SUSY-Sumrules and MT2 Combinatorics

David Curtin

In Collaboration with Maxim Perelstein, Monika Blanke

Cornell University
Cornell Institute for High Energy Phenomenology

Theory Seminar
University of Syracuse

Monday, December 6 2010
1. Motivation
   (i) The SUSY-Yukawa Sum Rule → The $\Upsilon$ Observable
   (ii) How can we use it at the LHC? → $\Upsilon'$

2. $M_{T2}$ Review

3. Collider Physics Investigation
   (i) Objective: measure $\Upsilon'$
   (ii) Measuring all the masses in $2 \tilde{g} \rightarrow 2\tilde{b}_1 + 2b \rightarrow 4b + 2\tilde{\chi}_1^0$
   (iii) Measuring $\tilde{t}_1$ mass
Introducing the SUSY-Yukawa Sum Rule
Hierarchy problem: In the SM, Higgs mass receives quadratically divergent corrections, most importantly from the top quark.

In SUSY, top contribution cancelled by stop.

This relies on both particle content and coupling relations. We want to test the coupling relations.
How to probe the Quartic Higgs Coupling?

\[
M_{\tilde{t}_{i\tilde{t}j}}^2 = \begin{bmatrix}
M_L^2 + \hat{m}_t^2 + g_u \hat{m}_Z^2 c_2 \beta & m_t (A_t + \mu \cot \beta) \\
m_t (A_t + \mu \cot \beta) & M_T^2 + \hat{m}_t^2 + g_R \hat{m}_Z^2 c_2 \beta
\end{bmatrix}
\]

\[
= \begin{bmatrix}
m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 & c_t s_t (m_{t1}^2 - m_{t2}^2) \\
c_t s_t (m_{t1}^2 - m_{t2}^2) & m_{t1}^2 s_t^2 + m_{t2}^2 c_t^2
\end{bmatrix}
\]

Extract this contribution to diagonal sfermion mass terms!
Consider stop/sbottom $LL$ mass terms at tree level:

\[
M^2_{\tilde{t}_L \tilde{t}_L} = M^2_L + \hat{m}_t^2 + g_{uL} \hat{m}_Z^2 \cos 2\beta = m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2
\]

\[
M^2_{\tilde{b}_L \tilde{b}_L} = M^2_L + \hat{m}_b^2 + g_{bL} \hat{m}_Z^2 \cos 2\beta = m_{b1}^2 c_b^2 + m_{b2}^2 s_b^2
\]

Soft masses Higgs Quartic Coupling D-term contributions measurable

(1) – (2) eliminates the soft mass:

\[
\hat{m}_t^2 - \hat{m}_b^2 = m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 - m_{b1}^2 c_b^2 - m_{b2}^2 s_b^2 - \hat{m}_Z^2 \cos^2 \theta_w \cos 2\beta
\]

We call this the **SUSY-Yukawa Sum Rule**: It has its origins in the same coupling relations that cancel higgs mass corrections.

**Testing this sum rule at a collider would constitute a highly nontrivial check on SUSY.**
How to test the sum rule?

**SUSY-Yukawa Sum Rule:**

\[
\hat{m}_t^2 - \hat{m}_b^2 = m_{t_1}^2 c_t^2 + m_{t_2}^2 s_t^2 - m_{b_1}^2 c_b^2 - m_{b_2}^2 s_b^2 - \hat{m}_Z \cos^2 \theta_W \cos 2\beta
\]

Define an observable \( \Upsilon \) for which the sum rule gives a definite prediction:

\[
\Upsilon \equiv \frac{1}{v^2} \left( m_{t_1}^2 c_t^2 + m_{t_2}^2 s_t^2 - m_{b_1}^2 c_b^2 - m_{b_2}^2 s_b^2 \right)
\]

