STRING FIELD $|\Phi\rangle \in \mathcal{H}$,

→ A STATE OF GHOST NO. 1 IN BCFT OF ghost + matter

String field theory action: should produce a number for every field configuration

$$\boxed{S = -\frac{1}{g^2}[\frac{1}{2}\langle\Phi|Q_B|\Phi\rangle + \frac{1}{3}\langle\Phi|\Phi * \Phi\rangle]}$$

g: a constant known as open string coupling constant on the D-p-brane

$$\boxed{g_s \sim g^2}$$

Q_B: the BRST charge

We shall now define $\langle A|B\rangle$ and $|A*B\rangle$ for $|A\rangle, |B\rangle \in \mathcal{H}$

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$\langle A|B\rangle$: BPZ inner product

$$\boxed{\langle A|B\rangle = \langle I \circ A(0) B(0)\rangle_{UHP}}$$

$A(x)$, $B(x)$: vertex operators associated with the states $|A\rangle$, $|B\rangle$

$A(0)|0\rangle = |A\rangle$
$B(0)|0\rangle = |B\rangle$

$\langle\cdot\rangle_{UHP}$: correlation function of BCFT in the upper half plane

$f \circ A(x)$: conformal transform of $A(x)$ by the map $f(x)$.

e.g. if $A(x)$ is a primary of dimension h then:

$$\boxed{f \circ A(x) = (f'(x))^h A(f(x))}$$

$$\boxed{I(z) = -1/z}$$

Note: $\langle A|B\rangle \neq 0$ only if the ghost numbers of $|A\rangle$ and $|B\rangle$ add up to 3.

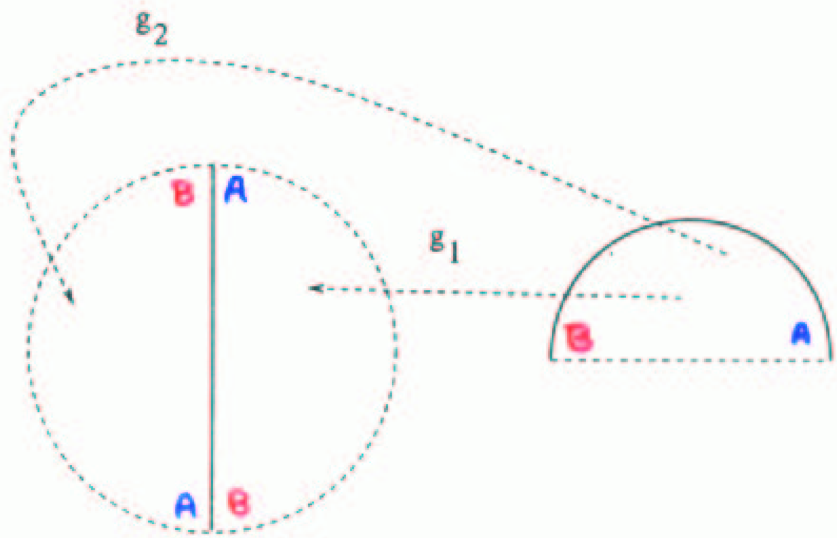
By a conformal transformation we can also write:

$$\boxed{\langle A|B\rangle = \langle g_2 \circ A(0) g_1 \circ B(0)\rangle_D}$$

$\langle\cdot\rangle_D$: correlation function on a unit disk

$g_1(z) = \dfrac{1+iz}{1-iz}$,	$g_2(z) = -g_1(z)$ $= g_1(I(z))$

Geometric description of the maps g_1 and g_2:



$|A * B\rangle$: Witten $*$-product

A map from $\mathcal{H}_m \otimes \mathcal{H}_n \to \mathcal{H}_{m+n}$

Thus ghost numbers add under star product.

We shall define $|A * B\rangle$ by specifying $\langle C|A * B\rangle$ for every state $|C\rangle$ in $\mathcal{H}$:

$$\boxed{\langle C|A * B\rangle \;=\; \langle h_1 \circ C(0) h_2 \circ A(0) h_3 \circ B(0)\rangle_D}$$

