SUSY-Yukawa Sum Rule at the LHC

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Phenomenology 2010 Symposium
Madison, Wisconsin

Monday, May 10 2010
**Hierarchy problem:** In the SM, Higgs mass receives quadratically divergent corrections, most importantly from the top quark.

In SUSY, top contribution cancelled by stop:

This relies on both particle content and coupling relations. **We want to test the coupling relations.**
How to probe the Quartic Higgs Coupling?

Look at diagonal sfermion mass terms!
SUSY-Yukawa Sum Rule

Look at stop/sbottom $LL$ mass terms at tree level:

\begin{align*}
M_{\tilde{t}_L\tilde{t}_L}^2 &= M_L^2 + m_t^2 + g_{uL} \hat{m}_Z^2 \cos 2\beta = m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 \\
M_{\tilde{b}_L\tilde{b}_L}^2 &= M_L^2 + m_b^2 + g_{bL} \hat{m}_Z^2 \cos 2\beta = m_{b1}^2 c_b^2 + m_{b2}^2 s_b^2
\end{align*}

Soft masses Higgs Quartic Coupling D-term contributions measurable

(1) – (2) eliminates the soft mass:

$$\hat{m}_t^2 - \hat{m}_b^2 = m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 - m_{b1}^2 c_b^2 - m_{b2}^2 s_b^2 - \hat{m}_Z^2 \cos^2 \theta_w \cos 2\beta$$

We call this the **SUSY-Yukawa Sum Rule**: It has its origins in the same coupling relations that cancel higgs mass corrections.

**We want to test this sum rule at a collider!**
Defining an observable to test the Sum Rule

- Assume SUSY-like particle content \( (\tilde{t}_L, \tilde{b}_L), \tilde{t}_R, \tilde{b}_R \) but not the SUSY coupling relations.

- Before EWSB, \( M_{t_L}^2 = \begin{pmatrix} M_{t_{1L}}^2 & M_{t_{2L}}^2 \end{pmatrix} \), \( M_{b_L}^2 = \begin{pmatrix} M_{b_{1L}}^2 & M_{b_{2L}}^2 \end{pmatrix} \)

- After EWSB, can parameterize quartic higgs coupling ‘model-independently’:

\[
(M_{t_L}^2)_{11} \to M_{t_{1L}}^2 + v^2 Y_{11}^t \quad , \quad (M_{b_L}^2)_{11} \to M_{b_{1L}}^2 + v^2 Y_{11}^b
\]

Define a new observable to probe the quartic higgs coupling:

\[
\gamma \equiv \frac{1}{v^2} \left( m_{t_1}^2 c_t^2 + m_{t_2}^2 s_t^2 - m_{b_1}^2 c_b^2 - m_{b_2}^2 s_b^2 \right) = Y_{11}^t - Y_{11}^b \text{ at tree level}
\]
SUSY prediction for $\Upsilon$ & Radiative Corrections

**Tree-Level Prediction for $\Upsilon$ from SUSY-Yukawa Sum Rule**

\[
\Upsilon_{\text{tree SUSY}} = \frac{1}{v^2} \left( \hat{m}_t^2 - \hat{m}_b^2 + m_Z^2 \cos^2 \theta_W \cos 2\beta \right)
\]

\[
= \begin{cases} 
0.39 & \text{for } \tan \beta = 1 \\
0.28 & \text{for } \tan \beta \to \infty \text{ (converges quickly for } \tan \beta \geq 5) 
\end{cases}
\]

- In a generic theory, only ‘requirement’ is $|\Upsilon| \lesssim 16\pi$.
- Radiative Corrections wash out SUSY tree-level prediction for $\Upsilon$.
- Worst case scenario ($\text{Suspect}$) →
- Can narrow predicted range with more measurements (see later).

