Particle Physics: Assignment # 4

Due Tuesday 2/28/15, before calss

1. The interaction Lagrangian for a neutral scalar particle H with field ϕ with two types of fermions, e with field ψ and μ with field χ, reads

$$\mathcal{L}_I = -\lambda_e \phi(x)\bar{\psi}(x)\psi(x) - \lambda_\mu \phi(x)\bar{\chi}(x)\chi(x)$$

   Using the Feynman rules for this interaction derive the Feynmann amplitude (at the lowest non-vanishing order) for the process

$$e^+(p1, s1)\, e^-(p2, s2) \to \mu^+(q1, r1)\, \mu^-(q2, r2)$$

   Hint: Notice that the vertices scalar-fermion-fermion in the Lagrangian do not mix electrons and muons so there is only one contribution.

2. In the chiral representation the 4-spinors for a fermion with momentum $\vec{p} = |\vec{p}|(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ with positive and negative helicity can be chosen as:

$$u^{1,2}(\vec{p}) = u^\pm(\vec{p}) = \begin{pmatrix} \sqrt{E \mp |\vec{p}|}\,\xi_p^\pm \\ \sqrt{E \pm |\vec{p}|}\,\xi_p^\pm \end{pmatrix} \quad \text{with} \quad \xi_p^+ = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\,\sin\frac{\theta}{2} \end{pmatrix} \quad \xi_p^- = \begin{pmatrix} -\sin\frac{\theta}{2} \\ e^{+i\phi}\,\cos\frac{\theta}{2} \end{pmatrix}$$

   and the 4-spinors for the anti-fermion are: $v^{1,2}(\vec{p}) = v^\pm(\vec{p}) = \pm \begin{pmatrix} \sqrt{E \pm |\vec{p}|}\,\xi_p^\mp \\ -\sqrt{E \mp |\vec{p}|}\,\xi_p^\mp \end{pmatrix}$

   Using these expresions evaluate by direct calculation the square of the Feynman derived in the previous problem for the each of the 16 possible helicity configurations in the COM (ie for $\vec{p}_1 = -\vec{p}_2$ and $\vec{q}_1 = -\vec{q}_2$). Give the answers as a function of the scattering angles between the incoming electron and the outgoing muon. Reason the answer (hint: notice that changing $\vec{p}$ to $-\vec{p}$ means changing θ to $\pi - \theta$ and ϕ to $\pi + \phi$).