Le Clair, Peskin, Preitschopf

$\langle \cdot \rangle_D$: correlation function on a unit disk

$$h_1(z) = \left(\frac{1+iz}{1-iz}\right)^{2/3},$$

$$h_2(z) = e^{-2\pi i/3} h_1(z), \qquad h_3(z) = e^{-4\pi i/3} h_1(z).$$

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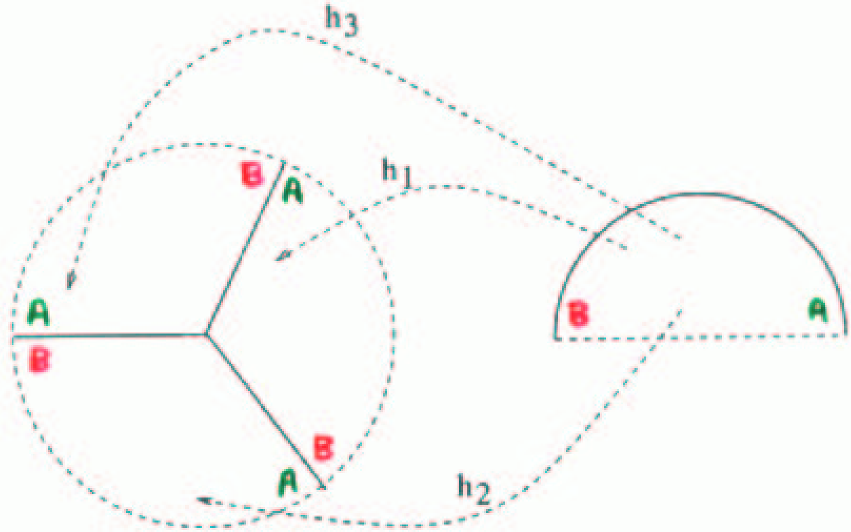
Geometric description of the maps h_1, h_2 and h_3:



Intuitive description of reality condition, $*$-product and inner product:

Collectively denote the set of basic fields in 2-dimensional CFT by $\{\varphi\}$

e.g. For flat space-time background φ includes the fields $X^0, \ldots X^{25}$ and the ghost fields.

Any open string state $|A\rangle$

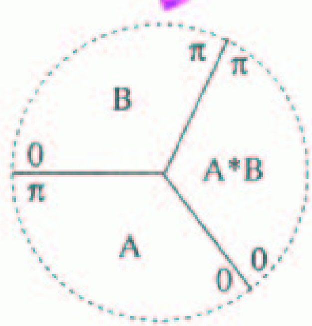
$\leftrightarrow$ an wave-functional $F_A(\{\varphi(\sigma\})$

$0 \leq \sigma \leq \pi$: parametrizes the open string.

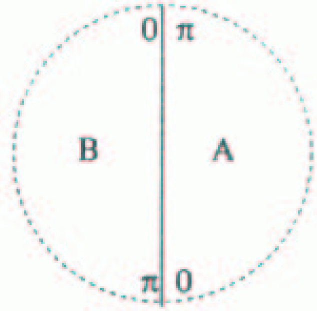
Reality condition: $\boxed{F_A^*(\{\varphi(\sigma\}) = F_A(\{\varphi(\pi - \sigma\})}$

Definition of $*$-product:

$$
\begin{aligned}
&F_{A*B}(\{\varphi(\sigma\}) \\
&= \int \prod_{\sigma=0}^{\pi} [\mathcal{D}\varphi_1(\sigma)][\mathcal{D}\varphi_2(\sigma)] F_A(\{\varphi_1(\sigma\}) F_B(\{\varphi_2(\sigma\}) \\
&\quad \times \prod_{\sigma=0}^{\pi/2} \{\delta(\varphi(\sigma) - \varphi_1(\sigma))\delta(\varphi(\pi-\sigma) - \varphi_2(\pi-\sigma)) \\
&\qquad \times \delta(\varphi_1(\pi-\sigma) - \varphi_2(\sigma))\}
\end{aligned}
$$



A*B



<A|B>

$$
\begin{aligned}
&\langle A|B\rangle \\
&= \int \prod_{\sigma=0}^{\pi} [\mathcal{D}\varphi_1(\sigma)][\mathcal{D}\varphi_2(\sigma)] F_A(\{\varphi_1(\sigma\}) F_B(\{\varphi_2(\sigma\}) \\
&\qquad \times \prod_{\sigma=0}^{\pi} \delta(\varphi_1(\pi-\sigma) - \varphi_2(\sigma))
\end{aligned}
$$

Q_B, $*$ and $\langle\cdot|\cdot\rangle$ satisfy some important identities:

1. $(Q_B)^2 = 0$

2. $\langle Q_B A|B\rangle = -(-1)^{n_A}\langle A|Q_B|B\rangle$

   n_A: ghost number of $|A\rangle$

3. Distributive property of Q_B:

   $Q_B|A*B\rangle = (Q_B|A\rangle)*|B\rangle + (-1)^{n_A}|A\rangle * Q_B|B\rangle$

4. $\langle A|B\rangle = \langle B|A\rangle$

5. Cyclicity: $\langle A|B * C\rangle = \langle C|A * B\rangle$

6. Associativity:

   $(|A\rangle * |B\rangle) * |C\rangle = |A\rangle * (|B\rangle * |C\rangle)$

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Gauge invariance: Using these identities one can show that the SFT action has a gauge invariance.

