{\mathcal{T}}_p$ for a non-BPS D-p brane.

$\mathcal{E}_p = 2\mathcal{T}_p$ for D-p – $\bar{\text{D}}$-p system.

Thus at $|T| = T_0$ the total energy density vanishes identically.

→ MORE NATURAL TO CALL $V(T) + \mathcal{E}_p$ THE TACHYON POTENTIAL

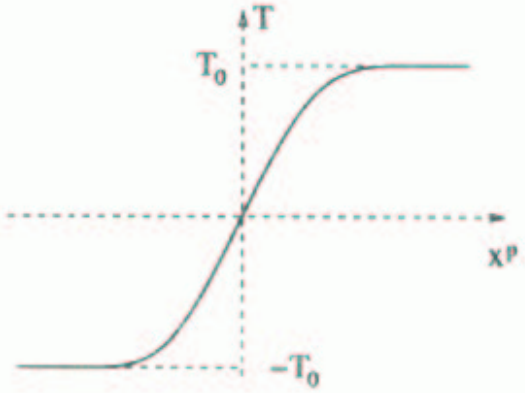
2. $|T| = T_0$ configuration describes the closed string vacuum without any D-brane.

   Thus around this minimum there are no physical open string excitations.

3. There are classical solutions of the equations of motion of T, representing lower dimensional D-branes.

   Examples:

   (a) On a non-BPS D-p-brane, a kink represents a D-$(p-1)$-brane. (BPS)



   Energy density is localized around a codimension 1 subspace ($x^p = 0$)

(b) On a D-p-$\bar{\text{D}}$-p pair, a kink represents a non-BPS D-$(p-1)$-brane.

In this case T is complex, but we consider a solution for which $\Im(T) = 0$ and $\Re(T)$ takes the form of a kink.

(c) A vortex on a D-p-$\bar{\text{D}}$-p pair represents a BPS D-$(p-2)$-brane.

$$T(x^0, \ldots x^p) = f(\rho) e^{i\theta}$$

$$x^p = \rho \cos\theta, \qquad x^{p-1} = \rho \sin\theta$$

$$f(\rho) \to T_0 \quad \text{for} \quad \rho \to \infty, \qquad f(0) = 0$$

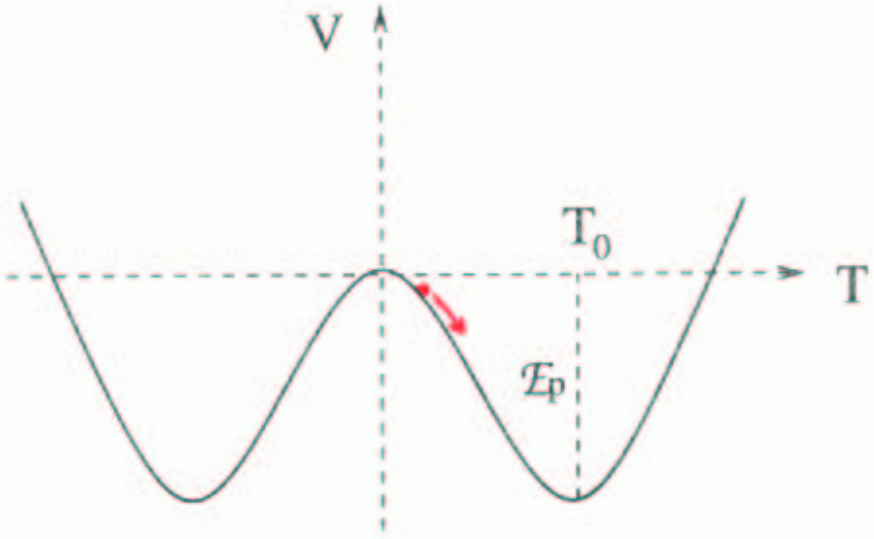
Energy density localized around $\rho = 0$

$\to$ a codimension 2 subspace ($x^p = x^{p-1} = 0$).

So far we have talked about time independent solutions.

