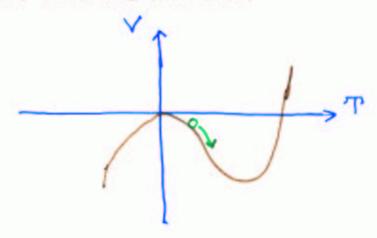
## JIME DEPENDENT SOLUTION;

FOR DEFINITENESS.



WE WANT TO STUDY SOLUTIONS

DESCRIBING ROLLING OF THE

TACHYON TOWARDS THE MINIMUM

OF THE POTENTIAL

RESULT: THE SYSTEM EVOLVES TO

A GAS OF ZERO PRESSURE AS THE

TACHYON ROLLS TOWARDS THE

MINIMUM OF VCT)



### Derivation of these results:

General strategy for studying time dependent solution in any theory:

- Begin with a static solution that depends on some spatial coordinate x.
- 2. Replace  $\underline{x}$  by  $\underline{i}\underline{x}^0$  where  $\underline{x}^0$  is the time coordinate.
- The new configuration is bound to be (formally) a solution of the equations of motion.
- 4. But there could be problems e.g.

The solution may not be real.

The solution may hit a singularity.

### BEGIN WITH LINEARIZED EQ. OF MOTION:

GIVES:

BOTH SOLUTIONS ARE VALID FOR



# WE SHALL FOCUS ON THE SOLUTION $T(\mathbf{x}^\circ) \cong \lambda \ \text{Cosh}(\mathbf{x}^\circ)$

#### FOR DEFINITENESS

PERTURBATION TO THE CFT:

PARAMETER LABELLING THE REAL

AXIS OF UPPER HALF PLANE

CAN WE FIND A SOLUTION TO THE
FULL EQUATIONS OF MOTION IN
STRING FIELD THEORY AS A POWER
SERIES IN A P

WE CAN, IF THE ABOVE BOUNDARY
PERTURBATION IS EXACTLY MARGINAL

IS IT & EXACTLY MARGINAL ?

(3)

CO TO THE WICK ROTATED THEORY

X°→ i X

PERTURBATION BECOMES

~ Sat cos (x(+))

THIS IS KNOWN TO BE AN EXACTLY
MARGINAL DEFORMATION

→ A SOLUTION OF SFT EQUATIONS OF MOTION

{ \$\phi\_{\mathbb{r}}(k) \rightarrow \interestarrow \{\bar{\phi}\_{\mathbb{r}}(\mathbb{x})\rightarrow \interestarrow \{\bar{\phi}\_{\mathbb{r}}(\mathbb{x})\rightarrow \interestarrow \intere

 $\widehat{\varphi}_{r}\left(\mathbf{x}^{\circ}\right) \equiv \widehat{\varphi}_{r}\left(-i\mathbf{x}^{\circ}\right) \quad \text{is A SOLUTION}$  OF THE . EQS. OF MOTION

THAT  $\hat{\Phi}_{r}(x^{e})$  IS REAL.



THIS SHOWS THE EXISTENCE OF A

FAMILY OF TIME DEPENDENT SOLUTIONS

PARAMETRIZED BY X.

IN STUDYING PROPERTIES OF THESE
SOLUTIONS, IT IS MORE CONVENIENT
TO DIRECTLY WORK WITH THE
DEFORMED BOUNDARY CFT

Consider the case  $T \simeq \lambda \cosh(x^0)$ 

Corresponds to a bondary deformation by:

$$\tilde{\lambda} \int dt \, \cosh(X^0(t))$$

$$\tilde{\lambda} = \lambda + \mathcal{O}(\lambda^2)$$

 $\tilde{\lambda}>0$  corresponds to pushing the tachyon in a direction in which the potential has a local minimum.

For  $\tilde{\lambda} > 0$ , the boundary term is positive definite and hence we might expect the theory to be sensible.