$\delta S = 0$ under:

$$\boxed{\delta|\Phi\rangle = Q_B|\Lambda\rangle + |\Phi\rangle * |\Lambda\rangle - |\Lambda\rangle * |\Phi\rangle}$$

$|\Lambda\rangle$: an arbitrary infinitesimal state in $\mathcal{H}_0$

(when we expand $|\Lambda\rangle$ as linear combination of the basis states in $\mathcal{H}_0$:

$$\boxed{|\Lambda\rangle = \sum_\alpha \lambda_\alpha |\chi_{0,\alpha}\rangle}$$

the coefficients of expansion λ_α are infinitesimal)

λ_α are the infinitesimal gauge transformation parameters

Equations of motion

Eqs. of motion obtained by requiring $\boxed{\frac{\delta S}{\delta|\Phi\rangle} = 0}$

$$\boxed{Q_B|\Phi\rangle + |\Phi\rangle * |\Phi\rangle = 0}$$

Thus at the linearized level:

$$\boxed{Q_B|\Phi\rangle = 0}$$

Linearized gauge transformation: $|\Phi\rangle$ is gauge equivalent to $Q_B|\Lambda\rangle + |\Phi\rangle$

→ gauge inequivalent solutions of linearized equations of motion of SFT

↔ elements of BRST cohomology

↔ physical states of the first quantized open string theory.

One can also show that the perturbative amplitudes derived from this action agree with the Polyakov amplitudes to all orders in the perturbation expansion.

Analogy with Chern-Simons theory in 3-dimensions:

$$\boxed{S_{CS} = \int_{M_3} Tr(\frac{1}{2} A \wedge dA + \frac{1}{3} A \wedge A \wedge A)}$$

M_3: a 3-dimensional manifold

A: (non-abelian) gauge field on the three dimensional manifold

$d \leftrightarrow Q_B,$	$\wedge \leftrightarrow *,$	$\int A \wedge B = \langle A \vert B \rangle$

degree of a form $\leftrightarrow$ ghost number of the state

Suppose $|\Phi_0\rangle$ represents a solution of the SFT equations of motion.

Define shifted field $|\Psi\rangle = |\Phi\rangle - |\Phi_0\rangle$

Express the action in terms of $|\Psi\rangle$:

$$\boxed{S(|\Phi\rangle) = S(|\Phi_0\rangle) + \widetilde{S}(|\Psi\rangle)}$$

$$\boxed{\widetilde{S}(|\Psi\rangle) = -\frac{1}{g^2}[\frac{1}{2}\langle\Psi|\mathcal{Q}|\Psi\rangle + \frac{1}{3}\langle\Psi|\Psi * \Psi\rangle]}$$

$$\boxed{\mathcal{Q}|A\rangle \equiv Q_B|A\rangle + |\Phi_0\rangle * |A\rangle \pm (-1)^{n_A}|A\rangle * |\Phi_0\rangle}$$

One can show that all the identities satisfied by Q_B hold if we replace Q_B by $\mathcal{Q}$.

Thus $\widetilde{S}(|\Psi\rangle)$ is invariant under the gauge transformation:

$$\boxed{\delta|\Psi\rangle = \mathcal{Q}|\Lambda\rangle + |\Psi\rangle * |\Lambda\rangle - |\Lambda\rangle * |\Psi\rangle}$$

The spectrum of physical open string states around the solution $|\Phi_0\rangle$ is given by the cohomology of $\mathcal{Q}$.

We shall now restate the tachyon condensation conjectures in the context of SFT

(For simplicity we shall restrict to the case of static D-p-branes in flat space-time, but many of the conjectures can be made more general)

1. There is a translationally invariant solution $|\Phi_0\rangle$ of the SFT equations of motion:

$$\boxed{Q_B|\Phi_0\rangle + |\Phi_0\rangle * |\Phi_0\rangle = 0}$$

such that

$$\boxed{-\frac{1}{V_{p+1}} S(|\Phi_0\rangle) + \mathcal{T}_p = 0}$$

V_{p+1}: world-volume of the D-brane

For a static D-brane $S(|\Phi_0\rangle) = -V_{p+1} \otimes$ potential

Thus the conjecture is equivalent to saying that the potential cancels the energy density of the original system.