**TeV-scale SUSY:** $|\Upsilon| \lesssim 1$. 

![Histogram](histogram.png)
Prospects at the LHC

- Fully measuring $\Upsilon$ requires lepton collider.
- Can make some progress at LHC in favorable regions of MSSM parameter space. $\Rightarrow$ Could then use $\Upsilon$ to constrain stop/sbottom parameters.
- Demonstrate feasibility of partial $\Upsilon$-measurement with a particular Benchmark Point:

**Parameters:**

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_3$</th>
<th>$\mu$</th>
<th>$M_A$</th>
<th>$M_{Q3L}$</th>
<th>$M_{tR}$</th>
<th>$A_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>100</td>
<td>450</td>
<td>450</td>
<td>400</td>
<td>600</td>
<td>310.6</td>
<td>778.1</td>
<td>392.6</td>
</tr>
</tbody>
</table>

**Spectrum: (GeV)**

<table>
<thead>
<tr>
<th>$m_{t1}$</th>
<th>$m_{t2}$</th>
<th>$s_t$</th>
<th>$m_{b1}$</th>
<th>$m_{b2}$</th>
<th>$s_b$</th>
<th>$m_{\tilde{g}}$</th>
<th>$m_{\tilde{\chi}_1^0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>371</td>
<td>800</td>
<td>-0.095</td>
<td>341</td>
<td>1000</td>
<td>-0.011</td>
<td>525</td>
<td>98</td>
</tr>
</tbody>
</table>
Measuring part of $\gamma$

- Small mixing angles and light $\tilde{t}_1, \tilde{b}_1 \implies$ rewrite

$$
\gamma = \frac{1}{v^2} \left( m_{t1}^2 - m_{b1}^2 \right) + \frac{s_t^2}{v^2} \left( m_{t2}^2 - m_{t1}^2 \right) - \frac{s_b^2}{v^2} \left( m_{b2}^2 - m_{b1}^2 \right)
$$

- Most of $\gamma = 0.423$ comes from $\gamma' = 0.350$. $\Delta \gamma_t \lesssim O(0.1)$ can be estimated. $\Delta \gamma_b$ can’t be measured at LHC.

- **We will measure $\gamma'$**
  - Need to determine $m_{t1}, m_{b1}$
  - Analyse gluino & stop pair production & decay
  - Extract kinematic- and $M_{T2}$-edges to get all the masses $\implies \gamma'$

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1. MP, Weiler 2008
Gluino Pair Production

- Analyze the process\(^2\)
  \[
  \tilde{g}\tilde{g} \rightarrow 2\tilde{b}_1 + 2b \rightarrow 4b + 2\tilde{\chi}_1^0.
  \]

- \(\sigma_{\tilde{g}\tilde{g}} \approx 11.6 \text{ pb} \) @ \(\sqrt{s} = 14 \text{ TeV}\).

- Impose basic \(p_T, \text{MET}-\)cuts and require \(4 b\)-tags.

- Use \(\mathcal{L} = 10 \text{ fb}^{-1}\). After cuts we are left with 4800 signal events.

- No SUSY-BG. SM-BG suppressed by \(b\)-tag requirement.

- Even with parton-level pure signal, full mass extraction is challenging!

\(^2\)MadGraph/Madevent & BRIDGE
To measure masses at hadron colliders with invisible massive particles in the final state, we go Edge Hunting!

Distributions of $M_{T2}$-subsystem-variables and $M_{bb}$ show edges which tell us mass combinations.

Big Problem: Combinatorial Error (especially for $M_{T2}$’s).

We are able to successfully measure $M_{bb}$, $M_{T2}^{210}(0)$ and $M_{T2}^{220}(0)$ edges:

<table>
<thead>
<tr>
<th>Mass</th>
<th>68% C.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{b1}$</td>
<td>$(316, 356)$</td>
</tr>
<tr>
<td>$m_{\tilde{g}}$</td>
<td>$(508, 552)$</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}^0_1}$</td>
<td>$(45^*, 115)$</td>
</tr>
</tbody>
</table>

$^3$Barr, Lester, Stephens, 2003; Cho, Choi, Kim, Park 2008; Burns, Kang, Matchev, Park 2009

* LEP bound
(II) Stop Pair Production

- Analyze the process $\tilde{t}_1^* \tilde{t}_1^* \rightarrow t\bar{t} + 2\tilde{\chi}_1^0$.

- $\sigma_{\tilde{t}_1^* \tilde{t}_1^*} \approx 2$ pb @ $\sqrt{s} = 14$ TeV.

- Impose standard cuts & use hadronic tops$^4$.

- Use $\mathcal{L} = 100$ fb$^{-1}$. After cuts: 1481 signal and 105 BG events.

