**Physics 103H Problem Set 1** Due: Friday, September 21, 2007, 4PM

Students who are interested in enrolling in Physics 105 should solve and hand in these problems. They will be graded and (except for Problem 5) will count towards your 105 grade. These problems should be done **in addition** to the normal 103 assignments.

If you are in a 103H precept, turn this in in precept on Friday or turn it in to the Undergraduate Physics Office in Jadwin 208 by 4:00 PM. If you are in a 103 precept, turn this in to Jadwin 208 by 4:00 PM. **Please write your name, the name of your precept instructor, and the time of your precept on your homework.**

**Problem 1.** Let $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ be unit vectors in the $x$-$y$ plane making angles $\theta$ and $\phi$ with the $x$ axis, respectively.

a) (K&K 1.7) Show that $\hat{\mathbf{a}} = \cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}$ and $\hat{\mathbf{b}} = \cos\phi\hat{\mathbf{i}} + \sin\phi\hat{\mathbf{j}}$, and using vector algebra prove that

$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi.$$

(Hint: use the dot product)

So, a unit vector in the $x$-$y$ plane can be written $\hat{\mathbf{a}} = \cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}$. You may also have seen a vector written by listing its components: $\hat{\mathbf{a}} = (\cos\theta, \sin\theta)$. This vector can also be written in the form of a $2 \times 1$ matrix, also known as a "column vector:"

$$\hat{\mathbf{a}} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}.$$

This is more than just notational intricacy – vectors and matrices are related. **Note:** don't worry if you haven't seen this before, it's **supposed** to be new! For a brief intro to matrices, look on the main 105 web page

`http://phy-page-g5.princeton.edu/~page/phy105/`

for a writeup called "Matrices and Matrix Multiplication." We'd be happy to discuss it further at office hours.

b) Show that when the column vector representing $\hat{\mathbf{a}}$ is multiplied by the $2 \times 2$ matrix

$$R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix},$$

using the standard rules of matrix multiplication, the result is a new column vector $\hat{\mathbf{c}}$

$$\hat{\mathbf{c}} = R(\alpha)\hat{\mathbf{a}}$$

where $\hat{\mathbf{c}}$ is just $\hat{\mathbf{a}}$ *rotated* by the angle $\alpha$. See the writeup for the scoop on matrix multiplication, with examples.

$R$ is called a *rotation matrix.* The notion of a matrix as an *operator* that does something to a vector is part of the language of quantum mechanics.

c) Show that the single matrix that does the combined operation of a rotation of $\alpha$ followed by a rotation of $\beta$ is the matrix product of the matrices for the two individual rotations.

**Problem 2.** A particle moves in the $xy$ plane, with position

$$x(t) = 3\,\mathrm{m} \times \sin\left(2\,\mathrm{rad/s} \times t\right) + 1\,\mathrm{m},$$
$$y(t) = (2\mathrm{m}^{1/2}/\mathrm{s} \times t - 1\mathrm{m}^{1/2})^2 - 1\mathrm{m},$$

with $x$ and $y$ in meters and $t$ in seconds.

a) What is the $x$ velocity of the particle at $t = 1$ second?

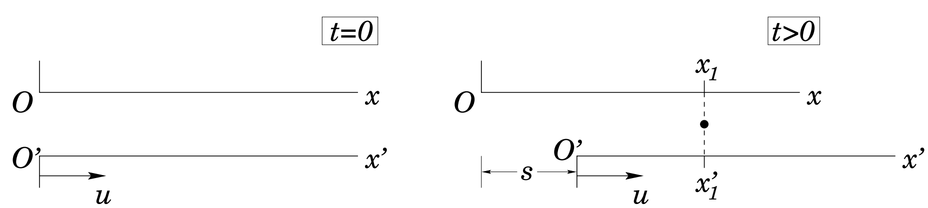
b) Where is the particle ($x$ and $y$) the first time it is (instantaneously) at rest in $x$ direction?

c) What is the particle's acceleration in $x$ direction at the time found in part b)?

d) What are the $x$ and $y$ velocities of the particle when it goes through $y = 0$ after initial time (at $t > 0$)?

**Problem 3.** Galilean relativity. Using calculus as a language, most of kinematics is just the application of a few definitions (velocity, acceleration. ...). A deeper understanding comes when we consider the same motion as viewed from different *frames of reference* or coordinate systems. This will be a recurring theme throughout Physics 103/5 and will form the basis of our discussion of Special Relativity.