2. For this $|\Phi_0\rangle$ let us define an operator $\mathcal{Q}$ such that

$$\boxed{\mathcal{Q}|A\rangle \equiv Q_B|A\rangle + |\Phi_0\rangle * |A\rangle \pm (-1)^{n_A}|A\rangle * |\Phi_0\rangle}$$

Cohomology of $\mathcal{Q}$ represents the spectrum of physical states around the solution $|\Phi_0\rangle$.

Thus $\mathcal{Q}$ must have vanishing cohomology.

3. $\forall k > 0$, there should be other solutions $|\Phi^k\rangle$ of the SFT equations of motion representing a D-$(p-k)$-brane:

$$\boxed{S(|\Phi^k\rangle) - S(|\Phi_0\rangle) = -V_{p-k+1}\mathcal{T}_{p-k}}$$

V_{p-k+1}: world-volume of the D-$(p-k)$-brane.

Note: $|\Phi^0\rangle \equiv 0$ represents the original D-p-brane whereas $|\Phi_0\rangle$ represents the tachyon vacuum solution.

Some definitions/normalization:

Define b_n, c_n through:

$$\boxed{c(z) = \sum_n c_n z^{-n+1}, \quad b(z) = \sum_n b_n z^{-n-2}}$$

$$\boxed{|k\rangle \equiv e^{ik.X(0)}|0\rangle}$$

Normalization of $|k\rangle$:

$$\boxed{\langle k|c_{-1}c_0c_1|k'\rangle = (2\pi)^{p+1}\delta(k+k')}$$

$$\to \boxed{\langle 0|c_{-1}c_0c_1|0\rangle = (2\pi)^{p+1}\delta(0) = V_{p+1}}$$

This suggests defining $\langle A|B\rangle' = \frac{1}{V_{p+1}}\langle A|B\rangle$

if $|A\rangle$ and $|B\rangle$ denote states carrying zero momentum.

Component form of the SFT action:

$|\chi_{1,\alpha}\rangle$: Basis of states in $\mathcal{H}_1$

$$|\Phi\rangle = \sum_\alpha \phi_\alpha |\chi_{1,\alpha}\rangle$$

$$S(|\Phi\rangle) = -\frac{1}{g^2}\left[\frac{1}{2}\mathcal{A}_{\alpha\beta}\phi_\alpha\phi_\beta + \frac{1}{3}\mathcal{C}_{\alpha\beta\gamma}\phi_\alpha\phi_\beta\phi_\gamma\right]$$

$$\begin{aligned}
\mathcal{A}_{\alpha\beta} &= \langle \chi_{1,\alpha}|Q_B|\chi_{1,\beta}\rangle \\
\mathcal{C}_{\alpha\beta\gamma} &= \langle h_1 \circ \chi_{1,\alpha}(0)\, h_2 \circ \chi_{1,\beta}(0)\, h_3 \circ \chi_{1,\gamma}(0)\rangle_D
\end{aligned}$$

$\mathcal{A}_{\alpha\beta}, \mathcal{C}_{\alpha\beta\gamma}$: computable coefficients

Recall, for a D-p-brane in flat space time,

$\{\alpha\} = \{\{k_\mu\}, r\}$

k_μ: momentum index, r: discrete index

Thus $\sum_\alpha \to \sum_r \int d^{p+1}k$

$\phi_\alpha \to \phi_r(k)$

In analysing the tachyon vacuum solution, we look for a solution of the equations of motion of the form:

$$\boxed{\phi_r(k) = \phi_r \prod_{\mu=0}^{p} \delta(k_\mu)}$$

This gives $\boxed{|\Phi\rangle = \sum_r \phi_r |\chi_{1,r}\rangle}$

$|\chi_{1,r}\rangle$: Basis of zero momentum states. Then

$$\boxed{S(|\Phi\rangle) = -\frac{1}{g^2}[\frac{1}{2}\mathcal{A}_{rs}\phi_r\phi_s + \frac{1}{3}\mathcal{C}_{rst}\phi_r\phi_s\phi_t]}$$

Note: both $\mathcal{A}_{rs}$ and $\mathcal{C}_{rst}$ are proportional to V_{p+1} since they involve correlation functions of zero momentum fields/states.