**Tree-Level Prediction for \( \Upsilon \) from SUSY-Yukawa Sum Rule**

\[
\Upsilon_{\text{SUSY}}^{\text{tree}} = \frac{1}{v^2} \left( \hat{m}_t^2 - \hat{m}_b^2 + m_Z^2 \cos^2 \theta_W \cos 2\beta \right)
\]

\[
= \begin{cases} 
0.39 & \text{for } \tan \beta = 1 \\
0.28 & \text{for } \tan \beta \to \infty \text{ (converges quickly for } \tan \beta \gtrsim 5) 
\end{cases}
\]
Radiative Corrections wash out SUSY tree-level prediction for $\Upsilon$

$$\Upsilon_{\text{meas}}^{\text{SUSY}} = \frac{1}{v^2} \left( m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 - m_{b1}^2 c_b^2 - m_{b2}^2 s_b^2 \right)$$

$$= \Upsilon_{\text{tree}}^{\text{SUSY}} + \Upsilon_{\text{loop}}^{\text{SUSY}} (m_{ti}, \theta_t, m_\tilde{g}, \tan \beta, \ldots) = ?$$

Properties of $\Upsilon_{\text{loop}}$:

- Kicks in at energies below $M_{\text{soft}}$.
- is a function of many SUSY-parameters, so could be theoretically constrained by measurements of those parameters.
- is very complicated\(^1\), but one can try to numerically estimate possible range with parameter scans.

\(^1\)Pierce, Bagger, Matchev, Zhang 1996
Numerically Estimate Radiative Corrections

- Tool: use \texttt{Suspect} to do MSSM Parameter Scan
- Examine a ‘Worst-case’ scenario: no BSM experimental constraints other than $M_{\text{SUSY}} < 2 \text{ TeV}$ and Neutralino LSP:

\[
\Upsilon_{\text{tree}}^{\text{SUSY}} \approx 0.3 \rightarrow |\Upsilon_{\text{SUSY}}| \lesssim 1
\]

(For comparison, the ‘generic’ perturbative theory prediction is $|\Upsilon| \lesssim 16\pi^2$.)
Introduce the SUSY-Yukawa Sum Rule,

\[ \hat{m}_t^2 - \hat{m}_b^2 = m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 - m_{b1}^2 c_b^2 - m_{b2}^2 s_b^2 - \hat{m}_Z^2 \cos^2 \theta_w \cos 2\beta \]

which relies on the same coupling relations that cancel contributions from stop & top loops to the higgs coupling.

Introduce new observable which can be measured at a collider:

\[ \Upsilon \equiv \frac{1}{v^2} \left( m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 - m_{b1}^2 c_b^2 - m_{b2}^2 s_b^2 \right) \]

SUSY-Yukawa Sum Rule \( \Rightarrow |\Upsilon| \lesssim 1 \)

Measuring \( \Upsilon \) constitutes a powerful nontrivial check that SUSY is the solution to the hierarchy problem.
How can we use the SUSY-Yukawa Sum Rule at the LHC?
To measure every ingredient of $\Upsilon$ (especially $\theta_b$) we probably need a lepton collider.

What good is the sum rule at the LHC? ⇒ Use SUSY-prediction for $\Upsilon$ to constrain ‘unmeasurable’ parameters!

Which parts can we measure? Often $\tilde{t}_2, \tilde{b}_2$ are too heavy for the LHC.

$$\Upsilon \sim \frac{1}{v^2} \left( m^2_{t1} - m^2_{b1} \right) \underbrace{\Upsilon' \text{ (maybe measurable)}}_{\text{known}} + \frac{s^2_t}{v^2} \left( m^2_{t2} - m^2_{t1} \right) - \frac{s^2_b}{v^2} \left( m^2_{b2} - m^2_{b1} \right) \underbrace{\text{unknown}}_{\text{unknown}}$$

We can try to measure $\Upsilon'$ at the LHC.
What does $\Upsilon'$ tell us?

$$\Upsilon = \Upsilon' + \frac{s_t}{v^2} \left( m_{t2}^2 - m_{t1}^2 \right) - \frac{s_b}{v^2} \left( m_{b2}^2 - m_{b1}^2 \right)$$

Even a rough measurement of $\Upsilon'$ gives strong constraints on the stop and/or sbottom mixing angles!
This gives us motivation to measure $m_{\tilde{t}_1}$, $m_{\tilde{b}_1}$.

