Frederik Denef, JAN. 12, 2009

WALL CROSSING & BLACK HOLES
— A REVIEW —

<u>Part I:</u> Various realizations of BPS States in String theory

How realized in supergravity

$g_s \to 0$ How they become D-branes

<u>Part II:</u> How you get wall-crossing and WCF. Concrete example D6D0.

1 line summary: WC = Decay of black hole molecules.

① BPS STATES IN STRING THEORY

Ⓐ $\mathcal{N}=2$, $D=4$ susy algebra Q_I

$I = 1, \ldots, 8$ $\quad Q_I$

$$\mathcal{H} = \mathcal{H}_B \oplus \mathcal{H}_F$$

$$[Q_I, H] = 0$$

If a <u>state</u> in the Hilbert space satisfies $Q_I|0\rangle = 0 \Rightarrow$ vacuum

$\Rightarrow H|0\rangle = 0 \Rightarrow E = 0, M = 0$

Not unique, labelled by

$$|0; \underbrace{t, g}_{\text{moduli}}\rangle$$

$t \in \mathcal{M}_V$ vectormultiplet

$g \in \mathcal{M}_H$ hypermultiplet.

Different moduli: Superselection sectors

Can't go $(t, g) \to (t, g)'$ with finite energy

From the susy algebra one can derive a BPS bound.

$$M \geq |Z(\Gamma, t)|$$

Γ is a charge

$Z(\Gamma, t)$ Central charge.

If there is a single photon in the theory $\Gamma = (P, Q) \in \mathbb{Z}^2$

P = magnetic charge

Q = electric charge.

Symplectic Product

$$\langle (P_1, Q_1), (P_2, Q_2) \rangle = P_1 Q_2 - P_2 Q_1$$

Then $Z(\Gamma, t) = Q \cdot t + P \cdot \dfrac{\partial \mathcal{F}}{\partial t}$

$\mathcal{F}$ = SOME HOLO FN. without gravity

= PREPOTENTIAL.

$\dim \mathcal{M}_V$ = # photons.

n PHOTONS $\Rightarrow$ $\Gamma = \mathbb{Z}^{2n}$

with gravity – some modifications.

always linear in Γ.

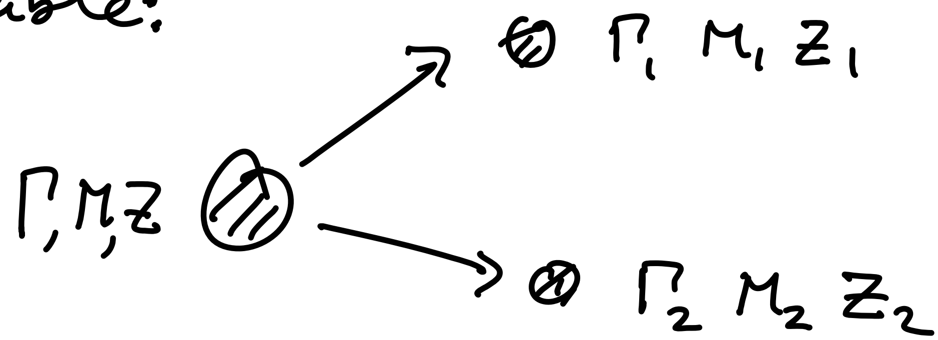
When BPS bound is saturated, we have a BPS state

$$M = |Z|$$

$\Longleftrightarrow$ $1/2$ of susy's annihilates

The state "$1/2$ susy is preserved"

At generic values, BPS states are stable:



Then

$$M \geq M_1 + M_2 + \text{kinetic energy}$$

but

$$\geq |Z_1| + |Z_2|$$

$$\underset{\text{generically}}{>} |Z_1 + Z_2| = M$$

Nonprimitive charges. Above is for

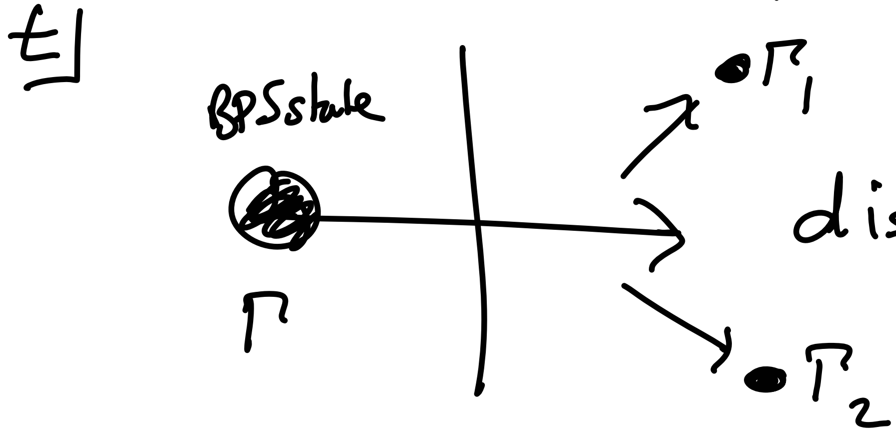
<u>Γ primitive</u>

<u>Notable exception:</u> $\Gamma \to \Gamma_1 + \Gamma_2$

Where phases do line up.

$\arg Z_1 = \arg Z_2 \Longrightarrow$ Cod. 1

Wall in moduli space



BPS INDEX

$$\Omega(\Gamma,t) \overset{?}{=} \dim \mathcal{H}^{B}_{BPS}(\Gamma,t) - \dim \mathcal{H}^{F}_{BPS}(\Gamma,t)$$

$$= \mathrm{Tr}_{\mathcal{H}_{BPS}(\Gamma,t)} (-1)^{2J_z}$$

$$= 0$$

$$\mathcal{H}_{BPS} = \left[0 \oplus 0 \oplus \left(\tfrac{1}{2}\right)\right] \otimes \mathcal{H}'_{BPS}$$

So just trace over $\mathcal{H}'_{BPS}$ with $(-1)^{2J_z}$

gives an index only receiving contributions from BPS states. invt under generic deformations.

Will not change under HM moduli such as g_s.

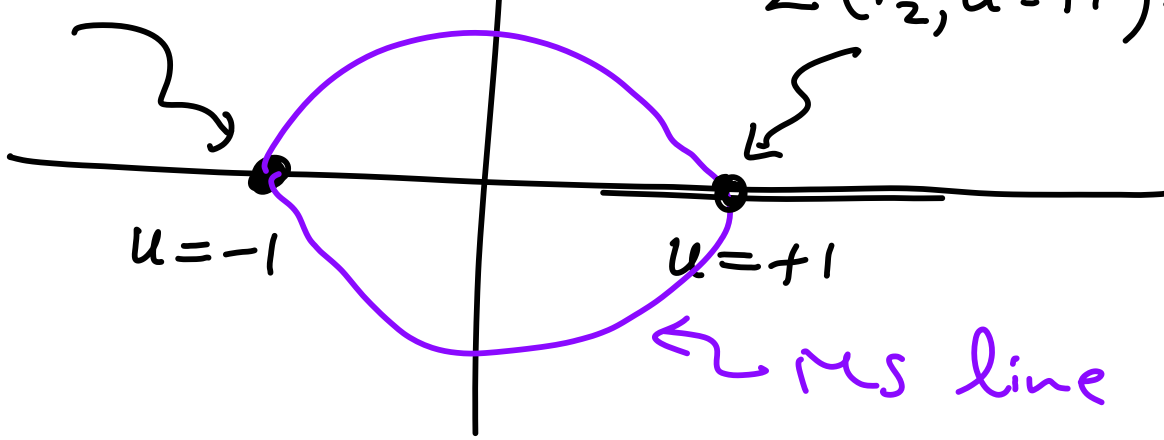
Will change across walls of MS

$\Delta\Omega \to$ w.c.f.

First example in 4 dimensions –
Seiberg-Witten Theory $SU(2)$, $\mathcal{N}=2$, has 1-dim'l VM moduli space $t=u$

$Z(\Gamma_1, u=-1)=0$

$Z(\Gamma_2, u=+1)=0$



Inside $\Omega(\Gamma_1) = 1$

$\Omega(\Gamma_2) = 1$

Outside: $\Omega(\Gamma_1+\Gamma_2) = -2.$

$$\Omega(n\Gamma_1+(n+1)\Gamma_2) = +1$$

$$\Omega((n+1)\Gamma_1+n\Gamma_2) = +1$$

$u \longrightarrow \infty$ $\Gamma_1+\Gamma_2$ exists by weak coupling

Monodromy maps $n\Gamma_1 + (n+1)\Gamma_2$ to Γ_2

First example in K+S paper.