$\rightarrow$ suggests that we factor out the overall factor of V_{p+1}:

$$\boxed{\mathcal{A}_{rs} \equiv V_{p+1} A_{rs}, \qquad \mathcal{C}_{rst} = V_{p+1} C_{rst}}$$

$$\boxed{\begin{aligned} S(|\Phi\rangle) &= -\frac{1}{g^2}[\frac{1}{2}\langle\Phi|Q_B|\Phi\rangle + \frac{1}{3}\langle\Phi|\Phi * \Phi\rangle] \\ &= -\frac{1}{g^2}V_{p+1}\,\mathcal{V}(|\Phi\rangle) \end{aligned}}$$

$$\boxed{\begin{aligned} \mathcal{V}(|\Phi\rangle) &= \frac{1}{2}\langle\Phi|Q_B|\Phi\rangle' + \frac{1}{3}\langle\Phi|\Phi * \Phi\rangle' \\ &= \frac{1}{2}A_{rs}\phi_r\phi_s + \frac{1}{3}C_{rst}\phi_r\phi_s\phi_t \end{aligned}}$$

$g^{-2}\mathcal{V}(|\Phi\rangle)$: The potential

Conjecture 1 gives: $\boxed{\frac{1}{g^2}\mathcal{V}(|\Phi_0\rangle) + \mathcal{T}_p = 0}$

On the other hand, one can show that

$$\boxed{\begin{aligned} &\mathcal{T}_p = 1/(2\pi^2 g^2) \\ &\approx \frac{1}{(2\pi)^p\, g_s} \end{aligned}}$$

Thus we need

$$\boxed{2\pi^2\mathcal{V}(|\Phi_0\rangle) + 1 = 0}$$

Expansion of the zero momentum string field:

$$|\Phi\rangle = \phi_0 c_1|0\rangle + \phi_1 c_0|0\rangle + \phi_2 c_{-1}|0\rangle + \phi_3 c_1 L^m_{-2}|0\rangle + \cdots$$

tachyon

L^m_{-n}: matter Virasoro generators

$$\begin{aligned}\mathcal{V}(|\Phi\rangle) \;&=\; \frac{1}{2}\langle\Phi|Q_B|\Phi\rangle' \\ &\quad+\frac{1}{3}\langle h_1\circ\Phi(0) h_2\circ\Phi(0) h_3\circ\Phi(0)\rangle'_D\end{aligned}$$

A drastic approximation:

$$|\Phi\rangle = \phi_0 c_1|0\rangle = \phi_0 c(0)|0\rangle$$

$$\begin{aligned}\mathcal{V}(\phi_0) \;&=\; \frac{1}{2}\phi_0^2\,\langle 0|c_{-1}Q_B c_1|0\rangle' \\ &\quad+\frac{1}{3}\phi_0^3\,\langle h_1\circ c(0) h_2\circ c(0) h_3\circ c(0)\rangle'_D \\ &=\; -\frac{1}{2}\phi_0^2 + \frac{1}{3}\left(\frac{3\sqrt{3}}{4}\right)^3\phi_0^3\end{aligned}$$

$(h_1'(0))^{-1}(h_2'(0))^{-1}(h_3'(0))^{-1}$

$\langle c(h_1(0))\, c(h_2(0))\, c(h_3(0))\rangle'_D$

$$\boxed{\mathcal{V}(\phi_0) = -\frac{1}{2}\phi_0^2 + \frac{1}{3}\left(\frac{3\sqrt{3}}{4}\right)^3 \phi_0^3}$$

$\partial\mathcal{V}/\partial\phi_0 = 0$ at $\phi_0 = (4/3\sqrt{3})^3$

At this minimum

$$\boxed{2\pi^2\mathcal{V}(\phi_0) = -(2\pi^2)(4/3\sqrt{3})^6/6 \simeq -.684}$$

$\rightarrow$ about 68% of the conjectured answer.

This is the beginning of a successive approximation scheme known as the level truncation scheme.

Kostelecky, Samuel
Kostelecky, Potting
A.S., Zwiebach
Moeller, Taylor

L_n^g : ghost Virasoro generators

L_n^m : matter Virasoro generators

$L_n \equiv L_n^g + L_n^m$: total Virasoro generators

Level of a state $\equiv$ Eigenvalue of $L_0 + 1$

Since $L_0 c_1 |0\rangle = -c_1|0\rangle$, $c_1|0\rangle$ has level 0

$c_0|0\rangle$ has level 1

$c_1 \alpha^\mu_{-1}|0\rangle \equiv c_1 \partial X^\mu(0)|0\rangle$ has level 1 etc.