Let’s talk Collider Physics!
$M_{T2}$ Review

Main $M_{T2}$ references used here:

- Barr, Lester, Stephens ’03 [hep-ph/0304226] (old-skool $M_{T2}$ review)
- Cho, Choi, Kim, Park ’07 [0711.4526] (analytical expressions for $M_{T2}$ event-by-event without ISR, $M_{T2}$-edges)
- Burns, Kong, Matchev, Park ’08 [0810.5576] (definition of $M_{T2}$-subsystem variables, analytical expressions for endpoints & kinks w. & w.o. ISR)
- Konar, Kong, Matchev, Park ’09 [0910.3679] (Definition of $M_{T2\perp}$ to project out ISR-dependence)
Warm-up: W-mass measurement

- Want to measure $m_W$ from $W \rightarrow \ell \nu$.
- We can reconstruct $p_{T}^{\nu}$ and hence calculate $m_T(\ell, \nu)$, assuming $m_\nu = 0$.
- Can measure $m_W$ from edge in $m_T$-distribution! $m_T^{\text{max}} = m_W$

Could we use a similar method for SUSY-like decays?

Two generalizations. LSP is
- massive
- always produced in pairs
Classical $M_{T2}$ Variable

$$M_{T2}(\vec{p}_{t1}^T, \vec{p}_{t2}^T, \tilde{m}_N) = \min_{\vec{q}_1^T + \vec{q}_2^T = \vec{p}^T} \left\{ \max \left[ m_T(\vec{p}_{t1}^T, \vec{q}_1^T, \tilde{m}_N), m_T(\vec{p}_{t1}^T, \vec{q}_2^T, \tilde{m}_N) \right] \right\}$$

- If $p_{N1}^T, p_{N2}^T$ were known, this would give us a lower bound on $m_x$.
- However, we only know total $\vec{p}^T$ ⇒ minimize wrt all possible splittings, get ‘worst’ but not ‘incorrect’ lower bound on $m_x$.
- We don’t even know the invisible mass $m_N$! Insert a testmass $\tilde{m}_N$.

For the correct testmass, $M_{T2}^{\text{max}} = m_x$ ⇒ Effectively get $m_x(m_N)$. 
$M_{T2 \text{max}}$ gives us $m_X$ as a function of $m_N$. 

$$M_{T2 \text{max}}(\chi) \text{ for } m_N = 98, \ m_X = 341 \text{ GeV without ISR}$$

$$M_{T2 \text{max}}(\tilde{m}_N) = \mu + \sqrt{\mu^2 + \tilde{m}_N^2}, \text{ where } \mu = \frac{m_X^2 - m_N^2}{2m_X}$$
Simple Example: Stop Pair Production without ISR

Find edge for zero testmass:

$$M_{T2}^{\text{max}}(0) = \frac{m_t^2 - m_\chi^2}{m_t}$$

⇒ Obtain stop mass as a function of LSP mass!
Longer Chains: $M_{T2}$-Subsystem Variables

\[
M_{T2}^{npc}(\tilde{M}_c) = \min_{p_{cT}^{(1)}+p_{cT}^{(2)}=\text{upstream}} \left\{ \max \left[ M_T^{(1)}, M_T^{(2)} \right] \right\}
\]

\[
M_{T2 \max}^{npc}(\tilde{M}_c) = M_p \quad \text{when} \quad \tilde{M}_c = M_c \quad \Rightarrow \quad \text{get} \quad M_p(M_c)
\]
Longer Chains: $M_{T2}$-Subsystem Variables

\[ M_{T2}^{npc}(\tilde{M}_c) = \min_{p_{cT}^{(1)} + p_{cT}^{(2)} = \text{upstream } p_T} \left\{ \max \left[ M_T^{(1)}, M_T^{(2)} \right] \right\} \]

\[ M_{T2}^{npc}(\tilde{M}_c) = M_p \quad \text{when} \quad \tilde{M}_c = M_c \quad \Rightarrow \quad \text{get} \quad M_p(M_c) \]
Dependence of endpoints on testmass & ISR more interesting → each contains unique information.