BPS States in 4D SUGRA:

Spherically symmetric BH

metric (r)

moduli $t(r)$

$t(r)$ follow gradient flow of $|Z|$

Can do same thing in field theory

$$S = \frac{A}{4} = \pi \cdot \min_{t} |Z(\Gamma; t)|^2$$

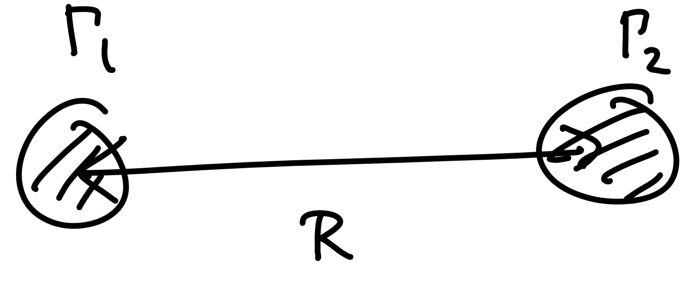
$$\sim \log \Omega(\Gamma; \cancel{t})$$

Suitable large charge Γ

$$\Gamma \to \lambda \Gamma$$

What about wall-crossing?

There are boundstates of two black holes Γ_1, Γ_2 which are BPS.



$$R = \frac{1}{2}\langle \Gamma_1, \Gamma_2 \rangle \frac{|Z_1 + Z_2|}{\mathrm{Im}(Z_1 \bar{Z}_2)} l_P$$

$R > 0$ so necessary condition for stability is

$$\mathrm{Im}\left(Z_1 \bar{Z}_2\right) \langle \Gamma_1, \Gamma_2 \rangle > 0$$

- Can write explicit solutions if you know $S(\Gamma)$.
- $\exists$ Conjectured 1-1 correspondence with attractor flow trees.

For a given total charge Γ -

$$\mathcal{H}_{BPS}(\Gamma; t) = \bigoplus_{\text{trees}}$$

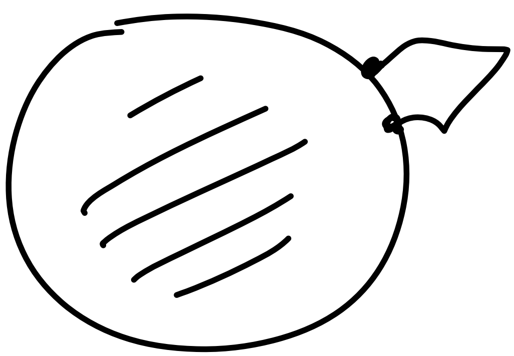
Above picture can be used to deduce a WCF.

How is all this related to D-branes?

(c) Go to $g_s \to 0$ regime where perturbation theory becomes valid. g_s doesn't affect Ω but does affect physical picture

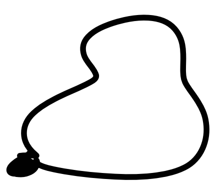
$$g_s = \frac{l_p}{l_s} \qquad g_s \to 0 \Rightarrow$$

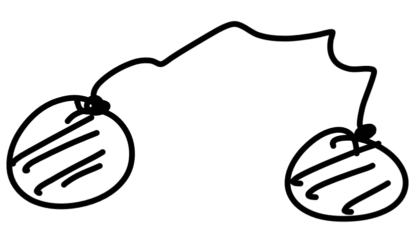
switch off grav. int.