Level (M, N) approximation to $\mathcal{V}(|\Phi\rangle)$:

1. Keep all fields $\{\phi_r\}$ with level $\leq M$. ($(L_0+1)|\chi_{1,r}\rangle = (h_r+1)|\chi_{1,r}\rangle$; $h_r + 1 \leq M$

2. Keep all terms in the action involving fields of level $\leq M$, for which the total level of all the fields is $\leq N$.

   e.g. at level (2,4) we keep interaction terms involving fields of level

   0-0-0, 0-0-2, 0-2-2 etc.

   but not of level 2-2-2

   $\rightarrow$ finite number of fields and finite number of terms for any given (M, N)

   $\rightarrow$ the equations can be solved (numerically).

3. Check for convergence as $M, N \rightarrow \infty$

From this viewpoint the previous approximation where we keep only the field ϕ_0 is level (0,0) approximation.

$\rightarrow$ gives $-2\pi^2\mathcal{V}(|\Phi_0\rangle) \simeq .684$

The result converges rapidly to 1 as we go to higher levels.

At level (10,20), we have $-2\pi^2\mathcal{V}(|\Phi_0\rangle) \simeq .9991$.

Some details:

1. First try to restrict the full zero momentum string field to a subspace which provides a consistent truncation of the SFT equations of motion

   → Eqs. of motion of fields outside the subspace is automatically satisfied if the string field lies in this subspace

   This allows us to restrict the string field to the

   a. Even level states

   b. The 'universal subspace', created from the vacuum $|0\rangle$ by the ghost oscillators and the matter Virasoro generators.

   In this subspace the D-brane action is universal for any D-brane in any background space-time.

2. Since we are dealing with a gauge theory, we need to 'gauge fix'

Standard choice: Siegel gauge

$$\boxed{b_0|\phi\rangle = 0}$$

3. Carry out level truncation

e.g. states at

level 0: $c_1|0\rangle$

level 2: $c_{-1}|0\rangle$, $c_1 L^m_{-2}|0\rangle$

level 4: $c_{-3}|0\rangle$, $c_{-1}L^m_{-2}|0\rangle$, $c_1 L^m_{-4}|0\rangle$,

$c_1 L^m_{-2} L^m_{-2}|0\rangle$, $c_1 b_{-3} c_{-1}|0\rangle$, $c_1 b_{-2} c_{-2}|0\rangle$

etc.

Note: The number of fields grows rapidly with level.

Level (10,20) → 102 fields

→ 99.91% OF THE CONJECTURED ANSWER

Verification of other conjectures

Conjecture 3: D-$(p-k)$-branes can be obtained as lump solution $|\Phi^k\rangle$:

$$\boxed{S(|\Phi^k\rangle) - S(|\Phi_0\rangle) = -V_{p-k+1}\mathcal{T}_{p-k+1}}$$

$\rightarrow$ requires studying string field configurations which depend on one or more spatial coordinates.

This can be studied using a generalization of the level truncation scheme.

$\rightarrow$ reproduces codimension 1 and 2 lump solutions with tensions in close agreement with the D-$(p-1)$-brane and D-$(p-2)$-brane tensions.

Moeller, A.S., Zwiebach
Moeller
de Mello Koch, Rodrigues

Codimension 1: 1% error.

Codimension 2: 13% error.

Analysis becomes complicated for higher codimension solutions.

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Conjecture 2: Absence of physical open string states around the tachyon vacuum

If $|\Phi_0\rangle$ is the vacuum solution, then define:

$$\boxed{\mathcal{Q}|A\rangle = Q_B|A\rangle + |\Phi_0\rangle * |A\rangle - (-1)^{n_A}|A\rangle * |\Phi_0\rangle}$$

We need to show that $\mathcal{Q}$ has vanishing cohomology:

1. Take the best available solution for $|\Phi_0\rangle$
2. Construct solutions of $\mathcal{Q}|A\rangle = 0$ by taking $|A\rangle$ to be arbitrary linear combinations of states up to certain level.
3. Show that there is a state $|B\rangle$ such that $|A\rangle = \mathcal{Q}|B\rangle$.

Complications: $\mathcal{Q}^2 \neq 0$ exactly with level truncated solution $|\Phi_0\rangle$.

$\rightarrow$ $|A\rangle \neq \mathcal{Q}|B\rangle$ exactly.