2+ steps ⇒ complete mass determination possible!
We can use the classical $M_{T2}$ variable for 1-step decays to find one of the masses as a function of the other.

For 2-step or longer chains, the $M_{T2}$-subsystem variables could allow complete mass determination.
We have not discussed what happens in realistic measurements with ISR. This is well understood. (Maybe say more later.)

There are two types of combinatorics problems when using $M_{T2}$:

(a) Distinguishing ISR from hard process final state (depends on the process).

(b) For subsystem variables: correctly placing each particle at its correct place in the long decay chain.

We will later discuss some strategies for addressing (b).
Collider Physics Investigation
Want to demonstrate that the SUSY-Yukawa Sum Rule can be used to measure stop & sbottom mixing angles at the LHC.

Choose a particular MSSM Benchmark Point with light $\tilde{t}_1$, $\tilde{b}_1$ and small mixing:

**Parameters:**

<table>
<thead>
<tr>
<th>$\tan\beta$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_3$</th>
<th>$\mu$</th>
<th>$M_A$</th>
<th>$M_{Q3L}$</th>
<th>$M_{tR}$</th>
<th>$A_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>100</td>
<td>450</td>
<td>450</td>
<td>400</td>
<td>600</td>
<td>310.6</td>
<td>778.1</td>
<td>392.6</td>
</tr>
</tbody>
</table>

**Spectrum: (GeV)**

<table>
<thead>
<tr>
<th>$m_{t1}$</th>
<th>$m_{t2}$</th>
<th>$s_t$</th>
<th>$m_{b1}$</th>
<th>$m_{b2}$</th>
<th>$s_b$</th>
<th>$m_{\tilde{g}}$</th>
<th>$m_{\tilde{\chi}_{1}^0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>371</td>
<td>800</td>
<td>-0.095</td>
<td>341</td>
<td>1000</td>
<td>-0.011</td>
<td>525</td>
<td>98</td>
</tr>
</tbody>
</table>
Required Mass Measurements

- **We will measure** $\gamma' = \frac{1}{v^2}(m_{t1}^2 - m_{b1}^2)$

- Parton-level Analysis with gaussian momentum smearing and no ISR. (More realistic analysis in progress, some preliminary comments later.)

- Gluino pair production $\Rightarrow m_{b1}$  
  (Bonus: $m_{\tilde{g}}$ and $m_{\tilde{\chi}_1^0}$)

- Stop pair production $\Rightarrow m_{t1}$

\[\begin{align*}
\tilde{g} & \quad b \quad \tilde{b}_1 \quad \tilde{b} \quad \tilde{\chi}_1^0 \\
\tilde{g} & \quad \tilde{b}_1 \quad \tilde{b} \quad \tilde{\chi}_1^0 \quad \tilde{t} \quad \tilde{t} \quad \tilde{\chi}_1^0
\end{align*}\]
Measuring all the masses in
Monte Carlo

- Simulate $\tilde{g}\tilde{g} \rightarrow 2\tilde{b}_1 + 2b \rightarrow 4b + 2\tilde{\chi}_1^0$ with MadGraph/MadEvent and BRIDGE at parton-level.

- $\sigma_{\tilde{g}\tilde{g}} \approx 11.6 \text{ pb} @ \sqrt{s} = 14 \text{ TeV}$. Use $\mathcal{L} = 10 \text{ fb}^{-1}$.

- **Signal**: $4b + \text{MET}$. We impose the following **Cuts**:
  - 4 b-tags
  - $p_T^{b\text{-jet}} > 40 \text{ GeV}$, $p_T^{\text{max}} > 100 \text{ GeV}$
  - MET > 200 GeV

  $\sigma_{\text{Signal}} = 480 \text{ fb after cuts} \Rightarrow 4800 \text{ signal events}.$

- **Backgrounds**:
  - no SUSY background due to choice of BP
  - SM backgrounds$^2$ suppressed by b-tag requirement

  $\sigma_{\text{BG}} \lesssim 30 \text{ fb after cuts: Ignore!}$

$^2$MGME & ALPGEN
We want to measure 3 masses: $m_{b1}$, $m_{\tilde{g}}$, $m_{\tilde{\chi}_1^0}$.

