

Elementary Particle Physics: Assignment # 2

Due **TUESDAY Feb 7th at 11:30 pm** (before starting class)

- (1) The momentum expansion of a free scalar complex field

$$\Phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} [a_p e^{-ipx} + b_p^{s\dagger} e^{ipx}] \quad \text{with } E_p = \sqrt{|\vec{p}|^2 + m^2}$$

a_p and b_p verify: $[a_p, a_q^\dagger] = [b_p, b_q^\dagger] = (2\pi)^3 \delta^3(p - q)$ and all other commutators vanish.

The propagator is defined

$$i\Delta_F(x-y) \equiv \langle 0|T[\Phi(x)\Phi^*(y)]|0\rangle \equiv \Theta(x_0-y_0)\langle 0|\Phi(x)\Phi^*(y)|0\rangle + \Theta(y_0-x_0)\langle 0|\Phi(y)^*\Phi(x)|0\rangle$$

Using the expansion of the fields and the commutation relations above show that

$$\Delta_F(x-y) = \Theta(x_0-y_0)\Delta_+(x-y) - \Theta(y_0-x_0)\Delta_-(x-y)$$

with

$$\Delta_\pm(x-y) = \mp i \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{\mp ip(x-y)}$$

where Δ_+ comes from the term $\langle 0|a_p a_p^\dagger|0\rangle$ and Δ_- comes from the term $\langle 0|b_p b_p^\dagger|0\rangle$

- (2) The Lagrangian and momentum expansion for the free Dirac field are

$$\mathcal{L} = N [\bar{\psi}(x) (i\gamma^\mu \partial_\mu - m) \psi(x)] \quad \psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s [u^s(p) c_p^s e^{-ipx} + v^s(p) d_p^{s\dagger} e^{ipx}]$$

where c_p^s and d_p^s verify: $\{c_p^s, c_q^{r\dagger}\} = \{d_p^s, d_q^{r\dagger}\} = (2\pi)^3 \delta^3(p - q) \delta^{rs}$ and all other anticommutators vanish.

2.1) Derive, starting from the Lagrangian above, the expression of the Hamiltonian and 3-momentum operators in terms of the fields ψ and $\bar{\psi}$.

2.2) Use the expansion of ψ above (and the corresponding for $\bar{\psi}$) and the anticommutation relations of d 's and c 's to show that

$$H = \int \frac{d^3p}{(2\pi)^3} \sum_s E_p [c_p^{s\dagger} c_p^s + d_p^{s\dagger} d_p^s]$$

2.3) What would you get if the fields c_p^s and d_p^s verified commutation (instead of anticommutation) relations?