$g_s \to 0$

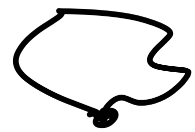
Size of BH becomes small





equilibrium

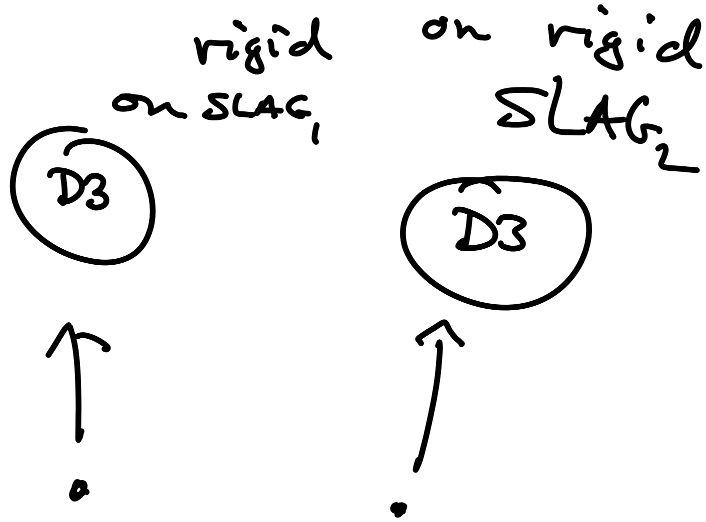
$g_s \to 0$



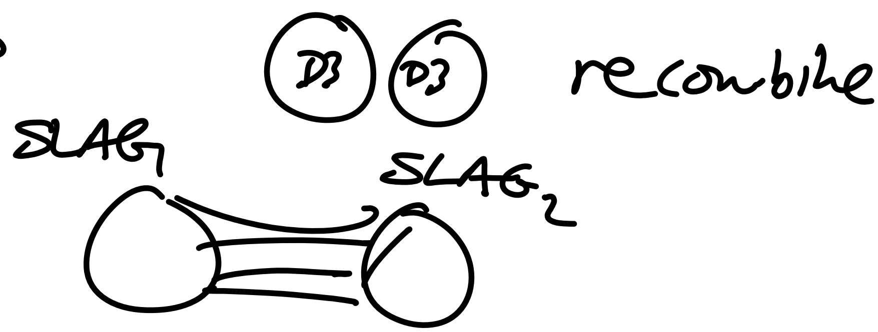
In visible space everything shrinks to a point.

But there is a 6 dim. internal space. . . .

Type IIB



$g_s \to 0$



# of reconnecting throats $= \langle \Gamma_1, \Gamma_2 \rangle$

Joyce transition.

IIB

$* \quad Z = \int \Gamma \wedge \Omega^{(3,0)}$ IIB

$= - \int \Gamma \wedge \frac{e^{-(B+iJ)}}{\sqrt{J^3}} + \cdots$ IIA

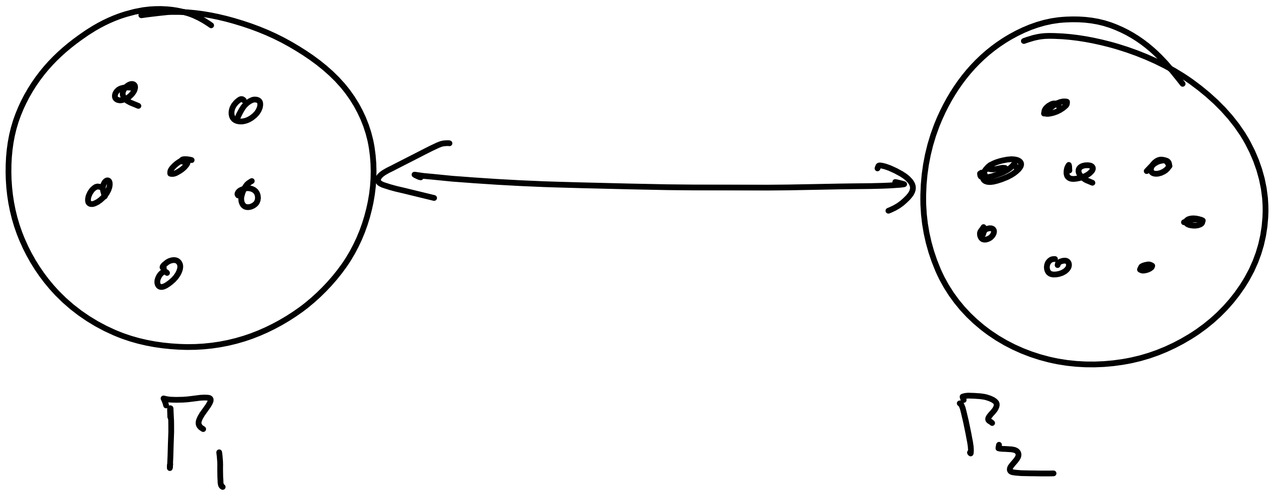
$* \quad \langle \Gamma_1, \Gamma_2 \rangle =$ standard $\cap$ IIB

$= \int \Gamma_1 \, \Gamma_2^*$ IIA

BPS states @ $g_s \to 0$ are coho. classes on moduli spaces of these objects.

