Color Superconductivity

Raúl Santos

Nov. 8 2010

Outline

Introduction

QCD is asymptotically free at large energies (T, μ)

- $T \gg \mu \rightarrow$ Quark Gluon Plasma (QGP), Appropriate regime for pQCD, testeable with current experiments, RHIC and LHC.
- $\mu \gg T \rightarrow$ New phase, Condensation of quarks, forming a Color superconductor (CSC)

Why it is important?

- Natural scenario in Neutron Stars,
  $\rightarrow$ Consequences of CSC in their evolution

Phase diagram in QCD

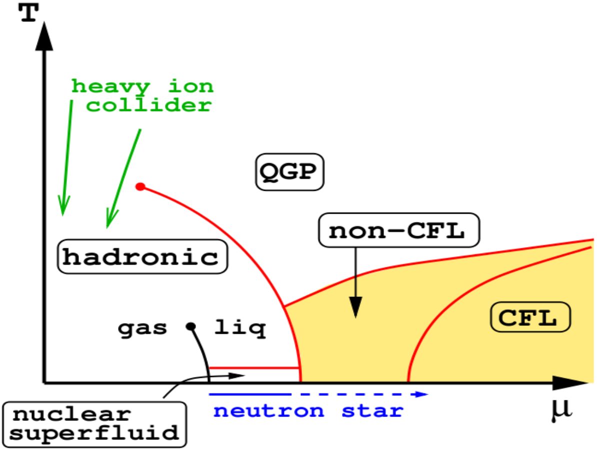
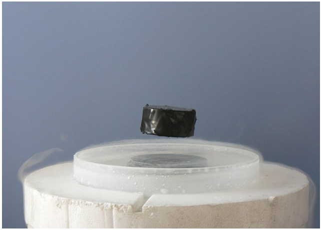


Figure: Different phases of nuclear matter (Schematic) [Alford]

Phenomenology of (usual) Superconductivity

- A superconductor can behave as if it had no measurable DC electrical resistivity $\gtrsim 2$ years.
- A superconductor can behave as a perfect diamagnet $\longrightarrow$ Meissner Effect.
- A superconductor usually behaves as if there were a gap in energy of width 2Δ centered about the Fermi energy, in the set of allowed one-electron levels. The energy gap Δ increases in size as the temperature drops.



BCS superconductivity revisited

Coupling between phonons and electrons can create an overall attractive interaction between fermions → Cooper pairs.

$$H = \sum_{\sigma k} \epsilon_k n_{k\sigma} - \frac{g}{L^d} \sum_{kk'} c^\dagger_{k+q\uparrow} c^\dagger_{-k\downarrow} c_{k'+q\downarrow} c_{-k'\uparrow} \tag{1}$$

Defining the order parameter $\Delta = \frac{g}{L^d} \sum_k \langle \Omega | c_{k\uparrow} c_{-k\downarrow} | \Omega \rangle$, and after some manipulations we can diagonalize the Hamiltonian, obtaining

$$H = \sum_{\sigma k} \lambda_k \left(\alpha^\dagger_{k\uparrow} \alpha_{k\uparrow} - \alpha_{-k\downarrow} \alpha^\dagger_{-k\downarrow} \right) \tag{2}$$

with $\lambda_k = \sqrt{\epsilon_k^2 + \Delta^2}$, and the self-consistency condition for Δ gives

$$\boxed{\Delta = \frac{\omega_D}{\sinh 1/g\nu(E_F)} \approx \omega_D \exp\left(\frac{-1}{g\nu(E_f)} \right)}$$

Inevitability of Color Superconductivity

Why we expect a similar phenomena in QCD at high μ?

- Atractive interaction in the antitriplet channel.

$$T^A_{ab} T^A_{a'b'} = -\frac{N_c+1}{4N_c}(\delta_{ab}\delta_{a'b'} - \delta_{aa'}\delta_{bb'}) + \frac{N_c+1}{4N_c}(\delta_{ab}\delta_{a'b'} + \delta_{aa'}\delta_{bb'}).$$

- Effective interaction mediated by Instanton vaccum.

$$L = -\frac{G}{16N_c(N_c-1)}[(\psi^T C\tau_2\lambda^n_A\psi)(\bar{\psi}\tau_2\lambda^n_A C\bar{\psi}^T) + (\psi^T C\tau_2\lambda^n_A\gamma_5\psi)(\bar{\psi}C\tau_2\lambda^n_A\gamma_5 C\bar{\psi}^T)]$$
$$+\frac{G}{32N_c(N_c+1)}(\psi^T C\tau_2\lambda^n_S\sigma_{\mu\nu}\psi)(\bar{\psi}\tau_2\lambda^n_S\sigma_{\mu\nu}C\bar{\psi}^T)$$

But, who pairs with whom?