What one shows is that there is a state $|B\rangle$ such that

$$\boxed{\Big| |A\rangle - \mathcal{Q}|B\rangle \Big| << 1}$$

$|\;|$: some arbitrary norm

If we take the

1) level (10,20) solution and

2) work with $\mathcal{Q}$ up to level $L = 6$ states of ghost number 1,

then all $\mathcal{Q}$-closed states with

$k^2 \leq (L-1)$

are $\mathcal{Q}$-exact to more than 99% accuracy. Ellwood, Taylor

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The analysis has also been extented to computation of $\mathcal{Q}$-cohomology among states of arbitrary ghost number ~~state~~ using an indirect method. Ellwood, Feng, He, Moeller

Identity string field $|\mathcal{I}\rangle$ is defined as

$$\boxed{|\mathcal{I}\rangle = \prod_{n\geq 2} \exp(-2^{1-n} L_{-2^n}) \exp(L_{-2})|0\rangle}$$

It satisfies:

$$\boxed{|\mathcal{I}\rangle * |A\rangle = |A\rangle = |A\rangle * |\mathcal{I}\rangle \qquad \forall\, |A\rangle}$$

If $|\mathcal{I}\rangle = \mathcal{Q}|K\rangle$ for some $|K\rangle$, then,

$$\mathcal{Q}|A\rangle = 0 \quad \to \quad |A\rangle = |\mathcal{I}\rangle * |A\rangle = \mathcal{Q}(|K\rangle * |A\rangle)$$

Thus if we can find a $|K\rangle$ such that $|\mathcal{I}\rangle = \mathcal{Q}|K\rangle$, then $\mathcal{Q}$ must have vanishing cohomology.

Using level truncation it has been checked that $\Big||\mathcal{I}\rangle - \mathcal{Q}|K\rangle\Big| / \Big||\mathcal{I}\rangle\Big| << 1$ for appropriate $|K\rangle$.

Superstring field theory:

Berkovits

Formulated in a similar manner but has interactions of arbitrarily high order:

$$\boxed{\langle \Phi | Q_B \Phi * \eta_0 \Phi * \Phi \cdots \Phi \rangle + \text{permutations}}$$

η_0: fermionic ghost zero mode operator

Fortunately, if we work with fields up to a given level, then only a finite number of terms contribute.

→ one can use level truncation in the same way.

Current best result for the tachyon vacuum solution: reproduces about 90% of the conjectured value of the potential.

Berkovits
Berkovits, A.S., Zwiebach
Smet, Raeymaekers

Guessing the analytical form of the solution

So far we have described numerical approach to determining various solutions.

Known numerical solutions:

1. Tachyon vacuum solution

2. Various lump solutions

3. Solutions representing marginal deformations of the BCFT (*e.g.* switching on a Wilson line)

Can we examine these solutions to discover some patterns?

Physical interpretation of many of these solutions involve a deformation of the matter part of the CFT leaving the ghost part unchanged.

Various solutions are related to each other by appropriate RG-flow in the matter part of the boundary CFT.

This suggests that in cubic open string field theory we have a large number of redundant fields.

Naive guess: different solutions should have some common ghost sector and different matter sector.

This is false as can be checked by explicitly examining the different known solutions.

Next question: Could some part of the solution be universal, common to all solutions?

Suppose some part of the solution is universal.

In that case there is a very simple way of deriving this part by looking at marginal deformations.

Marginal deformation: characterized by a deformation parameter λ and a matter primary state $|V\rangle_m$ of dimension 1

Solution:

$$\boxed{|\Phi\rangle \;=\; \lambda|\chi^{(1)}\rangle + \lambda^2|\chi^{(2)}\rangle + \mathcal{O}(\lambda^3)}$$

$$\boxed{|\chi^{(1)}\rangle \;=\; c_1|0\rangle_g \otimes |V\rangle_m}$$

One can easily verify that:

$$\boxed{Q_B|\chi^{(1)}\rangle = 0, \qquad |\Phi * \Phi\rangle = \mathcal{O}(\lambda^2)}$$

Thus $|\Phi\rangle$ satisfies the SFT equations $Q_B|\Phi\rangle + |\Phi\rangle * |\Phi\rangle = 0$ to order λ.

Equation to order λ^2:

$$\boxed{Q_B|\chi^{(2)}\rangle = -|\chi^{(1)} * \chi^{(1)}\rangle}$$

Work in the Siegel gauge $b_0|\Phi\rangle = 0$.

Solution for $|\chi^{(2)}\rangle$:

$$\boxed{|\chi^{(2)}\rangle = -\frac{b_0}{L_0}|\chi^{(1)} * \chi^{(1)}\rangle}$$

$\rightarrow$ can be computed explicitly.

