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QGP temperature in Au-Au collisions at RHIC reach temperatures of about $4 \times 10^{12}$ K, 2.5 $\times 10^5$ times hotter than the Sun’s core.

This is a many-particle system, so its dynamics are extremely complex.

It is strongly believed that the universe was composed of QGP (among other particles) at its very early stages.

QGP occurs in the non-perturbative region of QCD because the coupling is very strong.

Theoretical tools such as Lattice QCD or String Theory are required to perform predictions.
QCD is the theory that describes strong interactions. It is a gauge theory and its symmetry group is $SU(3)$

Its charges are called *colors*

The gauge bosons are called *gluons* and there are 8 different kinds

It has 2 peculiar physical properties:

1. *Asymptotic freedom*: quarks interactions become very weak at short distances (or high energies)
2. *Confinement*: The force between quarks does not decrease with the distance; there are no free quarks
Hydrodynamics... why?

- What do fluids have to do with nuclei collisions?
- If the mean free path is large compared to the size of the interaction region, then the produced particles do not respond to the initial geometry.
- If the mean free path is small compared to the transverse size of the nucleus, hydrodynamics is an appropriate framework to evaluate the response of the medium to the geometry.
- Calculations using ideal hydrodynamics reproduce the flow reasonably well.
Experimental results

- Elliptic flow: $v_2 := \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$

- Eccentricity: $\epsilon := \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$

- Experiments show that the elliptic flow seems to be bound.

- The ideal gas models overpredict the elliptic flow; no bound.

- Therefore, this fluid must be viscous.
The shear viscosity describes the reaction of a fluid to shear stress.

Suppose we have a plane symmetric fluid moving in the $x$ direction but whose velocity may depend on the $y$ direction. Then the velocity field is $\vec{v} = v_x(y) \vec{x}$.

Then we define the shear viscosity $\eta$ to be the proportionality coefficient that relates the pressure due to friction forces with the gradient along the $y$ direction:

$$P \equiv \eta \partial_y v_x.$$

There is a natural generalization of this coefficient to more complex geometrical distributions.
Hydrodynamic interpretation requires mean free paths and relaxation times to be small compared to the nuclear sizes and expansion rates.

In a fluid, it seems difficult to transport energy faster than a quantum time set by the temperature: \( \tau_{\text{quant}} \sim \frac{\hbar}{k_B T} \).

If we use \( \tau_R \) to denote the particle relaxation time, one can use thermodynamics, hydrodynamics and assume that diffusion occurs due to the viscosity in order estimate that

\[
\frac{\eta}{s} \sim \frac{\hbar}{k_B} \frac{\tau_R}{\tau_{\text{quant}}}
\]

Therefore \( \eta/s \) can be understood as the ratio between the medium relaxation time and the quantum time scale.
The shear viscosity to entropy ratio has been calculated using several different approaches. All of them present great uncertainty near the phase transition region, $T_c \approx 175\,\text{MeV}$.

It is very useful to have a strongly coupled theory where $\eta/s$ can be computed exactly.
The AdS/CFT correspondence

- Anti de Sitter spacetime has constant negative curvature. It satisfies the Einstein’s equations with negative cosmological constant. In 5 dimensions the infinitesimal distance looks like

\[ ds^2 = \frac{r^2}{R^2} (dt^2 - d\vec{x}^2) - \frac{R^2}{r^2} dr^2. \]

For any slice with \( r = \text{const} \) we are left with a 4D Minkowski space.

- Conformal Field Theories are QFTs that satisfy conformal symmetry. Essentially, conformal transformations are coordinate transformations that preserve the angle between vectors.

- It is strongly believed that String Theories constructed in a manifold which is cartesian product of an AdS space and a closed space (such as an sphere) are equivalent to CFTs at the boundary of that AdS space.
Eventually, we will be interested in describing particles in a 4 dimensional spacetime. Therefore our AdS space must be 5 dimensional.

We will assume supersymmetry (SUSY) is a good symmetry of nature, despite our understanding of strong interactions is based in QCD, which is not a supersymmetric theory.
CFT side
- Gauge theory with 1 gauge field, four Weyl fermions and six scalars.
- $\mathcal{N} = 4$ supersymmetries.
- 2 parameters: $N_c$ and $g$.

AdS side
- Target space is $\text{AdS}_5 \times S^5$:
  \[ ds^2 = \frac{R^2}{z^2} (dt^2 - d\vec{x}^2 - dz^2) - R^2 d\Omega_5^2 \]
  where $z = R^2 / r$.
- Type IIB ST with a finite # of massless fields and an infinite # of massive fields.
- 3 parameters: $R$, $l_s$ and $g_s$.
- When fields’ wavelengths $\gg l_s$, massive modes decouple and one is left with type IIB SUGRA in $N = 10$. 
SUSY $\mathcal{N} = 4$ model (3)

- **Connection**: the ST and the CFT parameters map to each other
  1. $g^2 = 4\pi g_s$
  2. $g^2 N_c = R^4 / l_s^4$ (=: $\lambda$, called t’Hooft coupling)

- Those relations imply
  1. ST weakly interacting $\implies$ small gauge coupling
  2. Large coupling in CFT $\implies$ $R \gg l_s \implies$ ST $\approx$ SUGRA

- Furthermore, if $g_s \ll 1$ and $R \gg l_s \implies$ ST $\to$ classical SUGRA
WE CAN PERFORM CALCULATIONS IN CLASSICAL SUPERGRAVITY TO LEARN ABOUT THE QUANTUM FIELD THEORY!!!
Hydrodynamical calculations using this model predict that, up to first order, \( \frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \)

The second order in perturbation theory is positive and proportional to \( \lambda^{-3/2} \). For small t’Hooft coupling the ratio diverges.

Therefore we can argue that

\[
\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}
\]

in all systems that can be obtained from a QFT by turning on temperatures and chemical potentials.

The plasma cannot be a perfect fluid.
Difficulties arise when trying to perform quantum field theoretical calculations.

Hydrodynamics are the correct framework to describe QGP.

The practical utility of the AdS/CFT correspondence comes from its ability to deal with strong coupling limit in gauge theory.

AdS/CFT sets a bound for the viscosity to entropy ratio, which implies that the plasma cannot be a perfect fluid.

So far, no fluid has been observed to break that bound.
References


Srednicki, *Quantum Field Theory*, Cambridge University Press
SUSY $\mathcal{N} = 4$ model (4bis)

- In AdS/CFT an operator $O$ of a CFT coupled to a source $J$ is put into correspondence with a "bulk field" $\phi$ in ST. In the SUGRA approximation this mathematical statement

$$Z_{4D}[J] = e^{iS[\phi_{cl}]} ,$$

where $Z$ is the partition function of the field theory and $S[\phi_{cl}]$ is the classical action of a field $\phi_{cl}$ that satisfies

$$\lim_{z \to 0} \frac{\phi_{cl}(x, z)}{z^\Delta} = J(x) .$$

$\Delta$ depends on the nature of the operator $O$ (spin and dimension)

- In the simplest case, $\Delta = 0$, we can compute the two-point function of $O$:

$$G(x - y) = -i \langle 0 | T [O(x) O(y)] | 0 \rangle = - \left( \frac{\delta^2 S[\phi_{cl}]}{\delta J(x) \delta J(y)} \right)_{\phi(z=0) = J} \phi(z=0) = J$$