Structure of the condensate

When $\mu \gg m_u, m_d, m_s$ we can treat them in equal footing.

$$\langle \psi_{iL}^{a\alpha}(p)\psi_{jL}^{b\beta}(-p)\rangle = -\langle \psi_{iR}^{a\alpha}(p)\psi_{jR}^{b\beta}(-p)\rangle = \Delta(p)\epsilon^{ab}\epsilon^{\alpha\beta\rho}\epsilon_{ij\rho}$$

- Solution of the consistency equation which minimizes the energy.
- *Local* minimum of the ground state energy functional.
- Global minimum? not rigorously proved (yet).
- This pattern involves quarks of all 9 color-flavor combinations, providing a gap for all of them. This makes the energy of such a ground state lower than that of the other possibilities, which leave some quarks ungapped.

Color Flavor Locked Phase

$$\langle \psi_i^{a\alpha}(p)\psi_j^{b\beta}(-p)\rangle \propto \Delta(p)\epsilon^{ab}\epsilon^{\alpha\beta\rho}\epsilon_{ij\rho} = \Delta(p)\epsilon^{ab}(\delta_i^\alpha\delta_j^\beta - \delta_j^\alpha\delta_i^\beta).$$

- Color and Flavor are 'Locked' together.
- Breaks chiral symmetry (up to arbitrarily high densities).
- It is a Color superconductor (Meissner Effect).
- It is a superfluid.

Structure of the symmetry breaking

$$[SU(3)_c] \times SU(3)_R \times SU(3)_L \times U(1)_B \rightarrow SU(3)_{c+L+R} \times \mathbb{Z}_2$$

Gauge symmetry breaking and electromagnetism

$$[SU(3)_c] \times SU(3)_R \times SU(3)_L \times U(1)_B \to SU(3)_{c+L+R} \times \mathbb{Z}_2$$

One of the generators of $SU(3)_{L+R}$ is the electric charge, which generates the $U(1)_Q$ gauge symmetry. This means

$$SU(3)_{L+R+c} \supset [U(1)_{\bar{Q}}]$$

which is unbroken and correspond to a simultaneous electromagnetic and color rotation.
7 gluons and one gluon-photon linear combination become massive via the Meissner effect. The mixing angle is

$$\cos\theta = \frac{g}{\sqrt{g^2 + 4e^2/3}}.$$

Consecuences of Sym. Breaking.

$$\cos\theta = \frac{g}{\sqrt{g^2 + 4e^2/3}}, \quad e \ll g \rightarrow \theta \sim 0.$$

$\bar{Q}$ photon $\sim$ original photon with a small admixture of gluon.

The $\bar{Q}$-electric and magnetic fields satisfy *Maxwell's* equations with a dielectric constant and index of refraction

$$n = 1 + \frac{e^2 \cos\theta^2}{9\pi^2}\left(\frac{\mu}{\Delta_{CFL}}\right)^2.$$

CFL phase is a transparent insulator,
Massive Gluons $\rightarrow$ Color Superconductor.

Intermediate densities

What about μ below m_s?.

2 Color pairing is the most symmetrical form of nuclear matter.

$$\langle \psi_i^\alpha C\gamma^5 \psi_j^\beta \rangle \propto \Delta_{2SC} \epsilon^{\alpha\beta 3}\epsilon_{ij3}$$

$$\begin{array}{ll} [SU(3)_c] \times \underbrace{SU(2)_R \times SU(2)_L \times U(1)_B \times U(1)_S} & \supset [U(1)_Q] \\ \rightarrow [SU(2)_{rg}] \times \underbrace{SU(2)_R \times SU(2)_L \times U(1)_{\tilde{B}} \times U(1)_S} & \supset [U(1)_{\tilde{Q}}]. \end{array}$$

2SC quark matter is therefore a color superconductor but is neither a superfluid nor an electromagnetic superconductor.