What about time dependent solutions?

We shall study time dependent solution that describes rolling of the tachyon away from the maximum.



Result:

The system evolves to a pressureless gas asymptotically.

(Zero pressure and non-zero energy density)

Case 1: The tachyon begins rolling at $T = \lambda$, $\partial_0 T = 0$:

Total energy of the system $< \tilde{T}_p$

Evolution of the energy momentum tensor:

$$T_{00} = \frac{\tilde{T}_p}{2}(1 + \cos(2\pi\tilde{\lambda}))$$

$$T_{i0} = 0, \qquad T_{ij} = -2f(x^0)\delta_{ij}$$

$\tilde{T}_p/2$

$$f(x^0) = \frac{1}{1 + e^{\sqrt{2}x^0}\sin^2(\tilde{\lambda}\pi)} + \frac{1}{1 + e^{-\sqrt{2}x^0}\sin^2(\tilde{\lambda}\pi)} - 1$$

$\tilde{\lambda} = \lambda + \mathcal{O}(\lambda^2)$: a parameter labelling the total energy density

As $x^0 \to \infty$, $f(x^0) \to 0$ $\Rightarrow$ $T_{ij} \to 0$

$\downarrow$

ZERO PRESSURE

Case 2: The system begins rolling with a non-zero velocity at the top of the potential.

$T = 0$, $\partial_0 T = \lambda_2$ initially

Total energy $> \tilde{T}_p$

$$T_{00} = \frac{\tilde{T}_p}{2}(1 + \cosh(2\pi\tilde{\lambda}))$$

$(\tilde{T}_{b/2}!)$

$$T_{i0} = 0, \qquad T_{ij} = -2f(x^0)\delta_{ij}$$

$$f(x^0) = \frac{1}{1 + e^{\sqrt{2}x^0}\sinh^2(\tilde{\lambda}\pi)} + \frac{1}{1 + e^{-\sqrt{2}x^0}\sinh^2(\tilde{\lambda}\pi)} - 1$$

Again as $x^0 \to \infty$, $f(x^0) \to 0$

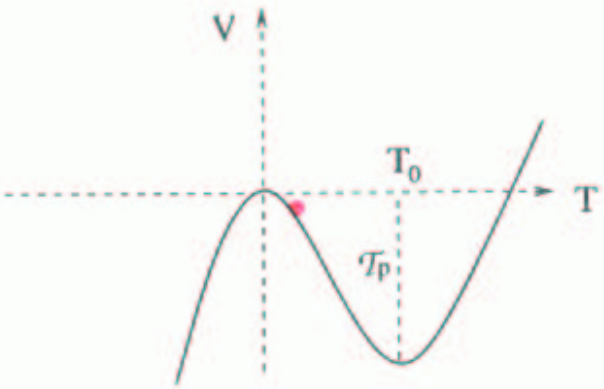
Note: in all these cases the final energy density is stored in the kinetic term for the tachyon field even though there is no physical particle associated with the tachyon.

Could such energy density be present in the universe today as dark matter?

Similar results also hold for D-branes in bosonic string theory.

Tachyon has mass$^2 = -1$ (in $\alpha' = 1$ unit)

Tachyon potential has a local minimum on one side, but is unbounded from below on the other side.



Initial condition $T = \lambda$, $\partial_0 T = 0$ at $x^0 = 0$

$\rightarrow$ $T \simeq \lambda \cosh(x^0)$ to linear order ($E < \mathcal{T}_p$)

Initial condition $T = 0$, $\partial_0 T = \lambda$ at $x^0 = 0$

$\rightarrow$ $T \simeq \lambda \sinh(x^0)$ to linear order ($E > \mathcal{T}_p$)

(25a)