 $\tilde{\lambda} < 0$  corresponds to pushing the tachyon in a direction in which the potential is unbounded from below.

For  $\tilde{\lambda} < 0$ , the boundary term is negative definite and hence we might expect the theory to be sick.

CAN WE SAY SOMETHING MORE QUANTITATIVE
FOR THIS BCFT?

E.g. HOW DOES THE ENERGY-MOMENTUM
TENSOR THE EVOLVE WITH TIME?

STUDY THE BOUNDARY STATE

WE SHALL STUDY THE BOUNDARY

STATE IN THE WICK ROTATED THEORY

AND THEM MAKE X - ZX REPLACEMENT

BOUNDARY STATE IB) ASSOCIATED WITH A BIFT

A CLOSED STRING STATE OF

GHOST NO. 3

COSED STRING VERTEX OF V(0)
 ON THE UNIT DISK
 DISK
 TO BCFT

V(0)

65

THE BOUNDARY STATE ASSOCIATED WITH A D-BRANE TELLS US WHAT TYPE OF SOURCE THE D-BRANE IS FOR THE CLOSED STRING FIELD

CLOSED STRING FIELD ITY :

A STATE OF THE FIRST

QUANTIZED CLOSED STRING OF GHOST

NUMBER 2

LINEARIZED EQ. OF MOTION OF 14.7

99

SINCE 187 CONTAINS SOURCE TERM

FOR how, IT HAS INFORMATION

ABOUT ENERGY MOMENTUM TENSORTIN

GENERAL FORM OF 18>:

CLOSED STRING FIELD EQ.

COMPUTE THE TIME.

A DIFFERENT METHOD:



THE BOUNDARY STATE IN THE WICK

ROTATED THEORY CAN BE COMPUTED

(Recknight, Schonerus)

WORLD - SHEET ACTION

LS LZZZ [ 22 × 22 × ] + X S dt Cos (×(+1))

+ ---- 25 OTHER SCALAR FIELD

RESULT:

$$T_{00}(x) = -\frac{1}{2} T_{b} (1 + \cos(2\tilde{\lambda}\pi))$$

$$T_{0i} = 0, \qquad T_{ij} = -T_{b} \hat{f}(x) \delta_{ij}$$

$$\hat{f}(x) = 1 + \sum_{n=1}^{\infty} (-1)^{n} (\sin\tilde{\lambda}\pi)^{n} \{e^{inx} + e^{-inx}\}$$

WICK ROTATE : X = - 1x°

$$\hat{f}(-ix^\circ) = 1 + \sum_{n=1}^{\infty} (-i)^n (\sin \lambda \pi)^n (e^{nx^\circ} + e^{-nx^\circ})$$

$$f(x^\circ) = \frac{1}{1 + e^{-x^\circ} \sin \lambda \pi} + \frac{1}{1 + e^{-x^\circ} \sin \lambda \pi} - 1$$

- GIVES THE RESULT QUOTED FARLIER

$$f(x^{\circ}) = \frac{1}{1 + e^{x^{\circ}} \sin \tilde{\chi} \pi} + \frac{1}{1 + e^{-x^{\circ}} \sin \tilde{\chi} \pi} - 1$$

As 
$$x^{\circ} \rightarrow \infty$$
,  $f(x^{\circ}) \rightarrow 0$ 

THE SYSTEM EVOLVES TO A PRESSURELESS GAS

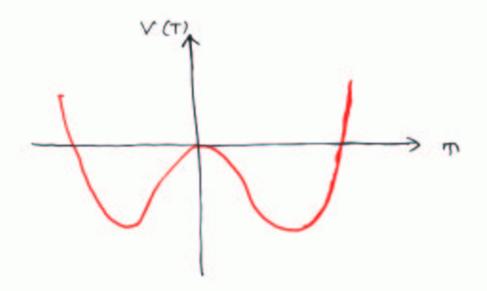
CASE OF PERTURBATION BY

RELATED TO THE PREVIOUS CASE

$$\Rightarrow f(x^{\circ}) = \frac{1}{1 + e^{x^{\circ}} \sinh(\widetilde{\chi}\pi)} + \frac{1}{1 - e^{-x^{\circ}} \sinh(\widetilde{\chi}\pi)}$$



