## The Limits of Knowledge: Philosophical and practical aspects of building a quantum computer

Simons Center November 19, 2010 Michael H. Freedman, Station Q, Microsoft Research • To explain quantum computing, I will offer some parallels between philosophical concepts, specifically from Catholicism on the one hand, and fundamental issues in math and physics on the other.

### Why in the world would I do that?

• I gave the first draft of this talk at the Physics Department of Notre Dame.

• There was a strange disconnect.

#### The audience was completely secular.

 They couldn't figure out why some guy named Freedman was talking to them about Catholicism.

• The comedic potential was palpable.

#### I Want To Try Again

#### With a rethought talk and broader audience

- Mathematics and Physics have been in their modern rebirth for 600 years, and in a sense both sprang from the Church (e.g. Roger Bacon)
- So let's compare these two long term enterprises:
  - Methods
  - Scope of Ideas

#### Common Points: Catholicism and Math/Physics

- Care about difficult ideas
- Agonize over systems and foundations
- Think on long time scales
- Safeguard, revisit, recycle fruitful ideas and methods

#### Some of my favorites





Aquinas

Dante



Lully







Tolkien





- Lully may have been the first person to try to build a computer.
- He sought an automated way to distinguish truth from falsehood, ٠ doctrine from heresy

- Philosophy and religion deal with large questions.
- In Math/Physics we also have great overarching questions which might be considered the social equals of: Omniscience, Free Will, Original Sin, and Redemption.

- $P/NP \leftrightarrow Omniscience$  -
- Quantum Mechanics ↔ Redemption
- Universality  $\leftrightarrow$  Original Sin
- Unicity ↔ Free Will



### (1) P/NP (Omniscience)

- The limits of knowledge
- The limits of computation
- The scaling of effort required to:
  - solve problems (factor numbers)
  - discover proofs



Kurt Gödel

### Quantum Mechanics (Redemption)

- Does it provide a complete framework for all physics? Does it redeem our understanding and give us a theanthropic perspective of the world?
- What about gravity?
- What about the classical world?
  - the measurement problem
  - where do unrealized probabilities go?



Schrodinger



#### Universality (Original Sin)





Ken Wilson

- Physical systems (when viewed from a distance) can be grouped into a small number of classes with identical scaling laws.
- "Curse of Universality." By looking at the emergent structure, one may never know what microscopics caused it.

## Unicity (Free Will)



"*The best of all possible worlds.*" – Gottfried Leibniz



Descartes

- Did our universe have to be **roughly** as it is?
  - 3+1 large dimensions
  - stable matter
  - weakly chaotic dynamics
- Did our universe have to be exactly as it is? Is it preordained?

#### The topic today is:



#### And also:



• Along the way we'll encounter our dancers:



Unicity



Gödel





Ken Wilson



Scott Aaronson

- What we can hope to **compute** is limited by the **scaling** of effort inherent in each type of problem.
- Obviously it is a lot more work to factor a large number than a small one.
- But exactly how fast does the work load grow?



The class Q depends on a different way, a quantum mechanical way, of storing numbers.





Quantum computation is a new paradigm in which computational work obeys different scaling laws than those that are known to hold in present day "classical" computers.

#### Modern Church-Turing (MCT) Thesis:

- There are only two physically realistic models of computation:
  - One based on Classical Physics
  - One based on Quantum Physics



Alonzo Church



Alan Turing

### Corollary of MCT

• All we will ever know (or at least compute) will lie in Q.

# Unicity (Free Will): Could the universe have been different?

• Is there a world where NP-complete problems can be solved efficiently?



 Many (Aaronson) think not – that just like perpetual motion, such worlds cannot be consistent.



# So perhaps there is only one possible theory governing our universe:

#### **Quantum Mechanics**



Q: What new power is conveyed by computing quantum mechanically?

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A: Superposition

 Superposition means that a general state ψ may be written as a linear combination of eigen-states ψ<sub>i</sub>, which typically are classical configurations

$$\psi = \sum \alpha_i \psi_i$$

 The coefficients α<sub>i</sub>, called amplitudes, are "square roots" of probabilities:

$$\sum |\alpha_i/2| = 1$$

- Square roots of probabilities are not intuitive.
- Nothing in our large scale clumsy world, nothing in our evolutionary experience, prepares our mind for superposition.
- Superposition was born amid mystery and seeming paradox in the period 1900-1927.





Plank

Born







Bohr

Heisenberg

Schrodinger

Radiation, Diffraction, Scattering, Atomic Spectra

• The double slit experiment shows amplitudes at work.





## How do amplitudes, opposed to probabilities, enhance computational power?

In a cleverly designed algorithm



Peter Shor

factoring

useless computational paths can often be arranged to cancel out – like the dark spots ("nodes") in the double slit experiment – and not consume computational resources.

