QGP, Hydrodynamics and the AdS/CFT correspondence

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Motivation

- QGP temperature in Au-Au collisions at RHIC reach temperatures of about 4 $10^{12}K$, 2.5 10^{5} times hotter than the Sun's core
- This is a many-particle system, so its dynamics are extremely complex
- It is strongly believed that the universe was composed of QGP (among other particles) at its very early stages
- QGP occurs in the non-perturbative region of QCD because the coupling is very strong
- Theoretical tools such as Lattice QCD or String Theory are required to perform predictions

Brief review of the Strong Interactions

- QCD is the theory that describes strong interactions. It is a gauge theory and its symmetry group is $SU(3)$
- Its charges are called *colors*
- The gauge bosons are called *gluons* and there are 8 different kinds
- It has 2 peculiar physical properties:
  1. *Asymptotic freedom*: quarks interactions become very weak at short distances (or high energies)
  2. *Confinement*: The force between quarks does not decrease with the distance; there are no free quarks

Hydrodynamics... why?

- What do fluids have to do with nuclei collisions?
- If the mean free path is large compared to the size of the interaction region, then the produced particles do not respond to the inital geometry
- If the mean free path is small compared to the transverse size of the nucleus, hydrodynamics is an appropiate framework to evaluate the response of the medium to the geometry
- Calculations using ideal hydrodynamics reproduce the flow reasonably well

Experimental results

- Elliptic flow: $v_2 := \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$

  Eccentricity: $\epsilon := \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$
- Experiments show that the elliptic flow seems to be bound
- The ideal gas models overpredict the elliptic flow; no bound
- Therefore, this fluid must be viscous

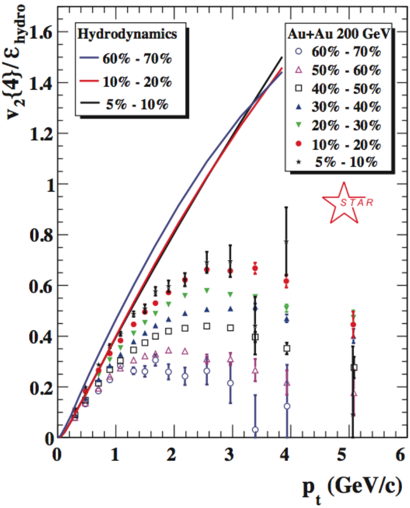

Figure: (see [1], p. 9)

The Shear Viscosity

- The shear viscosity describes the reaction of a fluid to shear stress.
- Suppose we have a plane symmetric fluid moving in the x direction but whose velocity may depend on the y direction. Then the velocity field is $\vec{v} = v_x(y)\,\vec{x}$.
- Then we define the shear viscosity η to be the proportionality coefficient that relates the pressure due to friction forces with the gradient along the y direction:

$$P \equiv \eta \partial_y v_x .$$

- There is a natural generalization of this coefficient to more complex geometrical distributions.

Properties and interesting quantities

- Hydrodynamic interpretation requires mean free paths and relaxation times to be small compared to the nuclear sizes and expansion rates
- In a fluid, it seems difficult to transport energy faster than a quantum time set by the temperature : $\tau_{quant} \sim \frac{\hbar}{k_B T}$
- If we use τ_R to denote the particle relaxation time, one can use thermodynamics, hydrodynamics and assume that diffusion occurs due to the viscosity in order estimate that

$$\frac{\eta}{s} \sim \frac{\hbar}{k_B} \frac{\tau_R}{\tau_{quant}}$$

- Therefore η/s can be understood as the ratio between the medium relaxation time and the quantum time scale

Theoretical predictions

- The shear viscosity to entropy ratio has been calculated using several different approaches. All of them present great uncertainty near the phase transition region, $T_c \simeq 175 MeV$.

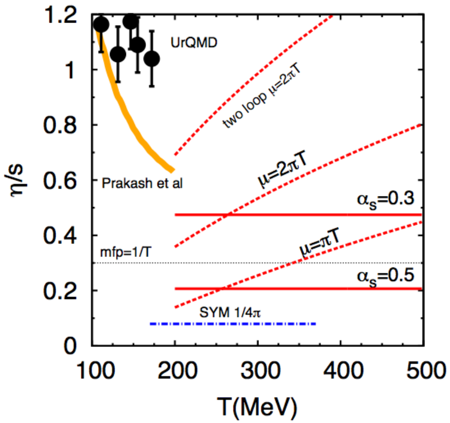

Figure: Calculations for η/s (see [1], p. 16)

- It is very useful to have a strongly coupled theory where η/s can be computed exactly.