Consider two coordinate systems, $O$ and $O'$. (We will consider only the $x$-axes in this problem.) At $t = 0$, the two systems coincide, but, relative to $O$, $O'$ is moving to the right with speed $u$ as shown:




At a later time $t$, $O'$ has moved (relative to $O$) by a distance $s = ut$, as shown. From the figure, it is easy to see that if a particle is located at position $x_1$ with respect to system $O$ at time $t$, then its position in system $O'$ is

$$x_1' = x_1 - s = x_1 - ut.$$

If the particle is moving parallel to the $x$-axis, say to the right with speed $v$, we can find its velocity in $O'$ by simply using the definition of velocity (we use the symbol "$\equiv$" to mean “is defined to be”) and taking the derivative of the above expression,

$$v' \equiv \frac{dx_1'}{dt} = \frac{d}{dt}(x_1 - ut) = \frac{dx_1}{dt} - u \equiv v - u.$$

This result is quite general. Using minus signs to represent motion to the right, it works for $u$ and $v$ positive or negative, for $u > v$, and so on. It works in three dimensions when vectors are used for velocities. On page 165, Knight presents this result (rearranged a bit) as obvious. We have just *proved* it. It is called the *Galilean transformation of velocity.*

a) Again in one dimension, find the Galilean transformation of acceleration. That is, if our particle has acceleration $a$ along the $x$-axis in system $O$, what is its acceleration $a'$ in $O'$? The fact that Newton's Second Law refers to acceleration (and not, for example, to velocity) makes this very important.

The Galilean transformations above work only if $O'$ is moving with **fixed velocity** with respect to $O$. A set of frames of reference, each moving with fixed velocity with respect to the others, is called a set of *inertial reference frames.* But what if $O'$ is *accelerating* with respect to $O$? Consider the case where $O'$ and $O$ coincide at $t = 0$, $O'$ is at rest with respect to $O$ at $t = 0$, but $O'$ has **constant acceleration** $\alpha$ with respect to $O$.

b) What is the expression for $s$, the position of the origin of $O'$ in $O$?

c) If at time $t$, the position, speed, and acceleration of a particle in $O$ are $x_2$, $v_2$, and $a_2$, what are the corresponding quantities $x_2'$, $v_2'$, and $a_2'$ measured in $O'$? Remember the last of these when you get to Problem VII of Learning Guide 2!

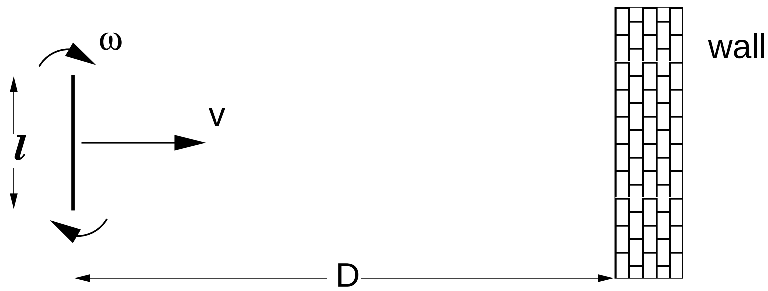
**Problem 4.** This is a problem that makes use of polar coordinates (K&K 1.9). A bead moves along the spoke of a wheel with a constant velocity $u = 2$m/s. The wheel rotates with uniform angular velocity $\omega = 1$rad/s. At time $t = 0$ the bead starts at the origin and the spoke is pointing in $y$ direction.

a) At time t, find the velocity of the bead in polar $(r, \theta)$ and cartesian $(x, y)$ coordinates. Write all quantities in vector form, as in K&K 1.9.

b) At time t, find the position of the bead in polar $(r, \theta)$ and cartesian $(x, y)$ coordinates.

**Problem 5.** Just to start stretching your brain for all the stuff Princeton wants to put in it, here is a **hard** problem that requires only the simplest of physics (distance = velocity $\times$ time), some math, and a **lot** of thinking. It will **not** count in your grade, but give it a try if you get a chance.

A stick of length $l = 10$ cm is sliding on a frictionless floor towards a wall. The stick at the moment shown is $D = 50$ cm from the wall and its center is moving directly toward the wall at $v = 10$ cm/s. The stick is also *spinning* about an axis through its center that is perpendicular to the plane of the picture. It spins at a constant rate of $\omega$ radians/s. (Hence, with $t = 0$ the moment shown in the picture, the angle the stick makes from its orientation at $t = 0$ is $\theta = \omega t$.)




For what values of $\omega$ will the stick hit the wall "flat-on," that is, the whole length of the stick hit the wall simultaneously? What makes this hard is that this is a **real** wall – no part of the stick can go through it!