→ Must measure the edges of at least 3 independent kinematic variables.

→ We will use Kinematic Edge $M_{bb}^{\text{max}}$ and $M_{T2}$-Subsystem Variables!

• Combinatorial Background! Final state is $4b + \text{MET}$
Measuring the Kinematic Edge

\[ M_{bb}^{\text{max}} = \sqrt{\frac{(m_g^2 - m_{b_1}^2)(m_{b_1}^2 - m_{\chi_1}^2)}{m_{b_1}^2}} \]

With known decay chain assignments get \((M_{b_1b_2}, M_{b_3b_4})\) for each event, plot \(M_{bb}\)-distribution \(\Rightarrow\) edge at 382 GeV.

Main problem:
Combinatorial Background!

Can reduce CB with \(\Delta R\) cuts and by dropping largest \(M_{bb}\)'s per event.

\[ M_{bb}^{\text{max, meas}} = 395 \pm 15 \text{ GeV} \]
Example: $M_{T2}^{210}(0)$

Combinatorial Background is more dangerous!
Different combinations for calculating $M_{T_2}^{210}(0)$

To calculate $M_{T_2}^{210}$, need to divide $4b$ into an upstream- and a downstream-pair:
6 possibilities
$M_{T2}^{210}(0)$ without combinatorial error reduction
Need to reduce Combinatorial Background!

This is an example of the $M_{T2}$ Combinatorics Problem (assignment of final states in the decay chain).

We will need two methods for reducing Combinatorial Background:

1. ‘Min’-Method
2. ‘Kinematic-Edge’-Method

Why two methods? Necessary consistency check!
Method 1: ‘Min’-Method

For each event, have 6 possibilities for $M_{T2}^{210}(0)$.

$\rightarrow$ 2 are particularly bad: if $b$’s from the same chain are assigned up-stream, the corresponding $M_{T2}$-distribution extends far beyond the correct edge.

Simple way to eliminate those maximally bad combinations:

Drop the largest 2 $M_{T2}$’s per event.

$M_{T2}^{210}(0)$, dropping the largest 2 possibilities per event

\[ K = (317.3 \pm 2.8) \text{ GeV} \]

(Can think of this as an extension of the min/maximization for calculating

\[ M_{T2}^{npc}(\tilde{M}_c) = \min_{\rho_{cT}^{(1)},\rho_{cT}^{(2)}} \left\{ \max \left[ M_T^{(1)}, M_T^{(2)} \right] \right\} \]

to ‘min’/min/max)
Method 2: ‘Kinematic-Edge’-Method

- We’ve already measured the $M_{bb}$-edge.
- Recall: For a given event with 4 $b$'s there are three possible decay chain assignments: $(M_{12}, M_{34}), (M_{13}, M_{24}), (M_{14}, M_{23})$
- For $\sim 30\%$ of events, situation like $M_{12} M_{14} M_{23} M_{34}$
  - Can deduce correct decay chain assignments!
- With this information we can reduce the number of possible $M_{T2}$'s for that event.

$M_{T2}^{210}(0)$ with known decay chain assignment $K = (310.7 \pm 3.7)$ GeV
Procedure for extracting $M_{T2}$-Edges

For each $M_{T2}$-subsystem variable:

1. Construct $M_{T2}$ distributions using both the Min-Method and the Kinematic-Edge-Method.

2. For each distribution, do unbinned ML fits using a linear kink trial PDF over many possible domains.

3. Only accept an edge measurement if
   - Both distributions have the same edge.
   - For each distribution, the edge-fit is stable under change of fit-domain.

This **double-check method** is vital for rejecting artifacts & fake edges due to low statistics!
Using this procedure, two $MT_2$-edges are recoverable.