RESULT

THE SYSTEM EVOLVES TO A PRESSURELESS GAS ASYMPTOTICALLY

CASE 1: $E < \gamma_p$

EVOLUTION OF $T_{\mu\nu}$:

$$T_{00} = \frac{\gamma_p}{2}\left(1+\cos(2\pi\tilde{\lambda})\right)$$

$$T_{i0} = 0, \qquad T_{ij} = -\gamma_p f(x^0)\,\delta_{ij}$$

$$f(x^0) = \frac{1}{1+e^{x^0}\sin(\tilde{\lambda}\pi)} + \frac{1}{1+e^{-x^0}\sin(\tilde{\lambda}\pi)} - 1$$

$\tilde{\lambda} = \lambda + O(\lambda^2)$: LABELS THE TOTAL ENERGY DENSITY

↓

INITIAL VALUE OF TACHYON FIELD T

$$f(x^0) = \frac{1}{1 + e^{x^0} \sin(\tilde{\lambda}\pi)} + \frac{1}{1 + e^{-x^0} \sin(\tilde{\lambda}\pi)} - 1$$

Note: for $\tilde{\lambda} > 0$, $f(x^0) \to 0$ as $x^0 \to \infty$

$\to$ Pressure vanishes

For $\tilde{\lambda} < 0$, $f(x^0)$ hits a singularity.

This corresponds to pushing the tachyon to the wrong side where the potential is unbounded from below.

Case 2: $E > \mathcal{T}_p$ OBTAINED BY $\tilde{\lambda} \to -i\tilde{\lambda}$, $x^0 \to x^0 + \frac{i\pi}{2}$

$$T_{00} = \frac{\mathcal{T}_p}{2}(1 + \cosh(2\pi\tilde{\lambda}))$$

$$T_{i0} = 0, \qquad T_{ij} = -2f(x^0)\delta_{ij}$$

$$f(x^0) = \frac{1}{1 + e^{x^0}\sinh(\tilde{\lambda}\pi)} + \frac{1}{1 - e^{-x^0}\sinh(\tilde{\lambda}\pi)} - 1$$

Again for $\tilde{\lambda} > 0$ and sufficiently small, $f(x^0) \to 0$ as $x^0 \to \infty$

The system evolves to pressureless gas.

On the other hand for $\tilde{\lambda} < 0$ and sufficiently small, $f(x^0)$ hits a singularity at finite x^0.

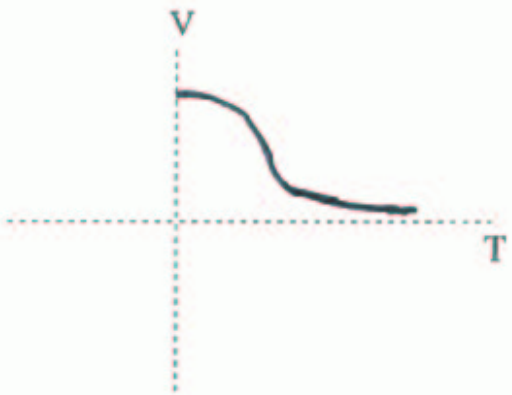
$\to$ corresponds to pushing in the direction in which the potential is unbounded from below.

An effective field theory description:

Take the following form of the tachyon action:

$$S = -\int d^{p+1}x\, V(T)\sqrt{1+\eta^{\mu\nu}\partial_\mu T\partial_\nu T}\,,$$

$V(T)$ has a maximum at $T=0$ and a minimum at ∞ (in this particular parametrization) where $V(T)=0$.



This gives the energy momentum tensor

$$T_{\mu\nu} = -V(T)\sqrt{-\det A}\,(A^{-1})_{\mu\nu}$$

$$A_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T\partial_\nu T$$

For spatially homogeneous time dependent solutions for which $\partial_i T = 0$, we have

$$A_{00} = -1 + (\partial_0 T)^2, \qquad A_{ij} = \delta_{ij}, \qquad A_{i0} = 0$$

This gives

$$\rho \equiv T_{00} = V(T)(1 - (\partial_0 T)^2)^{-1/2}, \qquad T_{i0} = 0$$

$$p\delta_{ij} \equiv T_{ij} = -V(T)\,(1 - (\partial_0 T)^2)^{1/2}\,\delta_{ij}$$

ρ is conserved

$\rightarrow$ as T approaches the minimum of the potential, $\partial_0 T$ must approach its critica. value 1

p vanishes in this limit

$\rightarrow$ precisely what we have observed in the boundary state analysis

Can we determine the form of $V(T)$?

