Full Mass Determination from $M_{T2}$ with Combinatorial Background

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This talk will be composed of three parts:

1. **$M_{T2}$ Review**
2. Introduction of SUSY-Yukawa Sum Rule
3. $M_{T2}$-subsystem analysis of $\tilde{g}\tilde{g} \rightarrow 2b_1 + 2b \rightarrow 4b + 2\tilde{\chi}^0_1$

Main $M_{T2}$ references:

- Barr, Lester, Stephens ’03 [hep-ph/0304226] (old-skool $M_{T2}$ review)
- Cho, Choi, Kim, Park ’07 [0711.4526] (analytical expressions for $M_{T2}$ event-by-event without ISR, $M_{T2}$-edges)
- Burns, Kong, Matchev, Park ’08 [0810.5576] (definition of $M_{T2}$-subsystem variables, analytical expressions for endpoints & kinks w. & w.o. ISR)
- Konar, Kong, Matchev, Park ’09 [0910.3679] (Definition of $M_{T2\perp}$ to project out ISR-dependence)
$M_{T2}$ Review
Warm-up: W-mass measurement

- Want to measure $m_W$ from $W \rightarrow \ell \nu$.

- We can reconstruct $p_T^\nu$ and hence calculate $m_T(\ell, \nu)$, assuming $m_\nu = 0$.

- Can measure $m_W$ from edge in $m_T$-distribution!

\[ m_T^{\text{max}} = m_W \]

- Could we use a similar method for SUSY-like decays?

- Two generalizations. LSP is
  - massive
  - always produced in pairs
Classical $M_{T2}$ Variable

$$M_{T2} (\vec{p}_{t1}^T, \vec{p}_{t2}^T, \chi) = \min_{\vec{q}_1 + \vec{q}_2 = \vec{p}} \left\{ \max \left[ m_T (\vec{p}_{t1}^T, \vec{q}_1^T, \chi), m_T (\vec{p}_{t1}^T, \vec{q}_2^T, \chi) \right] \right\}$$

- If $p_{N1}^T, p_{N2}^T$ were known, this would give us a **lower bound** on $m_X$.
- However, we only know total $\vec{p}$ ⇒ minimize wrt all possible splittings, get ‘worst’ but not ‘incorrect’ lower bound on $m_X$.
- We don’t even know the invisible mass $m_N$! Insert a **testmass** $\chi$.

For the **correct testmass**, $M_{T2}^{\text{max}} = m_X$ ⇒ Effectively get $m_X(m_N)$. 

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Simple Application: Stop Pair Production

For each choice of testmass $\chi$, plot $M_{T2}$ distribution and find endpoint (edge with BG)

$\implies$ Obtain stop mass as a function of LSP mass!
Longer chains: $M_{T2}$-Subsystem Variables

- Can generalize $M_{T2}$-idea to sub-chains of larger decay chains (even multi-step subchains).
- Require knowledge of positions in decay chain.
- Interpretation of endpoints more involved.

There are analytical expressions relating the endpoints to the unknown masses.

**Good:** Only need to measure edge for one testmass.

**Bad:** The expressions also depend on $p_T^{ISR}$!

(Same for classical $M_{T2}$, but there we didn’t really need the analytical expressions.)
ISR Dependence

- To use analytical expressions for edges, we must divide events into $p_T^{\text{ISR}}$-bins and find $M_{T2}$ edge for each bin. **Low Statistics per bin!**

- Some attempts to use $p_T^{\text{ISR}}$-dependence to extract additional information (not very successful).

- It would be great to project out $p_T^{\text{ISR}}$-dependence event-by-event! Then we could extract the edges with full statistics.

Solution: Define new variable $M_{T2\perp}$.

- For each event, $M_{T2\perp}$ is evaluated exactly like $M_{T2}$, except $p_T \rightarrow p_{T\perp}$ (component $\perp$ to $p_T^{\text{ISR}}$).

- Endpoints same as $M_{T2}$ with $p_T^{\text{ISR}} = 0$.

(We’re hoping this also works for $M_{T2}$-subsystem variables with different topologies from classical $M_{T2}$.)
We can use the **classical** $M_{T2}$ variable for 1-step decays to find one of the masses as a function of the other.

For 2-step or longer chains, the $M_{T2}$-subsystem variables could allow **complete mass determination.**
SUSY-Yukawa Sum Rule
**Hierarchy problem:** In the SM, Higgs mass receives quadratically divergent corrections, most importantly from the top quark.

