Physics 403, Spring 2011
Problem Set 10

due Thursday, May 5

1. **Irreps of SU(3)** [15 pts]: The quarks, anti-quarks, and gluons of QCD transform respectively under the fundamental, anti-fundamental and adjoint representations of SU(3).

   (a) In order for a quark to be able to absorb a gluon, there needs to be a fundamental representation in the tensor product of a fundamental and adjoint representation. Use the graphical method we discussed in class to express $\mathbf{3} \otimes \mathbf{8}$ as a direct sum of irreps of $\mathfrak{su}(3)$.

   (b) One way that QCD is very different from QED is that unlike photons, gluons can interact with each other. In order for a gluon to decay into two gluons, there needs to be an adjoint representation in the tensor product of two adjoint representations. Express $\mathbf{8} \otimes \mathbf{8}$ as a direct sum of irreps of $\mathfrak{su}(3)$.

2. **The Little Group of a Massless Particle** [15 pts]:

   (a) Consider a massless particle with momentum $k = (1, 1, 0, 0)$ and the 4×4 matrix where

$$g = \begin{pmatrix} 1+\zeta & -\zeta & \alpha & \beta \\ \zeta & 1-\zeta & \alpha & \beta \\ \alpha & -\alpha & 1 & 0 \\ \beta & -\beta & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{pmatrix} .$$

   For what value of ζ is g an element of $SO^+(3,1)$? Show that $gk = k$. Argue that elements of the type g constitute the little group of a massless particle.

   (b) Argue that an arbitrary element of the Lie algebra of the little group can be written as

$$\alpha A + \beta B + i\theta J_3 = \begin{pmatrix} 0 & 0 & \alpha & \beta \\ 0 & 0 & \alpha & \beta \\ \alpha & -\alpha & 0 & \theta \\ \beta & -\beta & -\theta & 0 \end{pmatrix} .$$

   Construct the commutators of A, B, and J_3.