Of course we cannot hope to have the effective action description to be able to describe the full stringy results, but one might hope that the late time behaviour of the system may be describable by an effective action.

At late time $\partial_0^n T \to 0$ for $n \geq 2$.

Thus if the action given above correctly accounts for all the terms involving $\partial_\mu T$, then it will be a valid description at late time.

Strategy:

- Study the x^0 dependence of p for large x^0.
- Find the potential $V(T)$ which reproduces this x^0 dependence of p.

$$(\partial_0 T)^2 = 1 + \frac{p(x^0)}{\rho}$$

$$V(T)^2 = -\rho\, p(x^0)$$

Thus given $p(x^0)$ we can find T and $V(T)$ as function of x^0.

Eliminate x^0 to find $V(T)$.

For large x^0:

$$p(x^0) = -Ke^{-\alpha x^0}$$

$\alpha = \sqrt{2}$ for superstring and 1 for bosonic string.

$\rightarrow$

$$V(T) = e^{-\alpha T/2}$$

Thus the proposed tachyon action near the minimum of the potential is:

$$\begin{aligned} S &= -\int d^{p+1}x\, e^{-\alpha T/2}\sqrt{-\det A} \\ &= -\int d^{p+1}x\, e^{-\alpha T/2}\sqrt{1+\eta^{\mu\nu}\partial_\mu T\partial_\nu T}\,, \end{aligned}$$

$$A_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \partial_\nu T$$

A Bonus:

We shall now show that this action automatically satisfies the condition that near the minimum there is no perturbative physical state.

(no plane-wave solution of the 'linearized' equations of motion)

First we expand the action in powers of derivatives and keep up to terms with two derivatives.

This gives

$$S = -\int d^{p+1}x\, e^{-\alpha T/2}(1 + \frac{1}{2}\eta^{\mu\nu}\partial_\mu T \partial_\nu T + \ldots)\,,$$

Define:

$$\phi = e^{-\alpha T/4}$$

This gives

$$S = \int d^{p+1}x\, \frac{16}{\alpha^2}(-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\alpha^2}{16}\phi^2 + \ldots)\,.$$

ϕ descibes a particle of mass $\alpha/2\sqrt{2}$

$\rightarrow$ related to the fact that there exists a plane-wave solution

$$\phi = a\, e^{ik_\mu x^\mu}$$

for $-\eta^{\mu\nu}k_\mu k_\nu = \alpha^2/8$.

We shall now show that this ceases to be a solution of the equations of motion when we include terms with higher powers of derivatives.

Full action written in terms of ϕ:

$$S = -\int d^{p+1}x\, \phi^2 \sqrt{1 + \frac{16}{\alpha^2}\phi^{-2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}$$

This is homogeneous of degree 2 in ϕ.

Equations of motion:

$$-\eta^{\mu\nu}\partial_\mu\left(\frac{\partial_\nu\phi}{\sqrt{1+\frac{16}{\alpha^2}\phi^{-2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}}\right) + \frac{\alpha^2}{8}\frac{\phi + \frac{8}{\alpha^2}\phi^{-1}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}{\sqrt{1+\frac{16}{\alpha^2}\phi^{-2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}} = 0$$

$\rightarrow$ homogeneous of degree 1

Look for a plane-wave solution $\phi = ae^{ik.x}$.

$$\frac{a}{\sqrt{1 - \frac{16}{\alpha^2}\eta^{\mu\nu}k_\mu k_\nu}} = 0 .$$

$\rightarrow$ no solution for finite k_μ.

(By expanding it in powers of k_μ and keeping up to second power we recover the previous condition $-k^2 = \alpha^2/8$)

Note: even if we had found a solution, it is not clear how we could superpose them to construct real solution since the equations are not linear (although homogeneous of degree 1)

Our analysis shows that even before we impose the reality condition, the plane-wave solution does not exist.

