Particle Physics: Assignment # 3

Due Tuesday 2/21/17, before class

1. Using that the creation-annihilation operators transform under Parity and Charge Conjugation as

$$\mathcal{P}\, a^\dagger_{\vec{p}}\, \mathcal{P}^{-1} = \alpha_P\, a^\dagger_{-\vec{p}} \qquad \mathcal{P}\, b^\dagger_{\vec{p}}\, \mathcal{P}^{-1} = \overline{\alpha_P}\, b^\dagger_{-\vec{p}}$$
$$\mathcal{C}\, a^\dagger_{\vec{p}}\, \mathcal{C}^{-1} = \alpha_C\, b^\dagger_{\vec{p}} \qquad \mathcal{C}\, b^\dagger_{\vec{p}}\, \mathcal{C}^{-1} = \overline{\alpha_C}\, a^\dagger_{\vec{p}}$$

show that for a scalar field it is possible to choose

$$\mathcal{P}\, \Phi(x)\, \mathcal{P}^{-1} = \alpha_P^\Phi\, \Phi(x_P)$$
$$\mathcal{C}\, \Phi(x)\, \mathcal{C}^{-1} = \alpha_C^\Phi\, \Phi^*(x)$$

with $\alpha^*_{P,C} = \overline{\alpha_{P,C}} = \alpha^\Phi_{P,C}$ $(x_P = (x_0, -\vec{x}))$.

2. Using the transformation properties of the fermion field given in class ($P = \gamma^0$ and $C = -i\gamma^2\gamma^0$),

$$\mathcal{P}\, \psi(x)\, \mathcal{P}^{-1} = \alpha_P^\psi\, P\, \psi(x_P) \qquad \mathcal{P}\, \bar\psi(x)\, \mathcal{P}^{-1} = \alpha_P^{\psi^*}\, \bar\psi(x_P)\, P$$
$$\mathcal{C}\, \psi(x)\, \mathcal{C}^{-1} = \alpha_C^\psi\, C\, \bar\psi^T(x) \qquad \mathcal{C}\, \bar\psi(x)\, \mathcal{C}^{-1} = -\alpha_C^{\psi^*}\, \psi^T(x) C^{-1}$$

a) Derive the transformation properties under Parity (P), Charge Conjugation (C) and the product of both (CP) of the following four bilinears (a and b are two type of fermions) (you can use that the γ^μ transform as a vector under parity).

1) $\bar\psi_a(x)\psi_b(x)$ 2) $\bar\psi_a(x)\gamma_5\psi_b(x)$
3) $\bar\psi_a(x)\gamma^\nu\psi_b(x)$ 4) $\bar\psi_a(x)\gamma^\nu\gamma^5\psi_b(x)$

b)With the resuts above check whether the following Lagrangiangs are invariant under C, P and CP

$$\mathcal{L} = -B\, \bar\psi_a(x)\gamma^\mu\psi_a(x)A_\mu$$
$$\mathcal{L} = -B\, \bar\psi_a(x)\gamma^\mu(1-\gamma^5)\psi_a(x)A_\mu$$
$$\mathcal{L} = -D\, \bar\psi_a(x)\gamma^\mu\psi_b(x)V_\mu + h.c.$$
$$\mathcal{L} = -D\, \bar\psi_a(x)\gamma^\mu(1-\gamma^5)\psi_b(x)V_\mu + h.c.$$

A^μ is a real vector field, while V^μ is a complex vector field. B is real (as required by reality of the Lagrangian). D can be complex.

3. Using Wicks theorem and the interaction $\mathcal{L}_I(x) = -\lambda\phi(x)\bar\psi(x)\psi(x)$ obtain the lowest order amplitude for $f\bar f \to H$ where $\bar f$ has momentum p_1 and helcity s_1, f has momentum p_2 and helicity s_2 and H has momentum k. Deduce the corresponding Feymman rules for incoming fermions and antifermions.