Elementary Particle Physics: Assignment # 2

Due TUESDAY Feb 9th at 11:30 pm (before starting class)

(1) The momentum expansion of a free scalar complex field

$$\Phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2Ep}} \left[a_p e^{-ipx} + b_p^{s\dagger} e^{ipx} \right] \quad \text{with} \quad E_p = \sqrt{|\vec{p}|^2 + m^2}$$

a_p and b_p verify: $\left[a_p, a_q^\dagger\right] = \left[b_p, b_q^\dagger\right] = (2\pi)^3 \delta^3(p-q)$ and all other commutators vanish.
The propagator is defined

$i\Delta_F(x-y) \equiv \langle 0|T\left[\Phi(x)\Phi^\star(y)\right]|0\rangle \equiv \Theta(x_0-y_0)\langle 0|\Phi(x)\,\Phi^\star(y)|0\rangle + \Theta(y_0-x_0)\langle 0|\Phi(y)^\star\,\Phi(x)|0\rangle$

Using the expansion of the fields and the commutation relations above show that

$$\Delta_F(x-y) = \Theta(x_0-y_0)\Delta_+(x-y) - \Theta(y_0-x_0)\Delta_-(x-y)$$

with

$$\Delta_\pm(x-y) = \mp i \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{\mp ip(x-y)}$$

where Δ_+ comes from the term $\langle 0|a_p a_p^\dagger|0\rangle$ and Δ_- comes from the term $\langle 0|b_p b_p^\dagger|0\rangle$

(2) In the chiral representation the 4-spinors for a fermion with momentum $\vec{p} = |\vec{p}|(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ with positive and negative helicity are:

$$u^{1,2}(\vec{p}) = u^\pm(\vec{p}) = \begin{pmatrix} \sqrt{E \mp |\vec{p}|}\; \xi_p^\pm \\ \sqrt{E \pm |\vec{p}|}\; \xi_p^\pm \end{pmatrix} \qquad \text{with} \quad \xi_p^+ = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix} \quad \xi_p^- = \begin{pmatrix} -e^{-i\phi}\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}$$

and the corresponding 4-spinors for the anti-fermion are: $v^{1,2}(\vec{p}) = v^\pm(\vec{p}) = \pm \begin{pmatrix} \sqrt{E \pm |\vec{p}|}\; \xi_p^\mp \\ -\sqrt{E \mp |\vec{p}|}\; \xi_p^\mp \end{pmatrix}$

<u>Using these expresions</u> evaluate <u>by direct calculation</u>

$$\bar{u}^s(\vec{p})v^r(-\vec{p}) \quad \text{for the four helicity combinations}$$

(3) The Lagrangian and momentum expansion for the free Dirac field are

$$\mathcal{L} \;=\; N\left[\bar{\psi}(x)\left(i\gamma^\mu\partial_\mu - m\right)\psi(x)\right] \qquad \psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2Ep}} \sum_s \left[u^s(p)c_p^s e^{-ipx} + v^s(p)d_p^{s\dagger} e^{ipx}\right]$$

where c_p^s and d_p^s verify: $\left\{c_p^s, c_q^{r\dagger}\right\} = \left\{d_p^s, d_q^{r\dagger}\right\} = (2\pi)^3\delta^3(p-q)\delta^{rs}$ and all other anticommutators vanish.

4.1) Derive, starting from the Lagrangian above, the expression of the Hamiltionian and 3-momentum operators in terms of the fields ψ and $\bar{\psi}$.

4.2) Use the expansion of ψ above (and the corresponding for $\bar{\psi}$) and the anticommutation relations of d's and c's to show that

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_s E_p \left[c_p^{s\dagger}c_p^s + d_p^{s\dagger}d_d^s\right]$$

4.3) What would you get if the fields c_p^s and d_p^s verified commutation (instead of anticommutation) relations?