Conclusion: The proposed action correctly describes the physics around the tachyon vacuum.

Use this to see what kind of classical solutions we have in the theory.

Need to work in the Hamiltonian formalism.

following Gibbons, Hori, Y

Momentum conjugate to $T(x)$:

$$\Pi(x) \equiv \frac{\delta S}{\delta(\partial_0 T(x))},$$

Hamiltonian

$$H = \int d^p x \mathcal{H}_* = \int d^p x \left[\Pi(x)\partial_0 T(x) - \mathcal{L}\right]$$

$$\mathcal{H} = T_{00} = \sqrt{\Pi^2 + e^{-\alpha T}}\sqrt{1 + \partial_i T \partial_i T}\,.$$

For large T we can ignore the $e^{-\alpha T}$ term

From this we get Hamilton's equation:

$$\partial_0 \Pi(x) = -\frac{\delta H}{\delta T(x)} = \partial_j \frac{\pi(x)\,\partial_j T(x)}{\sqrt{1+\partial_i T \partial_i T}}$$

$$\partial_0 T(x) = \frac{\delta H}{\delta \Pi(x)} = \sqrt{1+\partial_i T \partial_i T}$$

FOR $\pi(x) \geq 0$, $\partial_0 T(x) \geq 0$.

A possible solution:

Set $\partial_i T = \partial_0 \Pi = 0, \quad \partial_0 T = 1.$

Gives:

$$\Pi(x) = f(\vec{x}), \qquad T(x) = x^0$$

$f(\vec{x})$: any arbitrary function of the spatial co-ordinates.

$$\mathcal{H}(\vec{x}) = f(\vec{x})$$

→ existence of classical solution with arbitrary spatial distribution of energy density

(just like ordinary non-relativistic matter)

→ POSSIBLE CANDIDATE FOR DARK MATTER ???

IN FACT THE CLASSICAL EQUATION OF MOTION CAN BE REWRITTEN SO THAT THE SYSTEM DESCRIBES STREAMLINE MOTION OF PERFECT DUST

DEFINE: $u_\mu = \partial_\mu T$, $\rho(x) = \frac{\Pi(x)}{\partial_0 T(x)}$

IN TERMS OF THESE VARIABLES THE EQUATIONS OF MOTION HAVE THE FORM:

$$\eta^{\mu\nu} u_\mu u_\nu = -1 \; ; \quad \partial_\mu (\rho u^\mu) = 0$$

$$T_{\mu\nu} = \rho u_\mu u_\nu$$

→ STREALINE MOTION OF DUST WITH LOCAL 4-VELOCITY u_μ.

Conclusion:

The dynamics of the tachyon is similar to that of an ordinary scalar field theory near the top of the potential, but is very different from that of an ordinary scalar field near the bottom of the potential.

Hopefully a complete understanding of this dynamics will give us new insight into the cosmology in the early universe as well as in the present universe.

IN PARTICULAR, WE NEED TO STUDY HOW THE CLASSICAL ANALYSIS IS MODIFIED BY THE QUANTUM EFFECTS.

WE SHALL NOW DISCUSS DERIVATION/VERIFICATION OF THE RESULTS QUOTED SO FAR.

FIRST WE SHALL FOCUS ON THE STATIC PROPERTIES

3 CONJECTURES FOR $S_{eff}(T)$ ON NON BPS Dp-BRANE OR $D\text{-}p - \bar{D}\text{-}p$ SYSTEM IN IIA/IIB:

① $V(T_0) + \mathcal{E}_p = 0$

$V(T_0)$: MINIMUM OF $V(T)$

$\mathcal{E}_p$: ENERGY DENSITY OF THE BRANE SYSTEM AT $T=0$

② ABSENCE OF PHYSICAL OPEN STRING STATES AROUND $T=T_0$

③ LOWER DIMENSIONAL D-BRANES ARE SOLITON SOLUTIONS OF EQS. OF MOTION DERIVED FROM $S_{eff}(T)$.

