Homework 6: Due at the beginning of class on Friday April 19th

You must write out clear, logical, and complete answers to the homework questions. You may work together on solving the homework problems, but you must write up the final answer on your own.

The scoring of each homework problem will be out of 3 points: 3 points will be given for a complete, 100% correct solution; 2 points will be given for a largely correct (and still clearly explained) solution, 1 point will be given for a poorly explained, incorrect solution. Excellent solutions by students may be photocopied and used as homework solutions available to everyone.

- *Killing vectors and conserved quantities*
  Show that if a spacetime has a Killing vector field ξ (e.g. $\mathcal{L}_\xi g = 0$, then there is a conserved quantity along geodesics (e.g. $\xi_a v^a = const.$ along a geodesic with tangent v^a).
- Carroll Chapter 5 problem 3
- Carroll Chapter 5, problem 5
- *Light Bending*
  a) Derive a formula for the angular deflection of light ray that passes a spherically symmetric object of mass M with distance of closest approach given by b.
  b) Derive a *Newtonian* formula for the angular deflection of a light ray by assuming that the gravitational force on a photon due to an object of mass M is given by $\mathbf{F}_{Newton} = -GME/r^3\mathbf{r}$ where E is the energy of the photon.
  c) Evaluate your answers from a) and b) for an object of mass $M = M_{sun}$.
- *Gravitational Time Delay* (This problem is problem 6.5 in Wald)
  Suppose a radar signal is emitted from earth located at a Schwarzschild radial coordinate R_E from the sun. The signal passes near the sun with a radius of closest approach given by R_0, and is reflected off of a planet at coordinate R_p (on the opposite side of the sun from Earth) and returns to Earth. Show that the coordinate time elapsed between the time the signal is emitted on Earth and the time it returns to Earth is given by

$$\begin{aligned}\Delta t &= 2\left((R_E^2 - R_0^2)^{1/2} + (R_p^2 - R_0^2)^{1/2}\right) + 2GM\left(2\ln\left(\frac{R_E + (R_E^2 - R_0^2)^{1/2}}{R_0}\right) + \right. \\ &\left. 2\ln\left(\frac{R_p + (R_p^2 - R_0^2)^{1/2}}{R_0}\right) + \left(\frac{R_E - R_0}{R_E + R_0}\right)^{1/2} + \left(\frac{R_p - R_0}{R_p + R_0}\right)^{1/2}\right) \qquad (1)\end{aligned}$$