Quark-gluon plasma from AdS/CFT Correspondence

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Graduate Seminar

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November 1st, 2010

Outline

1. Motivation
   - Problems from strongly coupled quark-gluon plasma (QGP)
   - The AdS/CFT correspondence conjecture
   - Why QCD ought to have a string dual: The large N_c limit
2. The AdS/CFT Correspondence
   - Matching parameters of D3-branes
   - Inside the "*dictionary*"
3. Connection with QGP
   - Finite temperature AdS/CFT
   - Thermodynamics and hydrodynamics of $\mathcal{N} = 4$ SYM plasma
   - The viscosity/entropy ratio
   - QGP from RHIC
4. Summary

Part I

Motivation

What is QGP?

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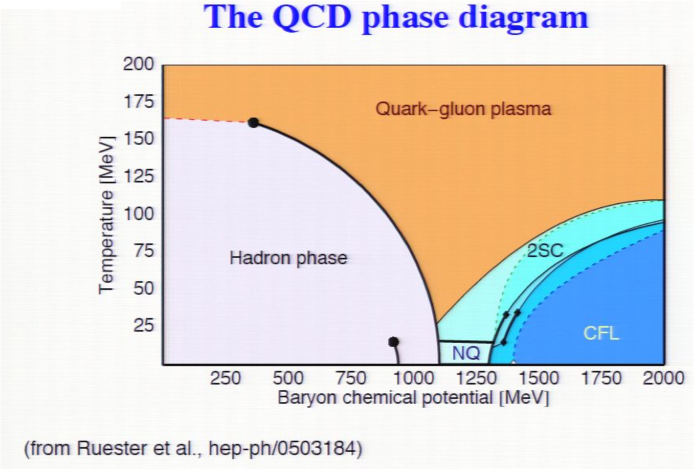

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$$\frac{\eta}{s} \sim \frac{\text{mean free path}}{\text{de Broglie wavelength}} \gg 1$$

  - Strong coupled plasma: collective flow.

$$\frac{\eta}{s} = ?$$

    Fundamental theory (QCD) is well known; perturbation theory does not work. What shall we do?

The conjectures

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> "...quantum gravity should be described entirely by a topological quantum field theory, in which all physical degrees of freedom can be projected onto the boundary."

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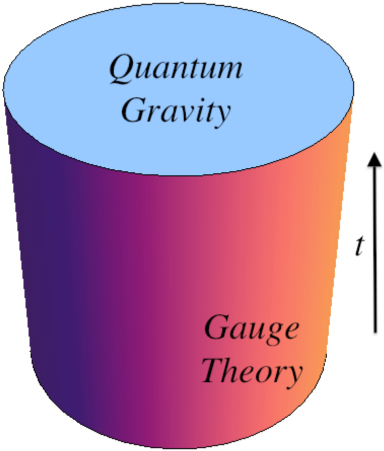

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  - $\mathcal{N} = 4$ SYM is a conformal field theory (CFT) living on the confromal boundary of Minkwoski spacetime.

  - Type IIB string theory here is a theory of quantum gravity living on asymptotically AdS spacetime (fixed background).

Why QCD ought to have a string dual?

- QCD is a gauge theory with gauge group $SU(3)$. QCD theory possesses no expansion parameter at low energy. In 1974, 't Hooft proposed a generalized QCD by replacing the gauge group $SU(3)$ with $SU(N_c)$. The generalized QCD has an expansion parameter $1/N_c$ under the limit $N_c \to \infty$. It is called 't Hooft's Large N limit.

- To keep the confinement scale Λ_{QCD} to be fixed in the large N_c limit, we need to impose a fixed parameter $\lambda = g_{YM}^2 N_c$, which is named as 't Hooft coupling.

- Feynman diagrams can be shown by the "double line notation", i.e. a gluon is associated as a pair of a quark and an antiquark. And the rank N_c is associated to the number internal lines.

