Homework 7: Due at the beginning of class on Friday May 3rd

You must write out clear, logical, and complete answers to the homework questions. You may work together on solving the homework problems, but you must write up the final answer on your own.

The scoring of each homework problem will be out of 3 points: 3 points will be given for a complete, 100% correct solution; 2 points will be given for a largely correct (and still clearly explained) solution, 1 point will be given for a poorly explained, incorrect solution. Excellent solutions by students may be photocopied and used as homework solutions available to everyone.

- *Reissner-Nordstrom solutions*

  Consider the Reissner-Nordstrom metric from class with just electric charge

  $$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right) dt^2 + \left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{1}$$

  a) Compute the components of the Einstein Tensor and determine the energy momentum tensor. Verify that this is the energy momentum tensor for a radial electric field around charge Q.

  b) Using the procedures we have discussed in class, find coordinates (u, v), analogous to the Kruskal coordinates, such that

  $$ds^2 = -f(r)(dv^2 - du^2) + r^2 d\Omega^2 \tag{2}$$

  where (u, v) are functions of r, t and $f(r)$ is nonsingular for $r > r_-$.

  c) Again following procedures for Kruskal coordinates, construct the conformal diagram for this spacetime (as drawn in class, but explicitly show the coordinate transformations and locations of $t = \pm\infty$, $r = \infty$, etc).

- Caroll Problem 6.2

- *Conformal Diagram for de Sitter space*

  This is a follow-up question to the midterm/homework question from a few weeks ago. On that problem set, you studied de Sitter space in a few coordinate systems. One of these gave a line element for de Sitter of

$$ds^2 = -d\tau^2 + \ell^2 \cosh^2(\tau/\ell)\left(d\chi^2 + \sin^2\chi d\theta^2 + \sin^2\chi \sin^2\theta d\phi^2\right) \tag{3}$$

where $\tau \in (-\infty, \infty)$, $\chi \in [0, \pi)$, $\theta \in [0, \pi)$, $\phi \in [0, 2\pi)$.

(a) Show that the coordinates (T, χ, θ, ϕ) with $\cosh(\tau/\ell) = 1/\cos T$ make the de Sitter metric conformal to the metric $ds^2_{\mathrm{Einstein-static}} = -dT^2 + d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)$. What is the range of T?

(b) Sketch the conformal diagram for de Sitter space, identifying past and future null infinity $\mathcal{I}^-$, $\mathcal{I}^+$ and the north and south poles (defined w.r.t. χ). Does an observer sitting at one of the poles at $\mathcal{I}^+$ have access to the whole spacetime? Is this the same or different from Minkowski space?