- Easy to extract $M_{T2}^{\text{max}}$ edge $\implies$ Gives $m_{t1}$ ($m_{\tilde{\chi}_1^0}$)

- Combine with (I) $\Rightarrow$

<table>
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<th>th.</th>
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<td>$m_{t1}$</td>
<td>371</td>
<td>(356, 414)</td>
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</tbody>
</table>

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$^4$ Meade, Reece 2006
Putting all these measurements together, we get

<table>
<thead>
<tr>
<th></th>
<th>th.</th>
<th>meas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma'$</td>
<td>0.350</td>
<td>0.525$^{+0.20}_{-0.15}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.423</td>
<td>—</td>
</tr>
</tbody>
</table>

The measurements of the $\tilde{b}_1, \tilde{t}_1, \tilde{g}, \tilde{\chi}^0_1$ masses also allow us to make the SUSY-prediction for $\gamma$ more precise:
Summary & Conclusions

- Confirmation of SUSY-Yukawa Sum Rule

\[ \hat{m}_t^2 - \hat{m}_b^2 = m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 - m_{b1}^2 c_b^2 - m_{b2}^2 s_b^2 - \hat{m}_Z^2 \cos^2 \theta_w \cos 2\beta \]

would be strong support for TeV-scale SUSY as the solution for hierarchy problem.

- Full measurement will have to wait for Lepton Collider.

- Can make significant progress at LHC in some regions of parameter space.

- We developed new techniques for reducing \( M_{T2} \)-combinatorial background, allowing us to measure \( \tilde{t}_1, \tilde{b}_1, \tilde{g}, \tilde{\chi}_1^0 \) masses at our benchmark point.
Gluino Pair Production: Kinematic Edge

\[ M_{bb}^{\text{max}} = \sqrt{\frac{(m_\tilde{g}^2 - m_{\tilde{b}_1}^2)(m_{\tilde{b}_1}^2 - m_{\tilde{\chi}_1}^2)}{m_{\tilde{b}_1}^2}} \]

With known decay chain assignments get \((M_{b_1 b_2}, M_{b_3 b_4})\) for each event, plot \(M_{bb}\)-distribution ⇒ edge at 382 GeV.

Main problem:
Combinatorial Background!

Can reduce CB with \(\Delta R\) cuts and dropping largest \(M_{bb}\)'s per event.

Discard pair with largest \(\text{Max}[M_{12}, M_{34}]\) and require \(\text{Max}[\Delta R_{12}, \Delta R_{34}] < 2.5\)

\[ M_{bb}^{\text{meas}} = 395 \pm 15 \text{ GeV} \]
The distributions of $M_{T2}$ subsystem variables also have edges we can measure. Look at $M_{T2}^{210}(0)$.

Combinatorial Background is more dangerous.

- To calculate $M_{T2}^{210}$, have to divide $4b$ into an upstream and downstream pair: 6 possibilities.
- The $M_{T2}$-distribution for wrong pairings is more featured than $M_{bb}$.

One way to reduce CB:
Drop largest 2 $M_{T2}^{210}$’s per event →

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5 Barr, Lester, Stephens, 2003; Cho, Choi, Kim, Park 2008; Burns, Kang, Matchev, Park 2009
Gluino Pair Production: $M_{T2}$-subsystem Edges

Another way to reduce CB:

Use Kinematic Edge Measurement!

Possible $M_{bb}$ pairs: $(M_{12}, M_{34}), (M_{13}, M_{24}), (M_{14}, M_{23})$

For ~30% of events, situation like:

Can deduce correct decay chain assignment!

For edge measurement, require two methods to agree!

<table>
<thead>
<tr>
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<th>th.</th>
<th>measurement</th>
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<tr>
<td>$M_{bb}$</td>
<td>382</td>
<td>$395 \pm 15$</td>
</tr>
<tr>
<td>$M_{T2}^{210}(0)$</td>
<td>321</td>
<td>$314 \pm 13 \text{ GeV}$</td>
</tr>
<tr>
<td>$M_{T2}^{220}(0)$</td>
<td>507</td>
<td>$492 \pm 14 \text{ GeV}$</td>
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$\Rightarrow$

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(Imposed $m_{\tilde{\chi}_1^0} > 45 \text{ GeV}$ bound from LEP measurement of invisible Z decay width.)