In SUSY, top contribution cancelled by stop.

This relies on both particle content and coupling relations. **We want to test the coupling relations.**
SUSY-Yukawa Sum Rule

Look at stop/sbottom $LL$ mass terms at tree level:

$$M_{\tilde{t}_L\tilde{t}_L}^2 = M_L^2 + \hat{m}_t^2 + g_{uL} \hat{m}_Z^2 \cos 2\beta = m_{t_1}^2 c_t^2 + m_{t_2}^2 s_t^2 \quad (1)$$

$$M_{\tilde{b}_L\tilde{b}_L}^2 = M_L^2 + \hat{m}_b^2 + g_{bL} \hat{m}_Z^2 \cos 2\beta = m_{b_1}^2 c_b^2 + m_{b_2}^2 s_b^2 \quad (2)$$

Soft masses Higgs Quartic Coupling D-term contributions measurable

$(1) - (2)$ eliminates the soft mass:

$$\hat{m}_t^2 - \hat{m}_b^2 = m_{t_1}^2 c_t^2 + m_{t_2}^2 s_t^2 - m_{b_1}^2 c_b^2 - m_{b_2}^2 s_b^2 - \hat{m}_Z^2 \cos^2 \theta_w \cos 2\beta$$

We call this the SUSY-Yukawa Sum Rule: It has its origins in the same coupling relations that cancel higgs mass corrections.

We want to test this sum rule at a collider!

$\Rightarrow$ First step: measure $m_{t_1}, m_{b_1} @ LHC$
$M_{T2}$-subsystem analysis of

to measure $\tilde{b}_1$ mass
Benchmark Point

- **Aim**: Measure $\tilde{b}_1$ mass.

**Benchmark Point:**

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_3$</th>
<th>$\mu$</th>
<th>$M_A$</th>
<th>$M_{Q3L}$</th>
<th>$M_{tR}$</th>
<th>$A_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>100</td>
<td>450</td>
<td>450</td>
<td>400</td>
<td>600</td>
<td>310.6</td>
<td>778.1</td>
<td>392.6</td>
</tr>
</tbody>
</table>

**Spectrum: (GeV)**

<table>
<thead>
<tr>
<th>$m_{t1}$</th>
<th>$m_{t2}$</th>
<th>$s_t$</th>
<th>$m_{b1}$</th>
<th>$m_{b2}$</th>
<th>$s_b$</th>
<th>$m_{\tilde{g}}$</th>
<th>$m_{\tilde{\chi}^0_1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>371</td>
<td>800</td>
<td>-0.095</td>
<td>341</td>
<td>1000</td>
<td>-0.011</td>
<td>525</td>
<td>98</td>
</tr>
</tbody>
</table>

All other sfermions at 1 TeV.

- This sets $\text{Br}(\tilde{g}\tilde{g} \rightarrow 2\tilde{b}_1 + 2b \rightarrow 4b + 2\tilde{\chi}^0_1) = 100\%$ and eliminates SUSY background.
Monte Carlo

- Simulate $\tilde{g}\tilde{g} \rightarrow 2\tilde{b}_1 + 2b \rightarrow 4b + 2\tilde{\chi}_1^0$ with MadGraph/MadEvent and BRIDGE at parton-level.

- $\sigma_{\tilde{g}\tilde{g}} \approx 11.6 \, \text{pb} @ \sqrt{s} = 14 \, \text{TeV}$. Use $L = 10 \, \text{fb}^{-1}$.

**Signal: $4b + \text{MET}$**. We impose the following **Cuts**:
  - 4 b-tags
  - $p_T^{b-jet} > 40 \, \text{GeV}$, $p_T^{\text{max}} > 100 \, \text{GeV}$
  - MET > 200 GeV

$\sigma_{\text{Signal}} = 480 \, \text{fb} \text{ after cuts} \Rightarrow 4800 \text{ signal events}$.

**Backgrounds**:
  - no SUSY background due to choice of BP
  - SM backgrounds\(^1\) suppressed by $b$-tag requirement

$\sigma_{\text{BG}} \lesssim 30 \, \text{fb} \text{ after cuts: Ignore}!$

\(^1\)MGME & ALPGEN
**Kinematic Edge**

\[ M_{bb}^{\text{max}} = \sqrt{\left( m_{g}^2 - m_{b_1}^2 \right) \left( m_{b_1}^2 - m_{\chi_1}^2 \right) / m_{b_1}^2} \]

- With known decay chain assignments get \((M_{b_1b_2}, M_{b_3b_4})\) for each event, plot \(M_{bb}\)-distribution
  \[ \Rightarrow \text{edge at 382 GeV}. \]

- Main problem: Combinatorial Background!