Planar diagrams and genus classification

- Examples

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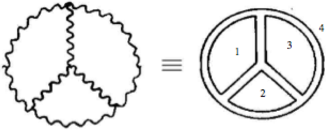

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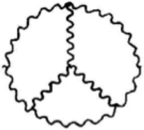

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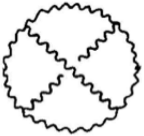

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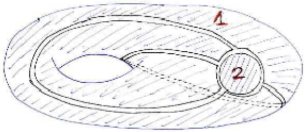


Links to string theory

- The expansion of the gauge theory amplitudes associated with the power of N_c^{χ}, where $\chi = 2 - 2g$. i.e. the amplitudes can be classified by genus number of Riemann surface. To be more specific, the scattering amplitude can be written as:

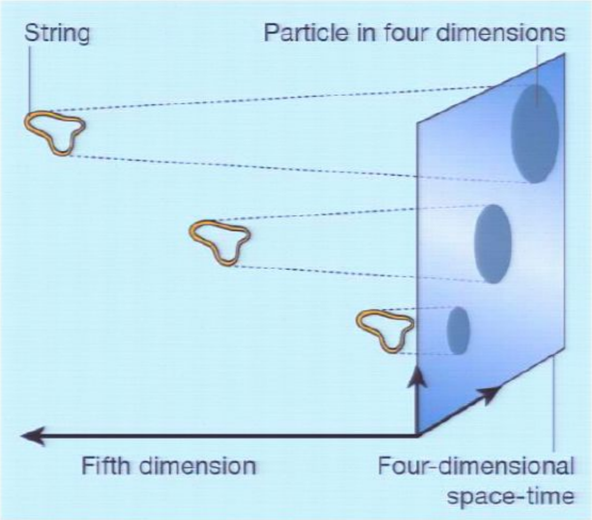
$$A = \sum_{g=0}^{\infty} N_c^{\chi} \sum_{n=0}^{\infty} c_{g,n} \lambda^n$$

- Considering the quark loops (N_f/N_c) with gluon loops (N_c) still gives amplitudes associated with the power of N_c^{χ}. But in this case, $\chi = 2 - 2g - b$. i.e the amplitudes can be classified by genus number and boundaries number.
- The first case is recognized as the loop expansion of closed string theory with coupling constant $g_s \sim 1/N_c$. The second case is recognized as the open string one with coupling constant $g_{open} \sim N_f/N_c$.

Part II

The AdS/CFT Correspondence

AdS/CFT correspondence



(from Maldacena, Nature 2003)

Matching the parameters

Gravitational radius of D3 branes in string length unit:

$$R^4/l_s^4 = 4\pi g_s N_c \sim g_{YM}^2 N_c = \lambda$$

Gauge theory		String theory
λ	$\longleftrightarrow$	R^4/l_s^4
N_c	$\longleftrightarrow$	R^4/l_p^4 ($G = l_p^2 \sim g_s^2 l_s^8$)
Large N_c ($N_c \gg 1$)	$\longleftrightarrow$	Classical gravity ($R/l_p \gg 1$)
$\hookrightarrow \lambda \ll 1$	$\longleftrightarrow$	Classical string theory ($g_s \to 0$)
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$\hookrightarrow \lambda \gg 1$ ☹	$\longleftrightarrow$	String field at $\partial AdS_5 \Rightarrow$
		Semiclassical supergravity ☺

How does gauge/gravity relate?

The gauge symmetries on the string side (type IIB string on $AdS_5 \times S^5$) are as large as the global symmetries on the gauge theory side ($\mathcal{N} = 4$ SYM theory on four dimensional Minkwoski spacetime). The symmetries are matching in this duality.

Metric of AdS_5 in Poincare coordinate

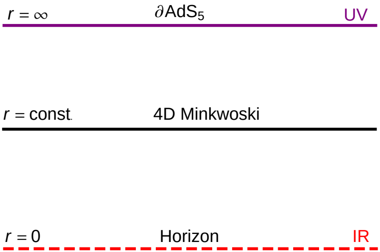
$$ds^2 = \frac{r^2}{R^2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{R^2}{r^2} dr^2$$

where coordinate x^i are on the D-branes and coordinate r and those of S^5 are transverse to them.

