Elementary Particle Physics: Assignment # 7

Due Tuesday March 22nd

1 The lagrangian for electromagnetic interactions of an electron ψ (charge -1 and mass m) and a scalar ϕ of charge e_i and mass m_s with an electric field (photon) A is

$$\mathcal{L} = \bar{\psi}\left(i\partial_\mu\gamma^\mu - e\gamma^\mu A_\mu - m\right)\psi + [(\partial_\mu + iee_iA_\mu)\phi][(\partial^\mu + iee_iA^\mu)\phi]^\dagger - m_s^2|\phi|^2$$

- With this Lagrangian the amplitude for $e^-(k,r) + s(p) \to e^-(k',r') + s(p')$ is

$$M = \frac{e^2 e_i}{q^2}\bar{u}^{r'}(k')(\not{p} + \not{p}')u^r(k)$$

- Obtain the unpolarized squared amplitude and the corresponding differential cross section $\frac{d\sigma}{dE'd\Omega}$ in the LAB system (where $p = (m_s, 0)$). Neglect the electron mass. As usual E' and Ω are the corresponding energy and solid angle of the outgoing electron.
- With the results above obtain the differential cross section $\frac{d\sigma}{dE'd\Omega}$ for the DIS $e^-p \to e^-X$ in a parton model with partons being scalars.
- Predict the expected scaling and relations between the form factors F_1^{ep} and F_2^{ep} in this scalar-parton model

2 If we define the variables $x = \frac{Q^2}{2M\nu}$ and $y = \frac{\nu}{E}$ show that in the LAB frame

$$E' = E(1-y)\ , \qquad \sin^2\frac{\theta}{2} = \frac{M\,x\,y}{2E(1-y)}\ , \qquad dE'd\Omega = 2M\pi\frac{y}{(1-y)}dxdy$$

Write the prediction of the parton model for

$$\frac{d\sigma}{dxdy}(ep \to eX)$$

in the lab frame in terms of the "x" and "y" variables.

3 Suppose that you are looking for a heavy 4th-generation fermion F with electric charge -1 and mass M which can be pair produced in

quark-antiquark collisions $q_i\bar{q}_i \to F\bar{F}$ via electromagnetic interactions. QED predicts the fundamental cross section to be

$$\sigma(\hat{s}) = \frac{4\pi\alpha^2}{3\hat{s}}\sqrt{1 - 4m_F^2/\hat{s}} \tag{1}$$

Compute the cross section $pp \to F\bar{F}X$ in nb (nanobarns) and for $p\bar{p} \to F\bar{F}X$ for $\sqrt{s} = 7$ TeV (center of mass energy of the hadron-hadron collision) for masses M=100, 1000 GeV. Suppose that the up and down valence quark distribution in the proton are given by $u_v(x) = 2d_v(x) = 6(1-x)^2$ and that all the sea are $u_s(x) = \bar{u}_s(x) = d_s(x) = \bar{u}_s(x) = (1-x)^3/(4x)$. Neglect the contribution of the strange quark. Discuss the difference between the result in pp and $p\bar{p}$ (see Hint behind)

Hint: You are going to need to evaluate first some integrals which can be done analitically. And then a second integral has to be done numerically (for example with Mathematica)

Here are the answers of the first integrals (you may also check them):

$$\begin{aligned}
I_1(\tau) = & \int_\tau^1 dx\frac{1}{x^3}(1-x)^2(x-\tau)^2 = 3(\tau^2-1) - (\tau^2+4\tau+1)\ln(\tau) \\
I_2(\tau) = & \int_\tau^1 dx\frac{1}{x^4}(1-x)^3(x-\tau)^2 = \frac{1}{\tau}\int_\tau^1 dx\frac{1}{x^3}(1-x)^2(x-\tau)^3 = \\
& \frac{1}{3}(-10\tau^2 - 9\tau + \frac{1}{\tau} + 18) + \left(\tau^2+6\tau+3\right)\ln(\tau) \\
I_3(\tau) = & \int_\tau^1 dx\frac{1}{x^4}(1-x)^3(x-\tau)^3 = \frac{11}{3}(\tau^3-1) + 9\tau^2 - \left(\tau^3+9\tau^2+9\tau+1)\ln(\tau)\right)
\end{aligned}$$