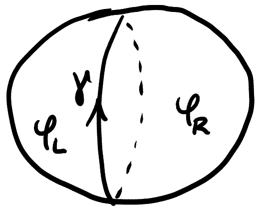
Lazaroiu – Topological defect theories + framed bicategories of topological defects

First appearance: special class of defects in $\mathcal{N}=2$ SCFT ($d=2$)



$$\mathcal{L} = \mathcal{L}_L(\varphi_L) + \mathcal{L}_R(\varphi_R) + \mathcal{L}_\gamma(\varphi_L|_\gamma, \varphi_R|_\gamma)$$

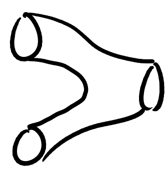
Task: Generalize Atiyah–Segal formalism to "TFT_{2d} with defects"
(on oriented RS w/o boundaries)

Atiyah–Segal define TFT_{2d} as a strongly monoidal functor

$$\Phi : (\mathrm{Cob}, \sqcup) \longrightarrow (\mathcal{V}, \otimes) \qquad (\mathcal{V}, \otimes) \text{ symmetric monoidal category}$$

$\mathrm{Ob}(\mathrm{Cob})$ = finite disjoint unions of oriented circles

$\mathrm{Mor}(\mathrm{Cob})$ = 2d oriented cobordisms
up to orientation-preserving diffeomorphisms



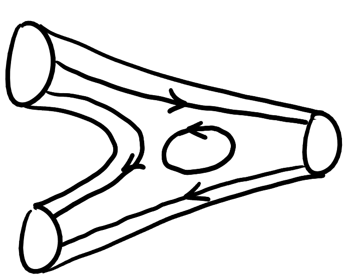
Composition: (smoothed-out) connected sum at the boundaries

Monoidal operation: the "obvious" $\sqcup$

To generalize this for defects (decorated):

Def: A protocategory $\mathcal{E}$ is the 1-step categorification of a unital magma.
(Like a category where the composition need not be associative.)

Defect cobordisms: e.g.



$\Gamma = \bigcup \gamma_i$

$\pi_0(\Sigma \setminus \Gamma) \longrightarrow \mathrm{Ob}(\mathcal{E})$ "coloring map"

Each defect carries a morphism between the colors on the right and the left

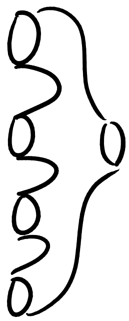
There is also a "parity" involution.

These are the morphisms in a category of defect cobordisms $Cobd_\Sigma$.
(Strictly symmetric monoidal.)

<u>Def.</u> An oriented 2d TFT with defects decorated in Σ is a strong monoidal functor $\phi : (Cobd_\Sigma, \sqcup) \longrightarrow (V, \otimes)$

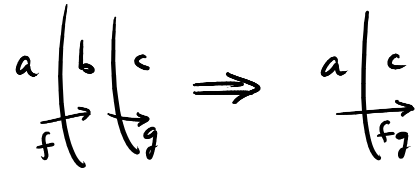
<u>e.g.</u> V. Jones planar algebras: $\Sigma = \bullet \rightleftarrows \bullet$

Restricting to



we get an <u>operad</u> (with some decorations, so more properly an <u>Σ-decorated operad</u> — enrichment of the usual notion?)

Say theory ϕ has <u>defect fusion</u> iff a certain associated planar poly-$\underbrace{\text{bicategory}}_{\text{" weak 2-category}}$ is <u>representable</u>

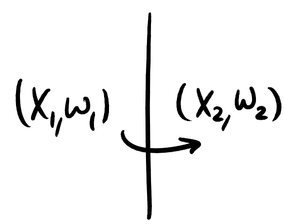


(compos. in Σ)

<u>Main examples</u>

- B-twisted LG models with defects

$$X_1 \xrightarrow{w_1} \mathbb{C} \qquad X_2 \xrightarrow{w_2} \mathbb{C}$$



$\vdots$