Similar conjectures exist for bosonic string theory in (25+1)-dimensions.

(Although bosonic string theory is not of physical interest at present, it nevertheless provides a simpler testing ground for these conjectures.)

Bosonic string theory has

1. Unoriented D-p-branes for every p with tension
$$\mathcal{T}_p = \frac{1}{(2\pi)^p g_s}$$
2. Each D-p-brane has a single tachyonic mode with mass$^2 = -1$.

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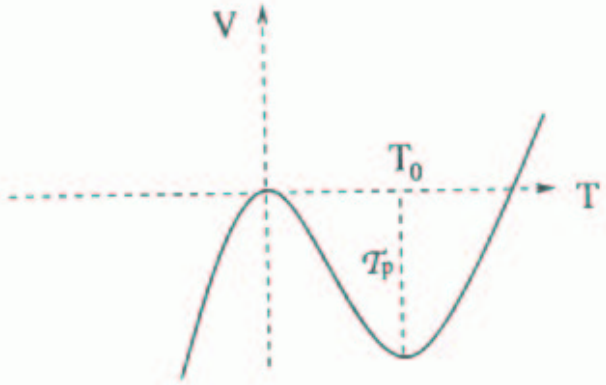
Conjectures:

1. The effective tachyon potential $V(T)$ has a local minimum at $T = T_0$ such that

$$V(T_0) + \mathcal{T}_p = 0$$



Thus the total energy density vanishes at $T = T_0$.

2. $T = T_0$ describes the closed string vacuum without any D-brane.

Thus there are no physical open string excitations around this vacuum.

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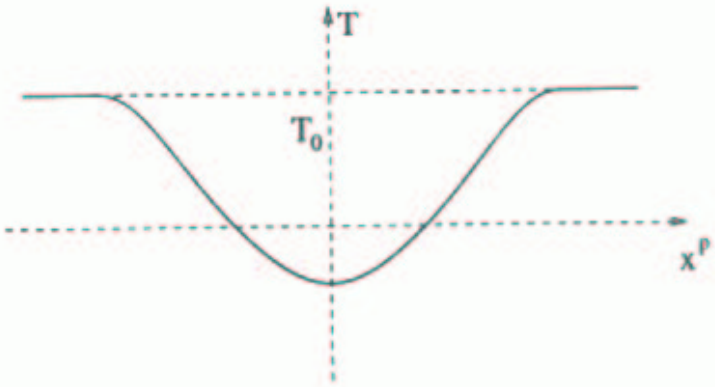
3. Classical lump solutions involving T represent lower dimensional D-branes.



Energy density concentrated around a codimension 1 subspace ($x^p = 0$)

Represents a D-$(p-1)$-brane.

Similar construction can be carried out for higher codimension solitons.

Verification of these conjectures:

Various approaches:

1. 2-dimensional conformal field theory
2. Renormalization group in 2-dim
3. Toy models, p-adic strings, ···
4. Non-commutative field theory
5. Boundary string field theory
6. Cubic string field theory
7. VACUUM STRING FIELD THEORY

IN OUR DISCUSSION ON THE STATIC PROPERTIES OF $S_{eff}(T)$ WE SHALL FOCUS EXCLUSIVELY ON:

① BOSONIC STRING THEORY

② ANALYSIS BASED ON CUBIC SFT

WHY BOSONIC STRING THEORY?

ALTHOUGH QUANTUM MECHANICALLY SUPERSTRING THEORY IS MUCH BETTER BEHAVED, THE CLASSICAL PROPERTIES OF UNSTABLE D-BRANES IN IIA/IIB ARE VERY SIMILAR TO THOSE OF BOSONIC STRING THEORY.