This is possible because amplitudes, unlike probabilities, can be negative (or even imaginary).

With quantum effects, "factoring" goes from exponential to polynomial time.

#### What did Shor do?

- Classical part: Order finding: factoring
  - Suppose  $f(x) = a^x \mod N$  has period r (even).
  - Then (usually):

$$(a^{r/2} + 1)(a^{r/2} - 1) = a^r - 1 = kN$$

will "separate" factors of *N*.

- Quantum Part: Study  $f(x) = a^x \mod N$ 
  - In Fourier space:  $\hat{f}(k) = M_{k,x}f(x), M_{k,x} = e^{2\pi kx i/M},$ *M* is approximately  $N^2$
  - Observing f returns *information on the periodicity of* f
  - The nodes or "dark spots" are the places where f(k) is small.

## What might a quantum computer do in the real world?

• (1) Wreak havoc: Break all classical codes



Panic on Wall Street

- (2) Allow physicists to explore exotic states of matter
  - Strongly correlated electron systems
    - High  $T_c$  superconductors
    - 2-dimensional electron gasses (2-DEGS)
    - Exotic magnets
- ? Compute string theories ? (good research problem!)

- (3) Sample from the solution space of exponentially large linear systems.
  - Many engineering applications:
    - Electrical engineering and communication
    - Optimization

• Fluid flow? 
$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = -\nabla p + \nabla \cdot \mathbb{T} + \mathbf{f},$$



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 (4) Allow chemists / pharmacologists to design drugs?


• (5) Artificial intelligence?





- In 1950, Alan Turing predicted :
  - Computing power would grow fast (it grew faster)
  - By 2000 we would have a hard time saying that machines were not thinking. (did not happen)
  - Quantum computing may give AI a second chance

#### Topology

- There is a topological approach to quantum computation that avoids local degrees of freedom: nuclear spin, electron spin, photon polarization, etc.
- We need a two dimensional electron gas (2DEG) – with a special property
- Majorana fermions localized in "vortices"



#### There are two prime candidates for Majoranas:

- Fractional Quantum Hall Effect (FQHE) at v = 5/2
- $p_x + ip_y$  superconductors

- We can execute operations ("gates") on system state by braiding Majoranas.
- Or "quasi particle interferometry."



### The Search for Majorana fermions



#### Observation would reveal something very profound about nature. We may be almost there.

See Wilczek, Nature Physics **5**, 614 (2009) Marcel Franz, Physics **3**, 24 (2010) Ady Stern Nature **464**, 187 (2010)

#### Properties of Majorana fermions

- Existence is topologically protected
- One Majorana = "half" a usual fermion



Majorana  
fermion  
$$E = 0$$

۱

- 2n Majoranas  $\Rightarrow$  2<sup>n</sup> degenerate ground states
- Exhibit non-Abelian braiding statistics

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#### Fractional Quantum Hall Effect

- 2DEG
- large B field (~ 10T)
- low temp (< 1K)
- gapped (incompressible)
- quantized filling fractions



$$v=\frac{n}{m}, \quad R_{xy}=\frac{1}{v}\frac{h}{e^2}, \quad R_{xx}=0$$

#### A topological state of matter: the quantum Hall state

$$\sigma_{xy} = n \frac{e^2}{h}$$

- Topological origin of the quantized Hall conductance:
- Bulk gap (Landau level gap)
- The first Chern number (Laughlin PRB 1981, Thouless, Kohmoto, Nightingale, den Nijs (TKNN), PRL 1982)

$$n = \int \frac{d^2k}{2\pi} f_{xy}(\mathbf{k})$$

• Chiral edge states on the boundary

OH

i = 2 kΩ 10 i = 3i = 4Ε ⊁ Х



- FQHE physics is topological, meaning that distance plays no role (or at least an inferior role).
- Topology is "rubber sheet geometry" and FQHE is "rubber sheet physics."
- Controlled by the Chern-Simons lagranian which does not mention distance!

$$S = \frac{k}{4\pi} \int \operatorname{tr} \left( a \wedge da + \frac{2}{3}a \wedge a \wedge a \right)$$

• It is topologically invariant

Topological-invariance is clearly not a symmetry of the underlying Hamiltonian.

$$H = \sum_{i} \frac{p_i^2}{2m_e} + \sum_{a} \frac{P_a^2}{2M} + \sum_{i>j} \frac{e^2}{|r_i - r_j|} + \sum_{a>b} \frac{Z^2 e^2}{|R_a - R_b|} - \sum_{i,a} \frac{Z e^2}{|r_i - R_b|}$$

#### How can Chern-Simons theory possibly describe the low energy physics of the above Hamiltonian?

The answer goes back to:

1970: Wilson, <u>Renormalization</u>: How does the Langrangian evolve when re-expressed using <u>longer length scales</u>, <u>lower frequencies</u>, <u>colder</u> <u>temperatures</u>?