SIMILAR RESULTS HOLD FOR UNSTABLE D-BRANES / DO SYSTEM#S IN SUPER STRING THEORY



Begin with the linearized equations of motion:

$$(\partial_0^2 + m^2)T = 0$$

$$m^2 = -1/2$$

Gives

$$T = Ae^{x^0/\sqrt{2}} + Be^{-x^0/\sqrt{2}}$$

1) 
$$T = \lambda$$
,  $\partial_0 T = 0$  at  $x^0 = 0$  gives 
$$T(x^0) = \lambda \cosh(x^0/\sqrt{2})$$

2) 
$$T = 0$$
,  $\partial_0 T = \lambda \cot x^0 = 0$  gives 
$$T(x^0) = \lambda \sinh(x^0/\sqrt{2})$$



We shall focus on the solution

$$T(x^0) \sim \lambda \cosh(x^0/\sqrt{2})$$

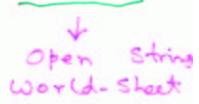
→ corresponds to adding a boundary pertur-

$$\tilde{\lambda} \int dt \, [\psi^0 \sinh(X^0/\sqrt{2})]_{boundary} \otimes \sigma_1$$

$$\tilde{\lambda} = \lambda + \mathcal{O}(\lambda^2)$$

 $\psi^0$ : world-sheet superpartner of  $x^0$ 

σ<sub>1</sub>: Chan-Paton factor



Can we find a solution to the full set of equations of motion in string field theory as a power series in  $\lambda$ ?

We can, if the above boundary perturbation is exactly marginal.

Is it exactly marginal?



Go to the Wick rotated theory

$$X^0 o i X$$
,  $\psi^0 o i \psi$ 

Perturbation becomes

$$\widetilde{\lambda} \int dt \, [\psi \sin(X/\sqrt{2})]_{boundary} \otimes \sigma_1$$

This is known to be an exactly marginal deformation via fermionization of X

On the boundary:

$$e^{iX/\sqrt{2}} \sim (\xi + i\eta)$$

 $\xi$ ,  $\eta$  are Majorana fermions.

Thus perturbation becomes proportional to:

$$\tilde{\lambda} \int dt \, [\psi \eta]_{boundary} \otimes \sigma_1$$

→ a solvable CFT

The relevant part of the boundary state for computation of  $T_{\mu\nu}$ :

$$|\mathcal{B}_{0}\rangle \propto \left[\psi^{\mu}_{-1/2}\bar{\psi}^{\nu}_{-1/2}A_{\mu\nu}(X^{0}(0)) + (\bar{\beta}_{-1/2}\gamma_{-1/2} - \beta_{-1/2}\bar{\gamma}_{-1/2})B(X^{0}(0))\right]|\Omega\rangle,$$

 $\beta, \gamma$ : bosonic superconformal ghosts

 $|\Omega\rangle$ : the ghost number 3, picture number 2 vacuum

$$|\Omega\rangle = (c_0 + \bar{c}_0)c_1\bar{c}_1e^{-\phi(0)}e^{-\bar{\phi}(0)}|0\rangle$$

 $\phi$ : bosonized ghost

Energy momentum tensor:

$$T_{\mu\nu}(x^0) = A_{\mu\nu}(x^0) + B(x^0)\eta_{\mu\nu}$$

(Follows from the requirement that  $Q_B|\mathcal{B}_0\rangle=0$  should reproduce  $\partial^\mu T_{\mu\nu}=0$ )



Results for  $A_{\mu\nu}$  and B can be obtained by picking up the appropriate terms in the boundary state of the Wick rotated theory and then performing an inverse Wick rotation.