BOSONIC STRING THEORY PROVIDES A SIMPLER SETTING FOR STUDYING UNSTABLE D-BRANE SYSTEMS, AS LONG AS WE FOCUS ON CLASSICAL DYNAMICS

Why SFT? What is SFT?

written

The dynamics of the tachyon on a D-brane cannot be studied without taking into account its coupling with infinite number of other fields living on the D-brane.

→ associated with infinite number of states of the open string.

Thus the classical dynamics of the D-brane is governed by a coupled system of infinite number of equations of motion for the infinite number of fields.

→ *string field theory*

The conjectures involving tachyon condensation should be interpreted as conjectures involving properties of the solutions of these infinite number of coupled equations.

Task:

1. Take a precise formulation of SFT.

2. Make the set of conjectures into a set of precise conjectures about classical SFT equations of motion.

3. Verify these conjectures.

We shall begin by describing the basic formulation of bosonic string field theory.

(good test problem for the more interesting case of superstrings)

First quantized bosonic string theory:

For a given space-time background which is a solution of the classical equations of motion we have a two dimensional conformal field theory (CFT) of the matter-ghost system on complex plane:

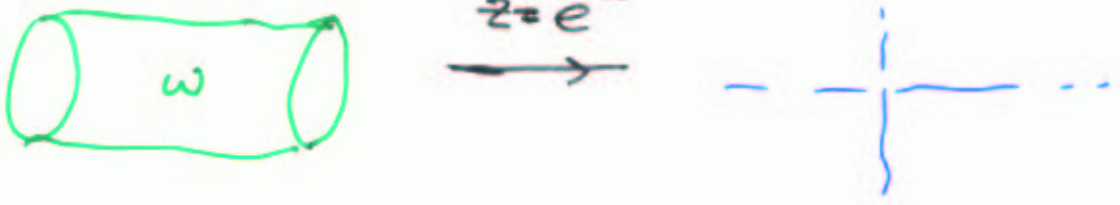
$CFT_{matter} \otimes CFT_{ghost}$ → Quantum theory of closed strings

CFT_{matter} has $c = 26$

For string theory in flat space-time this is a theory of 26 free scalar fields.

CFT_{ghost} has $c = -26$ and fields b, c, $\bar{b}$, $\bar{c}$

Conformal weights: $(2,0)$, $(-1,0)$, $(0,2)$, $(0,-1)$



A D-brane in this space-time background

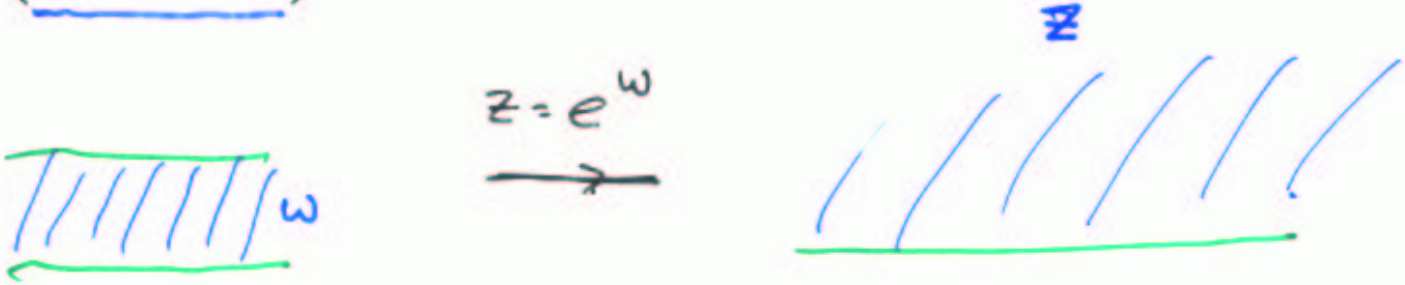
$\leftrightarrow$ a 2-dimensional CFT on the *upper half plane* with specific conformally invariant boundary condition on fields on the real axis

$\rightarrow$ boundary CFT (BCFT) $\rightarrow$ Quantum theory of open strings

Different D-branes correspond to different boundary conditions (b.c.).

e.g. for D-branes in flat space-time, the coordinates transverse to the brane has Dirichlet b.c., and the coordinates along the D-brane has Neumann b.c.