Answer:

$$\boxed{\begin{aligned} |\chi^{(2)}\rangle \;=\; & \alpha_0\Big(1 + \sum_{m,n=1}^{\infty} r_{m,n}c_{-m}b_{-n} + \sum_{m=1}^{\infty} s_m L^{(m)}_{-m} \\ & \qquad + \ldots\Big)c_1|0\rangle \\ & +\text{Tower over other matter primaries} \end{aligned}}$$

$r_{m,n}$, s_m: computable coefficients

$$\text{e.g.} \qquad \boxed{r_{m,n} = \frac{m}{m+n-1}\widetilde{N}^{11}_{nm}}$$

$\widetilde{N}^{11}_{nm}$: ghost Neumann coefficients computed by Gross and Jevicki

Some results:

$$r_{1,1} \simeq \frac{11}{27} = .407407, \quad r_{3,1} = -\frac{80}{729} \simeq -.109739$$

$$r_{5,1} = \frac{1136}{19683} \simeq .0577148,$$

$$r_{3,3} = \frac{2099}{98415} \simeq .021328$$

$$r_{2,2} = -\frac{19}{729} = -.026031$$

If any of these ratios $r_{m,n}$, s_m etc. is universal then that universal ratio must be equal to the ratio that appears in the expression for $|\chi^{(2)}\rangle$.

We now examine various known solutions and see if any of the ratios of coefficients appear to agree with the ratio that appears in the expression for $|\chi^{(2)}\rangle$.

Result: the ratios $r_{m,n}$ for m, n odd appear to be universal.

Gaiotto, Rastelli, A-S., Zwiebach

Results for the tachyon vacuum solution in $(L, 3L)$ approximation:

L	$r_{1,1}$	$r_{3,1}$	$r_{5,1}$	$r_{3,3}$
4	0.3750	-0.1025		
6	0.3866	-0.1047	0.05478	0.02084
8	0.3931	-0.1060	0.05544	0.02091
10	0.3972	-0.1067	0.05585	0.02099
∞	0.4115	-0.1095	0.05746	0.02121
$\chi^{(2)}$	0.4074	-0.1097	0.05771	0.02133

L	$r_{7,1}$	$r_{5,3}$
8	-0.03580	-0.014195
10	-0.03610	-0.0142012
∞	-0.03730	-0.01423
$\chi^{(2)}$	-0.03748	-0.01439

$r_{9,1}\|_{level10} = 0.02594$,	$r_{9,1}\|_{\chi^{(2)}} = 0.02706$
$r_{7,3}\|_{level10} = 0.1049$,	$r_{7,3}\|_{\chi^{(2)}} = 0.01059$
$r_{5,5}\|_{level10} = 0.006586$,	$r_{5,5}\|_{\chi^{(2)}} = 0.006619$

In contrast

$r_{2,2}\|_{level10} = -0.06404$,	$r_{2,2}\|_{\chi^{(2)}} = -0.02606$

Thus $r_{2,2}$ is not universal.

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Results for codimension one lump solution on a circle of radius R (level (4,8)):

R^2	1.05	1.5	2.9	$\chi^{(2)}$
$r_{1,1}$	0.4008	0.3859	0.3746	.4074
$r_{3,1}$	-0.1084	-0.1077	-0.1037	-.1097
$r_{2,2}$	-0.03123	- 0.04980	-0.05100	- 0.02606

Results for solutions representing marginal deformation with parameter λ (level (4,8)):

λ	.05	.2	.3	.32
$r_{1,1}$	0.4071	0.4020	0.3905	0.3857
$r_{3,1}$	-0.1097	-0.1084	-0.107	-0.1065
$r_{2,2}$	-0.02629	-0.03040	-0.03944	-0.04329

Possible origin of universality

$$\boxed{Q_B|\Phi\rangle = -|\Phi * \Phi\rangle}$$

$\rightarrow$ a non-linear equation.

Thus the 'universal coefficients' in $Q_B|\Phi\rangle$ receive contribution from non-universal coefficients in $|\Phi\rangle$ in computing $|\Phi * \Phi\rangle$.

From this point of view universality of some coefficients looks very surprising.

How is it possible to get such a universality?

A possible resolution: $|\Phi\rangle$ satisfies a set of linear equations.

How can we get such linear equations?

Result: Given any pair of Fock space states $|A\rangle$ and $|B\rangle$, we have

$$\boxed{c(\pm i)|A * B\rangle = 0}$$

since c has negative dimension.

We proceed by assuming that this property holds when either $|A\rangle$ or $|B\rangle$ or both are replaced by a 'good' string field configuration.

Then we have:

$$\boxed{c(\pm i)|\Phi * \Phi\rangle = 0, \quad \to \quad c(\pm i)Q_B|\Phi\rangle = 0}$$

$\to$ a set of linear equations for $|\Phi\rangle$.

There may be other linear equations of this type which will fix the universal ratios in the solution of SFT equations of motion.