Result: ( Following Recknagel, Schomerus; Gaberdiel,

$$A_{00} = g(x^0), \quad A_{ij} = -f(x^0)\delta_{ij}, \quad B = -f(x^0)$$

$$g(x^0) = 1 + \cos(2\pi\tilde{\lambda}) - f(x^0)$$

$$f(x^{0}) = 1 + \sum_{n=1}^{\infty} (-1)^{n} \sin^{2n}(\tilde{\lambda}\pi)$$

$$\times (e^{n\sqrt{2}x^{0}} + e^{-n\sqrt{2}x^{0}})$$

$$= \frac{1}{1 + e^{\sqrt{2}x^{0}} \sin^{2}(\tilde{\lambda}\pi)}$$

$$+ \frac{1}{1 + e^{-\sqrt{2}x^{0}} \sin^{2}(\tilde{\lambda}\pi)} - 1.$$

(Note:  $e^{n\sqrt{2}x^0}$  comes from inverse Wick rotation of  $e^{in\sqrt{2}x}$ )

This gives us back the answer quoted earlier.

The result for  $T(x^0) \sim \lambda \sinh(x^0/\sqrt{2})$  can be obtained by the replacement

$$\lambda \to -i\lambda, \qquad x^0 \to x^0 + i\pi/\sqrt{2}$$



Raslelli, A.s., Zueb W/ Giotto Gross, Taylor

### Vacuum string field theory (VSFT)

Suppose  $|\Phi_0\rangle$  denotes the tachyon vacuum solution

$$|\Psi\rangle \equiv |\Phi\rangle - |\Phi_0\rangle \in \mathcal{H}_1$$

$$S(|\Phi\rangle) = S(|\Phi_0\rangle) + \tilde{S}(|\Psi\rangle)$$

$$\tilde{S}(|\Psi\rangle) = -\frac{1}{g^2} \left[ \frac{1}{2} \langle \Psi | \mathcal{Q} | \Psi \rangle + \frac{1}{3} \langle \Psi | \Psi * \Psi \rangle \right]$$

$$\mathcal{Q}|A\rangle \equiv Q_B|A\rangle + |\Phi_0\rangle * |A\rangle - (-1)^{n_A}|A\rangle * |\Phi_0\rangle$$

Q satisfies a set of identities (E):

1) 
$$Q^2 = 0$$

2) 
$$\langle QA|B\rangle = (-1)^{n_A} \langle A|Q|B\rangle$$

3) 
$$Q(|A\rangle * |B\rangle) = (Q|A\rangle) * |B\rangle + (-1)^{n_A}|A\rangle * (Q|B\rangle)$$

 $\rightarrow$  guarantees gauge invariance of  $\tilde{S}(|\Psi\rangle)$ .





Conjecture 3: Q has vanishing cohomology

At present we know the form of  $|\Phi_0\rangle$  and hence of Q only numerically.

Strategy: Try to guess the form of Q which satisfies all the requirements.

Proposal: It is possible, via an appropriate field redefinition which leaves the cubic coupling unchanged, to bring Q into a form made purely of ghost fields.

Insight: beginning with any BCFT, we should end up in the same vacuum.

(vacuum without any D-brane)

Thus SFT around the tachyon vacuum should be insensitive to the choice of initial BCFT.

Deing made of ghosts → independent of the choice of BCFT.



Note however that  $|\Psi\rangle \in \mathcal{H}_1$ , — the vector space of states of BCFT.

→ there is still some implicit dependence on BCFT.

All physical results should be independent of the choice of BCFT (need to be shown)

Is there a choice of Q which is made purely of ghosts, satisfies the required identities E, and has vanishing cohomology?

Yes.

$$Q = \sum_{n} a_n (c_n + (-1)^n c_{-n})$$

 $a_n$ : arbitrary coefficients.

Each of these Q satisfies the identities E.