The AdS/CFT correspondence

- Anti de Sitter spacetime has constant negative curvature. It satisfies the Einstein's equations with negative cosmological constant. In 5 dimensions the infinitesimal distance looks like

$$ds^2 = \frac{r^2}{R^2}\left(dt^2 - d\vec{x}^2\right) - \frac{R^2}{r^2}dr^2.$$

  For any slice with $r = const$ we are left with a $4D$ Minkowski space.
- Conformal Field Theories are QFTs that satisfy conformal symmetry. Essentially, conformal transformations are coordinate transformations that preserve the angle between vectors.
- It is strongly believed that String Theories constructed in a manifold which is cartesian product of an AdS space and a closed space (such as an sphere) are equivalent to CFTs at the boundary of that AdS space.

SUSY $\mathcal{N} = 4$ model (1)

- Eventually, we will be interested in describing particles in a 4 dimensional spacetime. Therefore our AdS space must be 5 dimensional.
- We will assume supersymmetry (SUSY) is a good symmetry of nature, despite our understanding of strong interactions is based in QCD, which is not a supersymmetric theory.

SUSY $\mathcal{N} = 4$ model (2)

CFT side

- Gauge theory with 1 gauge field, four Weyl fermions and six scalars.
- $\mathcal{N} = 4$ supersymmetries.
- 2 parameters: N_c and g.

AdS side

- Target space is AdS $_5 \times S^5$:

$$ds^2 = \frac{R^2}{z^2}\left(dt^2 - d\vec{x}^2 - dz^2\right) - R^2 d\Omega_5^2$$

where $z = R^2/r$.

- Type IIB ST with a finite # of massless fields and an infinite # of massive fields.
- 3 parameters: R, l_s and g_s.
- When fields' wavelenghts $\gg l_s$, massive modes decouple and one is left with type IIB SUGRA in $N = 10$.

SUSY $\mathcal{N} = 4$ model (3)

- **Connection**: the ST and the CFT parameters map to each other
  1. $g^2 = 4\pi g_s$
  2. $g^2 N_c = R^4 / l_s^4$ ($=: \lambda$, called t'Hooft coupling)
- Those relations imply
  1. ST weakly interacting $\Longrightarrow$ small gauge coupling
  2. Large coupling in CFT $\Longrightarrow R \gg l_s \Longrightarrow$ ST $\approx$ SUGRA
- Furthermore, if $g_s \ll 1$ and $R \gg l_s \Longrightarrow$ ST $\rightarrow$ classical SUGRA

SUSY $\mathcal{N} = 4$ model (4)

WE CAN PERFORM CALCULATIONS IN CLASSICAL SUPERGRAVITY TO LEARN ABOUT THE QUANTUM FIELD THEORY!!!

SUSY $\mathcal{N} = 4$ model (5)

- Hydrodynamical calculations using this model predict that, up to first order, $\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$
- The second order in perturbation theory is positive and proportional to $\lambda^{-3/2}$. For small t'Hooft coupling the ratio diverges
- Therefore we can argue that

$$\boxed{\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}}$$

  in all systems that can be obtained from a QFT by turning on temperatures and chemical potentials
- The plasma cannot be a perfect fluid

Summary

- Difficulties arise when trying to perform quantum field theoretical calculations
- Hydrodynamics are the correct framework to describe QGP
- The practical utility of the AdS/CFT correspondence comes from its ability to deal with strong coupling limit in gauge theory
- AdS/CFT sets a bound for the viscosity to entropy ratio, which implies that the plasma cannot be a perfect fluid
- So far, no fluid has been observed to break that bound

References

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SUSY $\mathcal{N} = 4$ model (4bis)

- In AdS/CFT an operator O of a CFT coupled to a source J is put into correspondence with a "bulk field" ϕ in ST. In the SUGRA approximation this mathematical statement

$$Z_{4D}[J] = e^{iS[\phi_{cl}]},$$

where Z is the *partition function* of the field theory and $S[\phi_{cl}]$ is the classical action of a field ϕ_{cl} that satisfies

$$\lim_{z\to 0} \frac{\phi_{cl}(x,z)}{z^{\Delta}} = J(x).$$

Δ depends on the nature of the operator O (spin and dimension)

- In the simplest case, $\Delta = 0$, we can compute the two-point function of O:

$$G(x-y) = -i\langle 0 | T[O(x)O(y)] | 0\rangle = -\left(\frac{\delta^2 S[\phi_{cl}]}{\delta J(x)\,\delta J(y)}\right)_{\phi(z=0)=J}$$