N.B.

If $r = const.$, the metric is isometric to four dimensional Minkwoski spacetime. If $r \to \infty$, we are approaching the boundary ∂AdS_5. If $r \to 0$, we are approaching the horizon(which does not coincident with zero after adding temperature).

The dilatation



The geometric picture of AdS_5

$\mathcal{N} = 4$ SYM is a CFT which is invariant under the dilation operator

$$D : x^\mu \to \Lambda x^\mu, \; r \to r/\Lambda$$

where Λ is a constant. It means the short-distance gauge theory (UV limit) is associated to gravity theory near the boundary, whereas the long-distance gauge theory (IR limit) is associated to the gravity near the horizon.

The "*dictionary*" of gauge/gravity duality

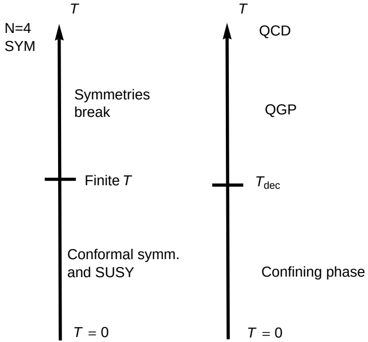
Gauge theory (D dim):	Gravity theory (D+1 dim):
• Operator O	• Field Φ
• Scaling dim of Operator Δ	• Mass of the field M
• Energy-momentum tensor $T_{\mu\nu}$	• Graviton $h_{\mu\nu}$
• RG flow	• Equations of motion at saddle points
• Conserved current	• Gauge field
• Global symmetry	• Gauge symmetry
...	...

Part III

Connection with Quark-gluon plasma

Is QCD equivalent to SYM?

- QCD is a confining theory at the ground states with a scale $\Lambda_{QCD} \simeq 200 MeV$. Heating the temperature around $T_{dec} \simeq 170 MeV$, QCD is believed to have a de-confined phase, i.e. QGP.
- $\mathcal{N} = 4$ SYM is a conformal theory without confining at ground states. Adding a finite temperature will break the SUSY/conformal invariance of the theory, which makes $\mathcal{N} = 4$ SYM plasma *look like* QGP.



Relation between N=4 SYM and QCD

Adding finite temperature

- It turns out that we can introduce finite temperature to AdS/CFT correspondence by modifying the metric to:

Metric of AdS_5 at a finite temperature in Poincare coordinate

$$ds^2 = \frac{r^2}{R^2}(-\mathbf{f}dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{R^2}{r^2\mathbf{f}}dr^2$$

where $f(r) = 1 - r_0^4/r^4$, and r_0 is a temperature associated constant.

- The metric has a coordinate singularity at $r = r_0$. i.e. the black branes' metric has a regular horizon at $r = r_0$ after introducing a finite temperature to its dual gauge theory.
- The Hawking temperature of the horizon is interpreted as the temperature of the dual conformal field theory. By identify the euclidean period under the constraint that avoiding the coordinate singularity of the metric, we can find the Hawking temperature as:

$$T = \frac{r_0}{\pi R^2}$$

Thermodynamics: the entropy density

In the semiclassical limit ($N_c \gg 1, \lambda \gg 1$), the entropy density of $\mathcal{N} = 4SYM$ is identified by the Bekenstein-Hawking entropy:

$$S_{BH} = \frac{A}{4G}$$

where A is the area of the horizon of the black branes

$$A = \int d^3x d^5\Omega \sqrt{\det g} = V \times \pi^3 R^5 \times \frac{r_0^3}{R^3}$$

and G is the Newton's constant in 10-dim (in string units):

$$16\pi G = (2\pi)^7 g_s^2 l_s^8$$

Imposing the expression of temperature and gravitational radius yields

The entropy density

$$s = \frac{S_{BH}}{V} = \frac{\pi^2}{2} N_c^2 T^3$$

The hydrodynamics: the shear viscosity

A hydrodynamics means an effective theory that describe a system with external long distance perturbations around its thermal equilibrium. The shear viscosity η, describing the reaction to applied shear stress, is an important hydrodynamic coefficients calculated by Kubo's formula:

$$\eta = -\lim_{\omega \to 0} \frac{1}{\omega} \Im G^R_{ij,ij}(q = 0)$$

where $q = (\omega, q_i)$ is the four velocity and G^R is the retarded Green's function of energy-momentum tensors.

