1. **Irreps of SU(3)** [15 pts]: The quarks, anti-quarks, and gluons of QCD transform respectively under the fundamental, anti-fundamental and adjoint representations of SU(3).

   (a) In order for a quark to be able to absorb a gluon, there needs to be a fundamental representation in the tensor product of a fundamental and adjoint representation. Use the graphical method we discussed in class to express $3 \otimes 8$ as a direct sum of irreps of $su(3)$.

   (b) One way that QCD is very different from QED is that unlike photons, gluons can interact with each other. In order for a gluon to decay into two gluons, there needs to be an adjoint representation in the tensor product of two adjoint representations. Express $8 \otimes 8$ as a direct sum of irreps of $su(3)$.

2. **The Little Group of a Massless Particle** [15 pts]:

   (a) Consider a massless particle with momentum $k = (1, 1, 0, 0)$ and the $4 \times 4$ matrix where

   \[
   g = \begin{pmatrix}
   1 + \zeta & -\zeta & \alpha & \beta \\
   \zeta & 1 - \zeta & \alpha & \beta \\
   \alpha & -\alpha & 1 & 0 \\
   \beta & -\beta & 0 & 1 \\
   \end{pmatrix}
   \begin{pmatrix}
   1 & 0 & 0 & 0 \\
   0 & 1 & 0 & 0 \\
   0 & 0 & \cos \theta & \sin \theta \\
   0 & 0 & -\sin \theta & \cos \theta \\
   \end{pmatrix}.
   \]

   For what value of $\zeta$ is $g$ an element of $SO^{+}(3,1)$? Show that $gk = k$. Argue that elements of the type $g$ constitute the little group of a massless particle.

   (b) Argue that an arbitrary element of the Lie algebra of the little group can be written as

   \[
   \alpha A + \beta B + i \theta J_3 = \begin{pmatrix}
   0 & 0 & \alpha & \beta \\
   0 & 0 & \alpha & \beta \\
   \alpha & -\alpha & 0 & \theta \\
   \beta & -\beta & -\theta & 0 \\
   \end{pmatrix}.
   \]

   Construct the commutators of $A$, $B$, and $J_3$. 

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