Note: the ghost fields always have the same b.c. (Neumann)



Define $\mathcal{H}$: vector space of states of:

$$BCFT_{matter} \otimes BCFT_{ghost}$$

$|0\rangle$: unique SL(2,R) invariant state of this BCFT

$\forall |\phi\rangle \in \mathcal{H}$, there is a unique local vertex operator $\phi(x)$ on the real line such that:

$$\phi(0)|0\rangle = |\phi\rangle$$

Definition of ghost number:

$b, \bar{b}$ has ghost no. -1

$c, \bar{c}$ has ghost no. 1

Matter sector operators have ghost no. 0

$|0\rangle$: has ghost no. 0

$\mathcal{H}_n$: subspace of $\mathcal{H}$ with ghost number n.

Physical states of the first quantized open string on a D-brane:

States $|\phi\rangle$ in $\mathcal{H}_1$ satisfying:

1. $Q_B|\phi\rangle = 0$

   Q_B: a nilpotent operator of ghost number 1 (BRST charge)

   CONSTRUCTED FROM $b, c, \bar{b}, \bar{c}$ AND MATTER VIRASORO GENERATORS

   $(Q_B)^2 = 0$

2. Two states $|\phi\rangle$ and $|\phi'\rangle$ are considered equivalent if:

$$|\phi\rangle - |\phi'\rangle = Q_B|\Lambda\rangle$$

   for some state $|\Lambda\rangle$ in $\mathcal{H}_0$.

Thus physical states $\leftrightarrow$ cohomology of Q_B

The perturbative S-matrix involving arbitrary number of external physical states can be computed using 'Polyakov prescription'.

Open String Field Theory (SFT): A field theory such that:

1. Inequivalent solutions of the linearized equations of motion are in $1 \leftrightarrow 1$ correspondence with the physical states of the open string.

2. S-matrix computed using the Feynman rules of SFT reproduces the S-matrix computed using Polyakov prescription to all orders in perturbation theory.

Formulation of cubic open string field theory:

In describing SFT one has to first specify what corresponds to a general off-shell string field configuration

Usually when we go from the first → second quantized formulation, the field configurations of the second quantized formulation can be identified as wave-functions / states of the first quantized theory.

(not just physical states *i.e.* wave-functions which satisfy Schroedinger equation)

In the case of SFT a general field configuration of SFT is taken to be a state $|\Phi\rangle$ in $\mathcal{H}_1$, not necessarily satisfying the physical state condition.

(a state of ghost no. 1 in $BCFT_{matter} \otimes BCFT_{ghost}$)

Why does $|\Phi\rangle$ represent fields in $(p+1)$-dimension?

Choose a basis of states in $\mathcal{H}_1$: $\{|\chi_\alpha\rangle\}$

Then we can expand:

$$|\Phi\rangle = \sum_\alpha \phi_\alpha |\chi_\alpha\rangle$$

basis

The index α: runs over all the states in $\mathcal{H}_1$

$\rightarrow$ includes the $(p+1)$ dimensional momentum of the open string states living on a D-p-brane.

Thus $\{\alpha\} = \{\{k_\mu\}, r\}$

$\{k_\mu\}$: $(p+1)$-dimensional momentum

r: infinite set of discrete labels.

Thus $\{\phi_\alpha\} = \{\phi_{\{k_\mu\},r}\} \equiv \{\phi_r(k_0, \cdots k_p)\}$

$\phi_{\{k_\mu\},r}$: Fourier transform of a field $\phi_r(x)$ in $(p+1)$-dimensions.

Reality condition on $|\Phi\rangle$: $|\Phi\rangle = |\Phi^c\rangle$

^{c}: an antilinear operation in $\mathcal{H}_1$

In terms of components:

$$\phi^*_\alpha = K_{\alpha\beta}\phi_\beta$$

for some suitable (known) matrix $K_{\alpha\beta}$.

TWIST