Large Extra Dimensions

Vinay Uppal

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Outline of the Talk

- A motivation towards studying the physics of extra dimensions.
- What is the Hierarchy Problem?
- Kaluza-Klein Formalism
- ADD Model
- Experimental Progress

Why Study Extra Dimensions?

- Kaluza-Klein model to unify gravity and electromagnetism.
- String Theory
- Cosmological Constant Problem
- Hierarchy Problem

Hierarchy Problem

- Electroweak Scale $\sim 1\,TeV$.
- Natural cut-off of our theory : Planck Scale$\sim 10^{18}\,GeV$.
- Grand Desert : $10^3\,GeV$ - $10^{18}\,GeV$.
- Quantum gravity effects expected at Planck Scale.
- No experimental test of physics at $10^{16}\,GeV$.

- Assume world has $(4+n)$ dimensions.
- Assume extra dimensions are compact.
- Simplest case : 1 extra dimension curled up in a circle of radius R .

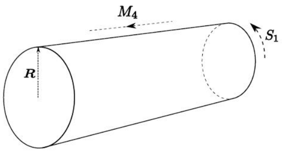
- 

Figure: Spacetime Geometry

- Consider a free massless scalar field

$$\Phi\left(x^{\mu},y\right)=\frac{1}{\sqrt{2\pi R}}\sum\phi^{(n)}\left(x^{\mu}\right)e^{\frac{iny}{R}} \tag{1}$$

- Action :

$$S=\int d^5x\frac{1}{2}\partial_M\Phi\left(x^{\mu},y\right)\partial^M\Phi\left(x^{\mu},y\right) \tag{2}$$

- Simplifies to :

$$S=\int d^4x\left[\frac{1}{2}\partial_\mu\phi^{(0)}\partial^\mu\phi^{(0)}+\sum_{n=1}^{\infty}\left(\partial_\mu\phi^{(n)}\partial^\mu\phi^{(n)}-\frac{n^2}{R^2}\phi^{(n)\dagger}\phi^{(n)}\right)\right] \tag{3}$$

Kaluza-Klein Formalism

- 4D action describes series of massive particles, $m = \frac{n}{R}$
- For a massive 5D scalar field, 4D KK particles have mass

$$m_{(n)}^2 = m_0^2 + \frac{n^2}{R^2} \tag{4}$$

- For higher dimensions compactified on a torus :

$$m_{(n)}^2 = m_0^2 + \frac{n_5^2}{R_6^2} + \cdots \tag{5}$$

- Now let us examine a guage field in KK formalism.

$$A_M(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum A_M^{(n)}(x^\mu)\, e^{\frac{iny}{R}} \tag{6}$$

- 5D Action :

$$S = \int d^5x \left(-\frac{1}{4} F_{MN} F^{MN} \right) \tag{7}$$

- Reduces to

$$\begin{aligned} S = \int d^4x \Big[&\tfrac{1}{2} \partial_\mu A_5^{(0)} \partial^\mu A_5^{(0)} - \tfrac{1}{4} F_{\mu\nu}^{(0)} F^{(0)\mu\nu} \\ &+ \textstyle\sum_{n=1}^{\infty} 2 \left(\tfrac{1}{4} F_{\mu\nu}^{(-n)} F^{(n)\mu\nu} - \tfrac{n^2}{R^2} \phi^{(n)\dagger} \phi^{(n)} \right) \Big] \end{aligned} \tag{8}$$

Kaluza-Klein Formalism

- We see the zero mode has a real scalar and a 4D guage field
- The non-zero modes are massive vector fields. There is no scalar field.
- 5D covariant derivative $D_M = \partial_M + i g_5 A_M$
- Expanding the guage field in it's fourier components, we have the relation between the 5D coupling and the 4D coupling,

$$g_4 = \frac{g_5}{\sqrt{2\pi R}} \tag{9}$$

- Theory to be treated as low energy effective theory with cut-off $\Lambda \sim \frac{1}{g_5^2}$.

ADD Model

- Arkani-Hamed-Dimopoulos-Dvali Model
- Assumption 1 : Geometry of spacetime : $M_4 \times S_n$
- Assumption 2 : All SM fields localized on the 3-brane M_4.
- Assumption 3 : Gravity propagates in all $4 + n$ dimensions.
- We expect gravity to become four-dimensional only for $r \gg R$.

- Consider Newtonian Gravity.
- Poisson's equation in higher dimensions :

$$\nabla^2 \Phi = 4\pi \frac{\rho}{M_{(4+n)}^{n+2}} \tag{10}$$

- We solve for a point mass
- For $r \ll R$ we have

$$\Phi(r) \sim \frac{1}{M_{(4+n)}^{n+2}} \frac{m}{r^{1+n}} \tag{11}$$

- For $r \gg R$ we have

$$\Phi(r) \sim \frac{1}{M_{(4+n)}^{n+2}} \frac{m}{(2\pi R)^n r^2} \tag{12}$$

ADD Model

- We expect it to be equal to the 4-dimensional gravity potential.

$$\frac{1}{M_{(4+n)}^{n+2}} \frac{m}{(2\pi R)^n r^2} \sim \frac{1}{M_4^2} \frac{m}{r^2} \tag{13}$$

$$\implies R \sim \left(\frac{M_4}{M_{4+n}^{\frac{n}{2}+1}} \right)^{2/n} \tag{14}$$

- Now the true fundamental scale is M_{4+n}, but the visible scale that we take to be the fundamental scale in 4D as is $M_4 \sim 10^{19} GeV$.

ADD Model

- Hypothesis : The true fundamental scale is near the Electroweak Scale i.e $M_{4+n} \sim 10\, TeV$.
- For $n = 1,\ R \sim 10^{12} cm$: inconsistent with Newton's laws.
- Single extra dimension is ruled out in ADD model.
- For $n = 2,\ R = 0.1\ mm$.
- Gravity has not been measured below $\sim$ 0.1mm
- Hence the scenario of 2 extra dimensions is allowed.

- We can derive the same result in general relativistic language.
- We have our action

$$S = \frac{M_{4+n}^{2+n}}{2} \int d^{4+n}x \sqrt{g_{4+n}} R_{4+n} \tag{15}$$

- As for scalar and guage field, we expand into KK mode and integrate out the extra dimensions

$$S = \frac{M_{4+n}^{2+n}}{2} (2\pi R)^n \int d^4x \sqrt{g_4} R_4 + \cdots \tag{16}$$

- Which should also equal the 4D gravitational action

$$S = \frac{M_4^2}{2} \int d^4x \sqrt{g_4} R_4 \tag{17}$$

ADD Model

- We have the previously established relation between the fundamental and the 4D Planck scale as

$$\left(\frac{M_4}{M_{4+n}^{\frac{n}{2}+1}} \right)^{2/n} \sim R \tag{18}$$

- No constraint that says there cannot be more than 2 extra dimensions.
- But in order to keep High Energy Physics as an experimentally falsifiable field, 2 extra dimensions bring down the fundamental scale to the level where new physics can be experimentally probed.
- SM fields are constrained to be localized on the 3-brane because experimentally the SM has been tested to much smaller distances than gravity.

Experimental Progress

- Current bound for 2 extra dimensions of the same length : $2\pi R \sim 37\mu m \implies M_6 > 1.4\,TeV$.
- Black holes expected to be produced at the LHC .
- BH will evaporate by Hawking Radiation and decay mostly to the 3-brane.
- Expected photon energy $\sim 100\,GeV$
- LHC expects to probe at $\sim 9\,TeV$

Bibliography

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- Thank You.