$*$ Spatial $J \sim$ Lefshetz $SU(2)$

$\Longrightarrow \quad \Omega = (-1)^{\dim_{\mathbb{C}} \mathcal{M}} \text{"} \chi(\mathcal{M}) \text{"}$

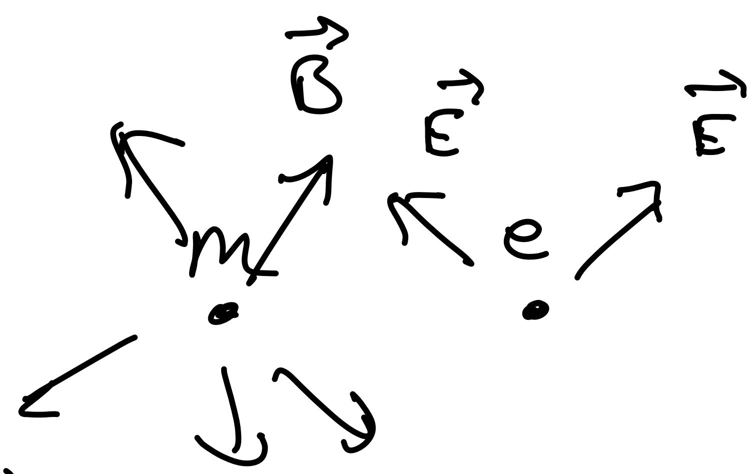


If Γ_1, Γ_2 are both primitive

$$\Delta\mathcal{H} = \underbrace{[J]}_{} \otimes \mathcal{H}_1 \otimes \mathcal{H}_2$$

arises from quantizing 2 particle system of particles of charges Γ_1, Γ_2.

groundstate is in nonzero spin multiplet



$\vec{E} \times \vec{B}$ = Poynting vector

$\vec{E} \times \vec{B}$ circles around the axis so there is some spin in the field

$$J = \frac{1}{2} \langle \Gamma_1, \Gamma_2 \rangle$$

quantum correction

$$J = \frac{1}{2} \langle \Gamma_1, \Gamma_2 \rangle - \frac{1}{2}$$

$\phi_1 \; \phi_2 \; \phi_3 \; \phi_4$

$$\sum |\phi_i|^2 = \text{const.}$$

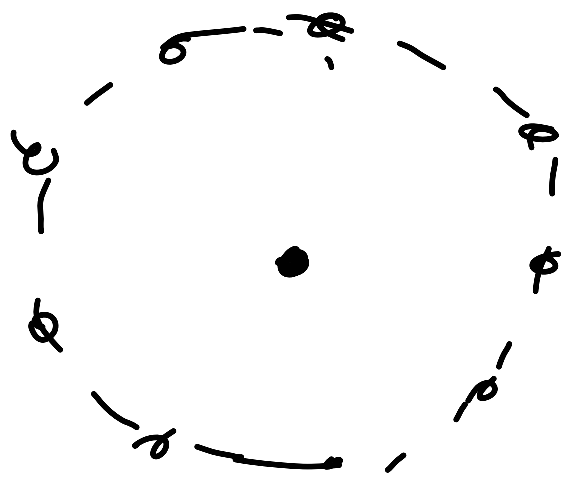
$$\Delta\Omega = (-1)^{2J} \, \Omega(\Gamma_1, t_{ms}) \, \Omega(\Gamma_2, t_{ms})$$

$(2J+1)$

electron moving on sphere in field of magnetic monopole.

One charge primitive other nonprimitive

$$\Gamma = \Gamma_1 + N\Gamma_2$$



Multiparticle system – noninteracting particles $\Longrightarrow$ Fock space construction

$$Z(q) = \sum_N \Delta\Omega(\Gamma_1 + N\Gamma_2) q^N$$

$$= \Omega(\Gamma_1) \prod_{n>0} \left(1 - (-1)^{nk} q^n\right)^{nk\Omega(n\Gamma_2)}$$

$$k = |\langle \Gamma_1, \Gamma_2 \rangle|$$

D6D0 system.

$\Gamma_1 = D6$, $\Gamma_2 = D0$ $\quad k = 1$

$$\Omega(n\Gamma_2) = (-1)^3 \chi(CY)$$

nD0 forms a boundstate.

$\Longrightarrow$ get $M(-q)^{-\chi}$

Can surely get the more refined formula of Szendröi by including spin.