The terms with the fewest derivatives dominate: This is because in momentum space, differentiation becomes multiplication by k and:

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- Chern-Simons Action:  $A \wedge dA + \frac{2}{3}(A \wedge A \wedge A)$  has one derivative,
- while kinetic energy  $p^2/2m$  is written with two derivatives.  $(p_i = 1/2 \ m \ d/dx_i)$
- Thus, in condensed matter at low enough temperatures, we may expect to see systems in which topological effects dominate and geometric detail becomes irrelevant.
- FQHE is such a system.

#### <u>A new proposal</u>:

Majoranas in a  $p_x + ip_y$  superconductor within a "conventional" semi-conductor device

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$$H = \int d^2 \mathbf{r} \psi^{\dagger} \bigg[ -\frac{\nabla^2}{2m} - \mu - i\alpha (\sigma^x \partial_y - \sigma^y \partial_x) \bigg] \psi$$

(Sau, Lutchyn, Tewari, & Das Sarma 2009)



$$H = \int d^2 \mathbf{r} \psi^{\dagger} \left[ -\frac{\nabla^2}{2m} - \mu - i\alpha(\sigma^x \partial_y - \sigma^y \partial_x) + V_z \sigma^z \right] \psi$$



$$\begin{split} H &= \int d^2 \mathbf{r} \psi^{\dagger} \bigg[ -\frac{\nabla^2}{2m} - \mu - i\alpha (\sigma^x \partial_y - \sigma^y \partial_x) + V_z \sigma^z \bigg] \psi \\ &+ \int d^2 \mathbf{r} (\Delta \psi_{\uparrow} \psi_{\downarrow} + h.c.) \end{split}$$



$$H = \int d^{2}\mathbf{k} \left\{ \left[ \epsilon_{+}(k)\psi_{+}^{\dagger}\psi_{+} + \epsilon_{-}(k)\psi_{-}^{\dagger}\psi_{-} \right] + \left[ \Delta_{s}(k)\psi_{+}(k)\psi_{-}(-k) + \Delta_{p+ip}(k)\psi_{+}(k)\psi_{+}(-k) + \Delta_{p-ip}(k)\psi_{-}(k)\psi_{-}(-k) + h.c. \right] \right\}$$

(Sau, Lutchyn, Tewari, & Das Sarma 2009)



<sup>(</sup>Sau, Lutchyn, Tewari, & Das Sarma 2009)

• In the topological approach, interferometers will play a key role:



## FQH interferometer



# Willett *et al.* 08 for v=5/2



A lot of theory [Bonderson, et al] has been devoted to using interferometers to:

- Measure topological charge
- Manipulate quantum information
- Simulate quasi particle braiding



- Interferometry creates probabilistic combinations of quasi-particle world lines.
- •Let me take you through some cartoons:



• Braiding, and hence calculation, can be simulated by measurement.



### **Measurement Simulated Braiding!**



 Bob Willett of Bell Labs has presented evidence of Majoranas in v=5/2 FQHE systems



**Robert Willett** 

**Bob Willett** 



## Conclusions:

- In building a topological quantum computer, universality is our friend.
  - It allows us to model and study exotic states of matter such as the fractional quantum Hall effect.

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 However, even if quantum computers are enormously successful and can compute LHC energy scale physics from string theory,



the same universality may cloak the true microscopics: Many competing theories may <u>all</u> work.

- Building a quantum computer will test quantum mechanics (QM) in new regimes.
  - We may find QM breaks down with complexity and cannot "redeem" the entire physical world.
  - If QM is fundamentally correct, it is now only a matter of technology to build a quantum computer. The mathematical theory is completely convincing.
  - Is the technology ready?



Charles Babbage 1792-1871



John von Neumann 1903-1957



Gordon E. Moore

- Moving from P to Q
  - The universe speaks to itself in the language of QM.
  - We are on the verge of fluency in this internal language of QM.
  - The quantum computer will give wings to our thoughts.



Athena

- But a doubt arises!
  - Why did biology not exploit this language first?

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 After all, we think biology and also neurobiology only processes information classically.





- Probably the universe is everywhere too warm for quantum mechanical thought.
  - The microwave background is approximately 3.8K.

- But now on Earth within dilution refrigerators it is colder than anywhere else (we know of) in space.
- The deep cold of a dilution refrigerator will be the home of our quantum computers. From this extreme stillness we will lift the ultimate tool out of our tool box.





• May we use it well!

