String Theory, Fall 2012
Problem Set 2

due Wednesday, October 31

1. **Tachyon Vertex Operator.** Verify that the normal ordered operator $:e^{ik\cdot X}:$ has conformal weights $h = \tilde{h} = \alpha' k^2/4$.

2. **The Schwarzian Derivative.** Given that the stress tensor transforms as

$$\left(\frac{\partial z'}{\partial z}\right)^2 T'(z') = T(z) - \frac{c}{12}\{z', z\} \ ,$$

where the Schwarzian derivative is defined to be

$$\{f, z\} \equiv \frac{2\partial_z^3 f \partial_z f - 3\partial_z^2 f \partial_z^2 f}{2\partial_z f \partial_z f} \ ,$$

verify

   a) the infinitesimal version of the transformation law

$$\delta T = -\frac{c}{12}\partial^3 v - 2(\partial v)T - v\partial T$$

   where $z'(z) = z + v(z)$.

   b) that the finite version of the transformation rule composes correctly.

3. **Commuting and Anticommuting Ghosts.** Calculate the singular terms in the $T(z)T(0)$ operator product expansion both for the bc system and for the $\beta\gamma$ system. Assume $h_b = h_\beta = \lambda$ and $h_c = h_\gamma = 1 - \lambda$.

4. **Linear Dilaton.** Consider the following modified energy momentum tensor for a scalar field:

$$\begin{aligned} T(z) &= -\frac{1}{\alpha'}:\partial X^\mu \partial X_\mu: + V_\mu \partial^2 X^\mu \ , \\ \tilde{T}(\bar{z}) &= -\frac{1}{\alpha'}:\bar{\partial} X_\mu \bar{\partial} X^\mu: + V_\mu \bar{\partial}^2 X^\mu \ . \end{aligned}$$

   a) Calculate the central charges c and $\tilde{c}$.

   b) Deduce the infinitesimal conformal transformation δX^μ from the OPE of the energy momentum tensor with X^μ.