What about the cohomology of Q?

Suppose  $a_m \neq 0$  for some m.

Define 
$$3b = \frac{1}{2}a_m^{-1}(b_m + (-1)^m b_{-m})$$

Then 
$$\{Q, B\} = 1$$

Thus if  $Q|A\rangle = 0$ , then  $|A\rangle = QB|A\rangle$ 

- $\rightarrow$  any  $\mathcal{Q}$ -closed state  $|A\rangle$  is also  $\mathcal{Q}$ -exact.
- → Q has trivial cohomology.

We now need to verify that:

The action

$$\widetilde{S}(|\Psi\rangle) = -\frac{1}{g^2} \left[ \frac{1}{2} \langle \Psi | \mathcal{Q} | \Psi \rangle + \frac{1}{3} \langle \Psi | \Psi * \Psi \rangle \right]$$

has a classical solution representing the original D-brane associated with BCFT.

 It has classical solution representing any other D-brane in the same space-time background, associated with some other boundary CFT BCFT'.

(the lump solutions)

Equations of motion:

$$Q|\Psi\rangle + |\Psi\rangle * |\Psi\rangle = 0$$

Look for factorized solution:

$$|\Psi\rangle = |\Psi_g\rangle \otimes |\Psi_m\rangle$$

$$g o ghost part, \qquad \underline{m} o matter part$$
 
$$\mathcal{Q}|\Psi_g\rangle \otimes |\Psi_m\rangle + |\Psi_g * \Psi_g\rangle \otimes |\Psi_m * \Psi_m\rangle = 0$$

This gives

$$Q|\Psi_g\rangle + \mathcal{N}|\Psi_g * \Psi_g\rangle = 0$$
$$|\Psi_m * \Psi_m\rangle = \mathcal{N}|\Psi_m\rangle$$

 $\mathcal{N}$ : arbitrary normalization constant

For such a solution the value of the action is

$$\widetilde{S}(|\Psi\rangle) = -\frac{1}{6g^2} \langle \Psi_g | \mathcal{Q} | \Psi_g \rangle \langle \Psi_m | \Psi_m \rangle$$

Universality ansatz: different D-branes are described by the same ghost part  $|\Psi_g\rangle$  but by different matter parts  $|\Psi_m\rangle$ .

$$BCFT$$
 D-brane  $\rightarrow |\Psi\rangle = |\Psi_g\rangle \otimes |\Psi_m\rangle$ 

$$BCFT'$$
 D-brane  $\rightarrow |\Psi\rangle = |\Psi_g\rangle \otimes |\Psi'_m\rangle$ 

Note: both  $|\Psi_m\rangle$  and  $|\Psi'_m\rangle$  belong to the same state space  $\mathcal{H}_1$  (of BCFT)

Ratio of the D-brane energies:

$$\frac{\widetilde{E}'}{E} = \frac{\widetilde{S}(|\Psi'\rangle)}{\widetilde{S}(|\Psi\rangle)} = \frac{\langle \Psi'_m | \Psi'_m \rangle_m}{\langle \Psi_m | \Psi_m \rangle_m}$$

 $\langle A|B\rangle_m$ : inner product in the matter sector only

Thus we can compute the ratio E'/E even without knowing the form of  $\mathcal{Q}$ , and compare with the known ratios of tensions of D-branes.





$$|\Psi_m * \Psi_m\rangle = \mathcal{N} |\Psi_m\rangle,$$

$$|\Psi'_m * \Psi'_m \rangle = \mathcal{N} |\Psi'_m \rangle$$

→ projectors of the \*-algebra in the matter sector.

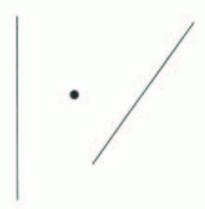
Candidate projectors describing different D-branes have been constructed.