$$G^R_{\mu\nu,\alpha\beta}(q) = -i \int d^4x e^{-iqx} \theta(t) \langle [T_{\mu\nu}(x), T_{\alpha\beta}(0)] \rangle$$

Again, the hydrodynamics of the gauge theory is identified with the hydrodynamics of black branes' horizon , which yields

The shear viscosity

$$\eta = \frac{\pi}{8} N_c^2 T^3$$

The viscosity/entropy ratio

- For $\mathcal{N} = 4$ SYM plasma
$$\frac{\eta}{s} = \frac{1}{4\pi}$$
Finite ratio at infinite strong coupling!
Very small (H_2O: $380/4\pi$; He: $8.8/4\pi$; N_2:$23/4\pi$)
- A more general argument has shown that $1/4\pi$ is a universal constant for a class of $N_c \gg 1, \lambda \gg 1$, finite temperature gauge theories with or without matter, with or without chemical potential.
- The reason for the universality of η/s is that those theory related universal properties of black branes' horizon. There is a argument that the shear viscosity is proportional to the black hole's horizon under very general conditions. Because the entropy density is also proportional to the black branes' horizon, the ratio keeps the same.
- There is a conjecture that $\eta/s \geq 1/4\pi$ is a universal bound of all relativistic physical systems. (motivations: Kats, Petrov, [arXiv:0712.0743]); Buchel, Myers, Sinha, [arXiv:0812.2521])

QGP from RHIC

- RHIC collides heavy ions like p-p collision at centre of mass energy around 200 GeV per nucleon. There are several evidences indicate that the QGP is strongly coupled at the temperature region $T_{dec} < T < 4T_{dec}$ and well equilibrated at the region $T_{dec} < T < 2T_{dec}$. It presents a robust collective flow behaviour rather than quasiparticle gas.

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- A initial hydrodynamic simulation suggested the ratio of QGP is $\eta/s \simeq (2-4) \times 1/4\pi$. A recent simulation suggested the ratio $\eta/s \simeq (0.4-2) \times 1/4\pi$. It is not clear that the universal bound conjecture $\eta/s \geq 1/4\pi$ is valid for QGP.

Part IV

Summary

Summary

- AdS/CFT correspondence shed lights on long-standing problems of strong coupled system.
  QGP"=" $\mathcal{N} = 4$ SYM plasma"=" Black branes

- AdS/CFT correspondence suggests $1/4\pi$ is a very interesting number of a class of plasma.
  Is $1/4\pi$ a real minimum?
  What's the ratio of real QGP?

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Thank you!

Appendix I: AdS space

- AdS_5 space is the hyperbolid of $\mathbb{R}^{2,4}$, which is described by

$$-x_{-1}^2 - x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^3 = -R^2$$

  where R is the curvature radius of AdS space.

- Re-parameterizing the AdS_5 metric in global coordinate yields

$$ds^2 = R^2 \cosh^2\rho\,(-dt^2 + d\rho^2/\cosh^2\rho + \tanh^2\rho\, d\Omega_3^2)$$

- Setting $\rho \to \infty \Rightarrow$, the metric gives

$$ds^2 = \frac{e^{2\rho}}{4}(-dt^2 + d\Omega_3^2)$$

  which means the boundary geometry of AdS_5 space is $\mathbb{R} \times S^3$, which will further transform into $\mathbb{R}^{1,3}$ after conformal transformation.