<table>
<thead>
<tr>
<th>edge</th>
<th>th.</th>
<th>measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{bb}$</td>
<td>382</td>
<td>$395 \pm 15 \text{ GeV}$</td>
</tr>
<tr>
<td>$M_{T2}^{210}(0)$</td>
<td>321</td>
<td>$314 \pm 13 \text{ GeV}$</td>
</tr>
<tr>
<td>$M_{T2}^{220}(0)$</td>
<td>507</td>
<td>$492 \pm 14 \text{ GeV}$</td>
</tr>
</tbody>
</table>
Obtaining Masses from Edge Measurements

Numerical Mass Extraction:

- Assume edge measurement errors are Gaussian.
- Assign each point in \((m_\tilde{g}, m_{\tilde{\chi}_1^0}, m_{\tilde{b}_1})\)-space a weight according to ‘distance’ from edge measurements.
- To get measured PDF of each mass, project to one axis:
- Extract confidence level intervals.
### Mass Measurements

<table>
<thead>
<tr>
<th>mass</th>
<th>th.</th>
<th>68 % c.l.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{b_1}$</td>
<td>341</td>
<td>(316, 356)</td>
</tr>
<tr>
<td>$m_{\tilde{g}}$</td>
<td>525</td>
<td>(508, 552)</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_1^0}$</td>
<td>98</td>
<td>(45, 115)</td>
</tr>
</tbody>
</table>

(Imposed $m_{\tilde{\chi}_1^0} > 45 \text{ GeV}$ bound from LEP measurement of invisible $Z$ decay width.)
Measuring $m_{\tilde{t}_1}$ from

\[ \tilde{t}_1 \quad | \quad t \quad | \quad \tilde{\chi}_1^0 \]

and determining $\gamma'$
Stop Pair Production and $m_{t_1}$ measurement

- Analyze the process $\tilde{t}_1 \tilde{t}_1^* \rightarrow \bar{t} + 2\tilde{\chi}_1^0$.

- $\sigma_{\tilde{t}_1 \tilde{t}_1^*} \approx 2$ pb @ $\sqrt{s} = 14$ TeV.

- Impose standard cuts & use hadronic tops$^3$.

- Use $\mathcal{L} = 100$ fb$^{-1}$. After cuts: 1481 signal and 105 BG events.

- Easy to extract $M_{T2}^{\max}$ edge $\rightarrow$ Gives $m_{t_1}(m_{\tilde{\chi}_1^0})$

- Combine with (I) $\Rightarrow$

<table>
<thead>
<tr>
<th>$m_{t_1}$</th>
<th>th.</th>
<th>68 % c.l.</th>
</tr>
</thead>
<tbody>
<tr>
<td>371</td>
<td></td>
<td>(356, 414)</td>
</tr>
</tbody>
</table>

---

3 Meade, Reece 2006
When determining $\gamma'$, correlations between mass measurements are vital! This is why $\gamma'$ must be defined as a separate observable.

<table>
<thead>
<tr>
<th></th>
<th>th.</th>
<th>meas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma'$</td>
<td>0.350</td>
<td>$0.525^{+0.20}_{-0.15}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.423</td>
<td>—</td>
</tr>
</tbody>
</table>

Unless there is a strong accidental cancellation, such a small $\gamma'$ measurement implies that both stop and sbottom mixing angles are small, $\lesssim 0.1$.

Compare to actual values:

- $s_t = -0.095$
- $s_b = -0.011$
Aside:
A More Realistic Investigation

(Preliminary!)
Let’s go beyond parton-level and do a (more) realistic investigation.

- Method for reducing Combinatorial Background carry over directly.
- To deal with ISR takes a bit more work, but it can be done.
**ISR dependence of** $M_{T2}^{\text{max}}$

The edges of the classical $M_{T2}$ and the $M_{T2}$-subsystem variable distributions depend on both the testmass and the amount of total $\rho_T^{\text{ISR}}$.