Computation of  $\langle \Psi'_m | \Psi'_m \rangle_m / \langle \Psi_m | \Psi_m \rangle_m$  gives the correct ratios of energies of different D-branes.

→ verification of conjectures 1 and 2.



One can also construct solutions representing a configuration of arbitrary number of Dbranes at arbitrary positions and orientations.



Strategy: Principle of superposition

If  $|\Psi_m\rangle$  and  $|\Psi_m'\rangle$  denote two different projectors describing two different D-branes then one can show that

$$|\Psi_m * \Psi'_m\rangle = 0,$$
  $\langle \Psi_m | \Psi'_m \rangle_m = 0$ 

Thus  $|\Psi_m\rangle + |\Psi_m'\rangle$  is also a projector and describes a solution representing the superposition of two D-branes.

This can be generalized to construct superposition of arbitrary number of D-branes. (08)

So far our analysis has been insensitive to the choice of Q.

What is Q?

Strategy for finding Q:

1. Take a general form

$$Q = \sum_{n=0}^{\infty} a_n (c_n + (-1)^n c_{-n})$$

2. Fix Siegel gauge  $b_0|\Psi\rangle = 0$ .

The equations of motion in this gauge take the form:

$$a_0|\Psi\rangle + b_0|\Psi * \Psi\rangle = 0$$

3. Then determine Q by requiring that the solution to the above equation satisfies the full set of equations of motion:

$$Q|\Psi\rangle + |\Psi * \Psi\rangle = 0$$

It turns out that among the class of operators linear in  $c_n$ , the only consistent choice is

$$Q = K[c_0 + \sum_n (-1)^n (c_{2n} + c_{-2n})] = \frac{K}{2i} [c(i) - c(-i)]$$

K: a normalization constant

We can determine K by requiring that the solution  $|\Psi\rangle$  of the form  $|\Psi_g\rangle\otimes|\Psi_m\rangle$  representing the original D-p-brane must have

$$S(|\Psi\rangle) = -\mathcal{T}_p V_{p+1}$$

It turns out that in order to satisfy this condition we must have  $K = \infty$ .

Conclusion: Vacuum string field theory must be related to the original SFT by a singular field redefinition.



Can we identify what field redefinition relates VSFT to the original SFT?

A complete answer will require the knowledge of the solution  $|\Phi_0\rangle$  which represents the tachyon vacuum in the original SFT.

However one can guess the singular part of the field redefinition that is responsible for the singular nature of Q in VSFT.

Consider a reparametrization of the string that is symmetric about the mid-point.

$$\sigma \to f(\sigma)$$
 with  $f(\pi - \sigma) = \pi - f(\sigma)$ .

Suppose further that the effect of this reparametrization is to squeeze most of the string to its mid-point.

i.e. 
$$f(\sigma) \simeq \pi/2$$
 for most of the range of  $\sigma$ .

Under such a reparametrization the coefficient of any negative dimension operator at the midpoint will grow large.





Thus if initially Q contained a piece of the form

$$\int d\sigma g(\sigma)c(\sigma)$$

for some finite function  $g(\sigma)$ , then after this reparametrization the dominant contribution to Q will be of the form

$$K(c(i)-c(-i))$$

with infinite coefficient K.

This mechanism not only explains the origin of the singularity in the VSFT action, but also provides a natural mechanism for obtaining a  $\mathcal Q$  made of pure ghosts.

Even if to begin with  $\mathcal Q$  contained matter pieces, after this reparametrization the ghost part involving  $c(\pm i)$  gives the dominant contribution.



### CONCLUSION,

DYNAMICS OF UNSTABLE D-BRANE SYSTEMS IN STRING THEORY EXHIBIT MANY NOVEL FEATURES

FURTHER STUDY OF THIS DYNAMIC IS LIKELY TO IMPROVE OUR UNDER STANDING OF FORMAL ASPECTS OF STRING THEORY AND ALSO POSSIBLE COSMOLOGICAL ASPECTS OF STRING THEORY.