- Can reduce CB with \(\Delta R\) cuts and by dropping largest \(M_{bb}\)'s per event.

\[ M_{bb}^{\text{meas}} = 395 \pm 15 \text{ GeV} \]
**$M_{T2}$-subsystem Edges**

- We need more edge measurements to fix all masses in this decay. Use $M_{T2}$ subsystem variables $^2$.

- Example: $M_{T2}^{210}(0)$.

- Combinatorial Background is more dangerous.
  - To calculate $M_{T2}^{210}$, have to divide $4b$ into an upstream and downstream pair: 6 possibilities.
  - The $M_{T2}$-distribution for wrong pairings is more featured than $M_{bb}$.

- We will reduce Combinatorial Background in **two** independent ways.
Method 1: Min-Method

- For each event, have 6 possibilities for $M_{T1}^{210}(0)$.
  → 2 are particularly bad: if $b$’s from the same chain are assigned up/down-stream, the corresponding $M_{T2}$-distribution extends far beyond the correct edge.

- Simple way to eliminate those maximally bad combinations: Drop the largest 2 $M_{T2}$’s per event.

$M_{T2}^{210}(0)$, dropping the largest 2 possibilities per event

$K = (317.3 \pm 2.8)$ GeV
Method 2: Kinematic-Edge-Method

- We’ve already measured the $M_{bb}$-edge.

- Recall: For a given event with 4 $b$’s there are three possible decay chain assignments: $(M_{12}, M_{34}), (M_{13}, M_{24}), (M_{14}, M_{23})$

- For $\sim 30\%$ of events, situation like $M_{12} M_{14} M_{23} M_{34}$

  $\Rightarrow$ Can deduce correct decay chain assignments!

- With this information we can reduce the number of possible $M_{T2}$’s for that event.

$M_{T2}^{210}(0)$ with known decay chain assignment

$K = (310.7 \pm 3.7)$ GeV

From graph:
- $M_{T2}$ distribution for $K = 310.7$ GeV
Procedure for extracting $M_{T2}$-Edges

For each $M_{T2}$-subsystem variable:

1. Construct $M_{T2}$ distributions using both the Min-Method and the Kinematic-Edge-Method.

2. For each distribution, do unbinned ML fits using a linear kink trial PDF over many possible domains.

3. Only accept an edge measurement if
   - Both distributions have the same edge.
   - For each distribution, the edge-fit is stable under change of fit-domain.

This double-check method is vital for rejecting artifacts & fake edges due to low statistics!
Using this procedure, two $M_{T2}$-edges are recoverable.

<table>
<thead>
<tr>
<th>edge</th>
<th>th.</th>
<th>measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{bb}$</td>
<td>382</td>
<td>395 ± 15</td>
</tr>
<tr>
<td>$M_{T2}^{210}(0)$</td>
<td>321</td>
<td>314 ± 13 GeV</td>
</tr>
<tr>
<td>$M_{T2}^{220}(0)$</td>
<td>507</td>
<td>492 ± 14 GeV</td>
</tr>
</tbody>
</table>

This allows us to determine all the masses:

<table>
<thead>
<tr>
<th>mass</th>
<th>th.</th>
<th>68 % c.l.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{b1}$</td>
<td>341</td>
<td>(316, 356)</td>
</tr>
<tr>
<td>$m_{\tilde{g}}$</td>
<td>525</td>
<td>(508, 552)</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_0^0}$</td>
<td>98</td>
<td>(45, 115)</td>
</tr>
</tbody>
</table>

(Imposed $m_{\tilde{\chi}_0^0} > 45$ GeV bound from LEP measurement of invisible $Z$ decay width.)
**Conclusion**

- $M_{T2}$ and its variants are powerful tools for mass determination at hadron colliders for theories with invisible (massive) particles in the final state.

- We also proposed the SUSY-Yukawa Sum Rule, which represents a powerful check on SUSY as the solution to the hierarchy problem.

- We have shown that complete mass determination using $M_{T2}$-subsystem variables is possible even with combinatorial background.