How can the endpoint of a distribution depend on ISR, which is defined event-by-event? → Only plot the $M_{T2}$ distribution for events with a certain $\rho_T^{\text{ISR}}$ and find edge, this gives $M_{T2}^{\text{max}}(\tilde{m}_N, \rho_T^{\text{ISR}})$

**Useful properties to keep in mind (both classical & subsystem):**

- $M_{T2}^{\text{max}}(\tilde{m}_N, 0) < M_{T2}^{\text{max}}(\tilde{m}_N, \rho_T^{\text{ISR}})$
- **edge shift** = $M_{T2}^{\text{max}}(\tilde{m}_N, \rho_T^{\text{ISR}}) - M_{T2}^{\text{max}}(\tilde{m}_N, 0) < \rho_T^{\text{ISR}} / 2$
- When $\tilde{m}_N = m_N$, $M_{T2}^{\text{max}}(\tilde{m}_N) = m_X$ indep. of ISR.
One Measurement Method: Lump it all together

This method does not take advantage of any of the analytical expressions we have for $M_{T2}^{\text{max}}$ and is extremely tedious, but should work with ISR.

1. Pick a test mass $\tilde{m}_N$
2. Calculate $M_{T2}(\tilde{m}_N)$ for each event.
3. Plot $M_{T2}$ distribution of all events (regardless of ISR) & extract edge. We have now obtained $M_{T2}^{\text{max}}(\tilde{m}_N)$

→ Repeat 1. - 3. for many different test masses. (Ouch!)

- Plot $M_{T2}^{\text{max}}$ vs $\tilde{m}_N$: still gives $m_X(m_N)$ since $M_{T2}^{\text{max}}(\tilde{m}_N = m_N) = m_X$
- The edge will be more washed out for test masses far away from the actual $m_N$ due to ISR.
A Simpler Method: ISR bins

Wouldn’t it be nice to just do one $M_{T2}^{\text{max}}$ measurement at zero test mass (like for the stop example), and hence extract $\mu = \frac{m_X^2 - m_N^2}{2m_X}$?

Simplest Idea: Could only keep events with $p_T^{\text{ISR}} < p_T^{\text{ISR}, \text{max}}$.

- Pretend that there is no ISR for that sample and extract $M_{T2}^{\text{max}} (\tilde{m}_N = 0, p_T^{\text{ISR}} = 0) \to \mu$. Easy!

- Reduces statistics! (Depends on choice of $p_T^{\text{ISR}, \text{max}}$).

- Introduces Systematic Error!
  
  e.g. choose $p_T^{\text{ISR}, \text{max}} = 40 \text{ GeV}$
  
  $\to$ edge shift $< 20 \text{ GeV} \to$ systematic error $\approx 10 \text{ GeV}$

Works OK!
New Idea: Project out ISR Dependence

- Using **ISR** bins reduces statistics in each bin.
- It would be great to project out $p_T^{\text{ISR}}$-dependence event-by-event! Then we could extract the edges with full statistics.

**Solution: Define new variable $M_{T2\perp}$.

- For each event, $M_{T2\perp}$ is evaluated exactly like $M_{T2}$, except $p_T \rightarrow p_T\perp$ (component $\perp$ to $p_T^{\text{ISR}}$).
- Endpoints same as $M_{T2}$ with $p_T^{\text{ISR}} = 0$.

This works very well, but makes the edges shallower:
Summary of realistic $M_{T2}$ measurement methods

Three main approaches to doing realistic measurements with ISR:

1. **Lump it all together** (find edge for each test mass individually, very tedious, we don’t like it)
2. **ISR bins** (reduced statistics)
3. $M_{T2\perp}$ (shallower edge, especially for $M_{T2}^{220}$)

**We do do both 2 + 3 and combine measurements.**
Conclusions
Confirmation of the SUSY-Yukawa Sum Rule

\[ \hat{m}_t^2 - \hat{m}_b^2 = m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 - m_{b1}^2 c_b^2 - m_{b2}^2 s_b^2 - \hat{m}_Z^2 \cos^2 \theta_w \cos 2\beta \]

(possibly at a lepton collider) would be strong support for TeV-scale SUSY as the solution for hierarchy problem.

At the LHC, the sum rule provides powerful constraints on stop and sbottom mixing angles (hard to come by otherwise) using only a mass measurement.

We developed new techniques for reducing $M_{T2}$-combinatorial background, allowing application to $4b + \text{MET}$ final states.

→ Were able to measure or strongly constrain $m_{t1}, m_{b1}, m_{\tilde{g}}, m_{\tilde{\chi}_1^0}, \theta_t, \theta_b$ from analyzing just two processes.