Color Superconductivity in Neutron Stars

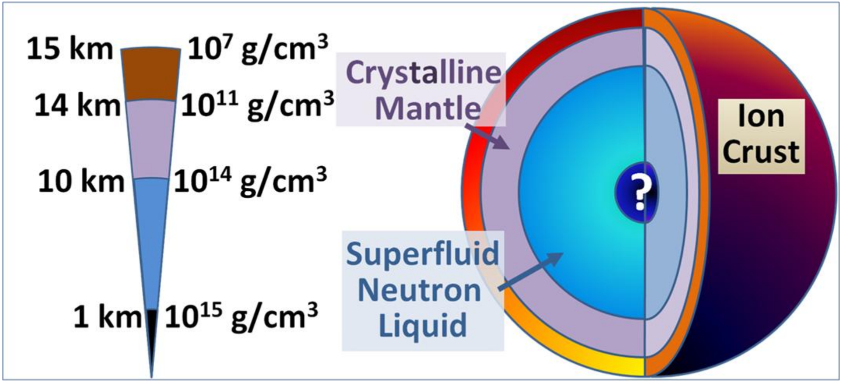
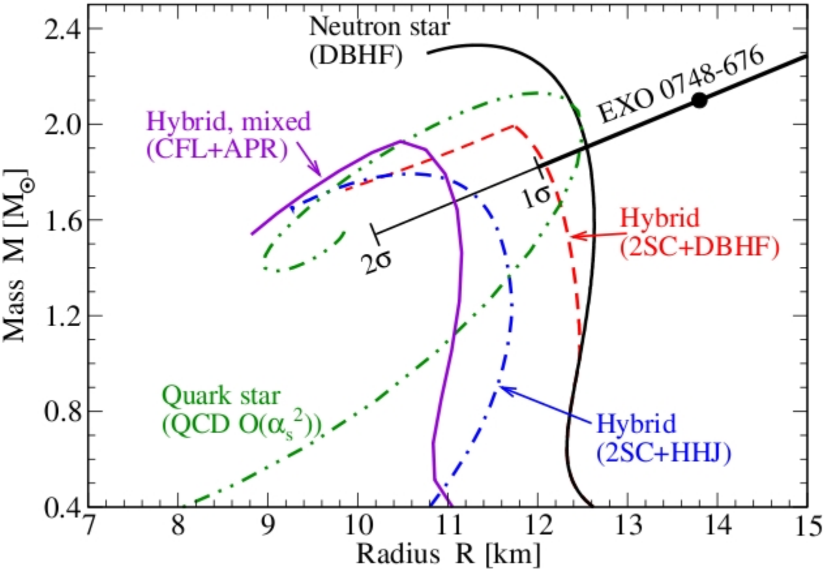


Figure: Neutron Star expected structure

$\rho_0 \sim 2.8 \times 10^{14} g\ cm^{-3}$ density of atomic nucleus.

Mass- Radius Relation



Cooling

Neutrino Emissivity

Ordinary dense matter (proton fraction higher than 0.1).

$$\epsilon_\nu^I \simeq (4 \cdot 10^{25}\, erg\, cm^{-3}\, s^{-1}) \frac{\alpha_s}{0.5} \left(\frac{\mu}{500 MeV}\right)^2 \left(\frac{T}{10^9 K}\right)^6 .$$

Ordinary dense matter $\longrightarrow$ proton fraction lower than 0.1.

$$\epsilon_\nu^{II} \simeq (1.2 \cdot 10^{20}\, erg\, cm^{-3}\, s^{-1}) \times \left(\frac{n}{n_0}\right)^{2/3} \left(\frac{T}{10^9 K}\right)^8$$

CFL phase

$$\epsilon_\nu^{III} \propto \left(\frac{T}{10^9 K}\right)^{15}$$

Summary

- High Density in nuclear matter $\rightarrow$ new phases of matter: Color Superconductivity.
- $\mu \gg m_s$, Color flavor locked phase
- $\mu \sim m_s$, 2 Flavor Color Superconductivity
- Effects in Neutron star evolution.

Open problems

- Nuclear equation of state.
- Explicit determination of transition density.
- Lattice approach, new algorithms.

References

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